Magnetic polarisability of the neutron from lattice QCD

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Overview

- Aims
- Polarisabilities
- Lattice QCD
- Chiral effective field theory
  - Leading-order loop integrals
  - Finite-volume corrections
- Intrinsic scales
- Results
- Conclusion and outlook

Aims

- To estimate the magnetic polarisability of the neutron using effective field theory and lattice QCD.
- To specify the accurate handling of finite-volume corrections to lattice QCD results.
- To perform a robust chiral extrapolation from outside the power-counting regime.
- To examine the role of 'intrinsic scales' in an extended effective field theory.

Polarisabilities

Nucleon Compton scattering

- Nucleon Compton scattering tells us about the internal structure of hadrons.
- In the low photon-energy limit, the forward Compton amplitude, f_1, is described by the static electric and magnetic polarisabilities, α and β, respectively.
- The total photoabsorption cross section may also be described in terms of the forward Compton amplitude:

$$\sigma_T(\omega) = \frac{4\pi}{\omega} \mathrm{Im}\, f_1(\omega^2).$$

Experimental estimates

- Experimentally, the sum $\alpha + \beta$ is known more precisely than the individual polarisabilities.
- For the neutron, $\alpha_n + \beta_n = 15.8 \pm 0.5 \times 10^{-4}$ fm^3.
- Values of the magnetic polarisability of the neutron include:
  - $4.1\,(1.8)\,(0.4)\,(0.8) \times 10^{-4}$ fm^3 (Grießhammer, *et. al.*);
  - $3.7\,(20) \times 10^{-4}$ fm^3 (PDG); and
  - $2.7\,(1.8)^{+1.3}_{-1.6} \times 10^{-4}$ fm^3 (Kossert *et. al.*).
- These experimental values will be compared to the chiral extrapolation from lattice QCD.

Lattice QCD

Lattice QCD

- Lattice QCD is a non-perturbative approach to QCD.
- It is performed at a finite volume, $L = aN$.
- The momenta only take discrete values defined in the box:

$$\vec{k} = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3.$$

- Lattice calculations are typically performed at pion masses larger than the physical value of $m_\pi = 140$ MeV. Therefore, an extrapolation is necessary.

Lattice QCD

- Finite-volume effects can become significant in the chiral regime (small pion mass).
- It is important to handle finite-volume corrections accurately prior to chiral extrapolation.
- The chiral extrapolation must take into account the effects of chiral loops, which can be derived from chiral perturbation theory.

Background field method

- The static nucleon properties: magnetic moment ($\vec{\mu}$) and polarisability (β), of a nucleon can be directly obtained by introducing a background magnetic field B on the lattice.

- This is done by multiplying each gauge link by a certain phase factor, resulting in the energy shift:

$$E(B) = M_N - \vec{\mu} \cdot \vec{B} + \frac{e|B|}{2M_N} - 2\pi\beta B^2 + \mathcal{O}(B^3).$$

Boundary conditions

- The magnetic field B is set up to be along a single spatial axis on the lattice (the z direction):

$$B_z = \partial_x A_y - \partial_y A_x.$$

- It can be made uniform by handling discontinuities at the lattice boundaries with a choice of gauge field:

$$A_y(x, y) = \begin{cases} 0, & \text{for } y/a < N_y - 1 \\ N_y B x, & \text{for } y/a = N_y - 1 \end{cases}$$

Boundary conditions

- The issue of the double boundary, $x/a = N_x - 1$ and $y/a = N_y - 1$ leads to the quantisation condition:

$$qBa^2 = \frac{2\pi n}{N_x N_y}, \quad n \in \mathbb{Z}.$$

- Therefore, the choices of magnetic field strength are limited, based on the lattice size.

Lattice results

- The CSSM lattice results to be analysed are based on PACS-CS configurations obtained through the ILDG.

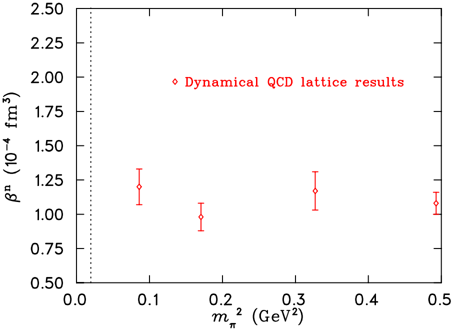


Figure: Magnetic polarisability of the neutron, extracted from lattice QCD simulations at multiple values of pion mass.

Chiral effective field theory

Polarisabilities in effective field theory

- In chiral effective field theory, the Compton amplitude is related to the tensor field with four momentum-dependent parameters:

$$\Theta_{\mu\nu} = e^2[g_{\mu\nu}A(s) + q_\mu q_\nu B(s) + (p_\mu q_\nu + p_\nu q_\mu)C(s) + p_\mu p_\nu D(s)].$$

- p is the initial nucleon momentum, q is the photon momentum, and $s \equiv (p+q)^2$.
- Of the four parameters, only two are independent, and the polarisabilities take the form:

$$\alpha + \beta = -\frac{e^2 m}{2\pi}\frac{\partial^2 A(s)}{\partial s^2}\bigg|_{s=m^2}, \quad \beta = -\frac{e^2}{4\pi m}B(s=m^2).$$

Chiral loop integrals at leading order

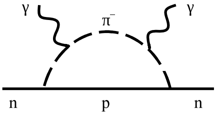
- The chiral loops that contribute to the polarisabilities may be arranged using power-counting.
- In the forward limit, $q \cdot q' \to 0$, we can expand the polarisabilities in quark mass or pion mass ($m_q \propto m_\pi^2$):

$$\beta = \frac{\chi_1}{m_\pi} + c_0 + \chi_2 \log(m_\pi/\mu) + \mathcal{O}(m_\pi).$$

- This is called the chiral expansion. The nonanalytic terms in quark mass are obtained from the chiral loops. c_0 is the leading order low-energy coefficient.
- μ is a mass scale associated with the logarithm, which we can freely set to $\mu = 1$ GeV. The χ coefficients are constant products of known couplings.

Chiral loop integrals at leading order

- The leading-order loop diagram for the neutron takes the following form for heavy baryons:

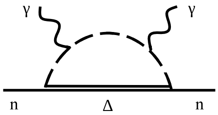


$$\beta_n^{\pi N} = \frac{e^2}{4\pi}\frac{g_A^2}{144\pi^3 f_\pi^2}\int \mathrm{d}^3k\,\frac{\vec{k}^2}{(\vec{k}^2+m_\pi^2)^3} = \frac{\chi_1}{m_\pi}.$$

- At chiral order $\mathcal{O}(\log m_\pi)$ in the expansion, the loop integrals are convergent.
- This provides a testing ground for a variety of difference schemes (massless regularisation, covariant, finite-range regularisation).

Chiral loop integrals at leading order

- One may also include a Δ baryon transition, which takes the following form for the neutron in heavy-baryon theory:



$$\beta_n^{\pi\Delta} = \frac{e^2}{4\pi}\frac{1}{288\pi^3 f_\pi^2}\frac{16}{27}\mathcal{C}^2\int \mathrm{d}^3k\, \frac{\omega_{\vec{k}}^2\Delta(3\omega_{\vec{k}}+\Delta)+k^2(8\omega_{\vec{k}}^2+9\omega_{\vec{k}}\Delta+3\Delta^2)}{16\omega_{\vec{k}}^5(\omega_{\vec{k}}+\Delta)^3}$$

$$= \chi_2 \log(m_\pi/\mu), \qquad \omega_{\vec{k}} = \sqrt{\vec{k}^2+m_\pi^2}, \quad \Delta \equiv M_\Delta - M_N.$$

Finite-volume corrections

- Finite-volume corrections can be estimated by calculating the difference between the loop integral and the finite sum of available lattice momenta.

$$\delta_L[\beta^{\mathrm{loops}}] = \left[\frac{(2\pi)^3}{L_x L_y L_z} \sum_{k_x, k_y, k_z} - \int \mathrm{d}^3 k \right] \mathcal{I}(k, m_\pi^2) \, .$$

- At this chiral order, regularisation of the loop integrals/sums is less important, as the integrals are convergent.
- $\delta_L[\beta^{\mathrm{loops}}]$ saturates to an asymptotic value, which can be obtained for any fixed suitably large U.V. momentum cutoff.

Intrinsic scales

Power-counting regime

- When applying chiral effective field theory to lattice QCD results, one must be aware of the power-counting regime.
- Lattice results invariably extend outside the power-counting regime ($m_\pi \lesssim 200$ MeV). When using those results to perform an extrapolation, the chiral expansion is divergent.
- One should then use a finite-range regulator (FRR), which cuts off the divergence, and resums the higher-order terms of the expansion.

Extended effective field theory

- In extended effective field theory, one models the higher-order terms of the expansion, which become significant outside the power-counting regime.
- Inside the power-counting regime, FRR effective field theory and chiral perturbation theory are identical.
- Outside the power-counting regime, standard chiral perturbation theory should not be used.
- Using FRR, the best choice of regularisation scale is the intrinsic scale. This is important when working outside the power-counting regime.

Intrinsic scales

- The intrinsic scale is the scale at which the low-energy coefficients of the chiral expansion are independent of the pion mass.
- Within the power-counting regime, where all regularisation schemes are equivalent, the range of suitable cutoff scales is very broad.
- Outside the power-counting regime, only a small range of the regularisation scale yields the correct values of the expansion coefficients.
- The intrinsic scale of the nucleon is typically $\Lambda \sim 1$ GeV, if one uses a dipole FRR in the loop integrals/sums:

$$u(k) = \left(1 + \frac{k^2}{\Lambda^2}\right)^{-2}.$$

Intrinsic scales

- As an example: analysis of the leading-order coefficient of the magnetic moment expansion yields a unique crossing-point.

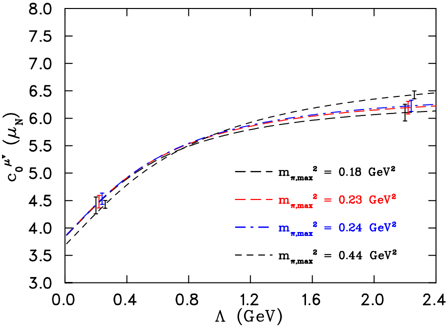


Figure: Nucleon magnetic moment analysis of QCDSF lattice results yields $\Lambda^{\rm scale} = 0.87^{+0.42}_{-0.36}$ GeV.

Intrinsic scales

- By considering a variety of static nucleon observables (mass, magnetic moment, electric charge radius) a weighted average yields an intrinsic scale of:

$$\bar{\Lambda}^{\text{scale}} = 0.99(27)\ \text{GeV}.$$

- Chiral extrapolations from FRR effective field theory using the intrinsic scale will be compared to a variety of typical massless schemes.

Results

Results

- The finite-volume corrections and chiral extrapolation of the magnetic polarisability of the neutron will now be applied to the recent CSSM lattice simulation results, using the fit formula:

$$\beta_n = a_0 + a_2 m_\pi^2 + \beta_n^{\pi N} + \delta_L[\beta_n^{\pi N}] + \beta_n^{\pi \Delta} + \delta_L[\beta_n^{\pi \Delta}].$$

- We add a linear term, $a_2 m_\pi^2$, to the chiral expansion, with fit parameter a_2, to account any residual curvature in the polarisabilities.
- This can account for any variation in obtaining a plateau in the extraction of β from the lattice calculation at each value of m_π^2.

Chiral extrapolation

- An extrapolation at the finite box size of the lattice, $L = 3$ fm, shows fairly weak chiral curvature.

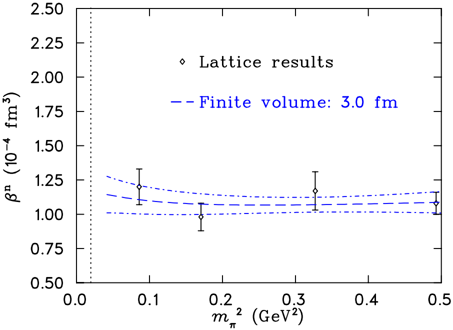


Figure: Extrapolation of β_n at $L = 3.0$ fm, corresponding to the volume of the lightest point in m_π^2. The constraint $m_\pi L > 3$ is used.

Chiral extrapolation

- The expectation of the lattice calculation for current and future box sizes; boxes as large as 7 fm are required to reach within 5% of the infinite-volume result at the physical point.

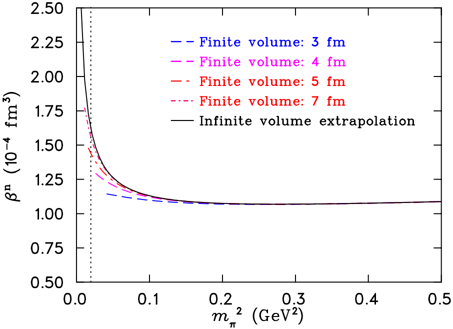


Figure: Extrapolation of β_n for a variety of box sizes, including infinite volume.

Chiral extrapolation

- FRR suppresses the curvature at large m_π.
- The differences among the curves indicate that the lattice results extend outside the power-counting regime.

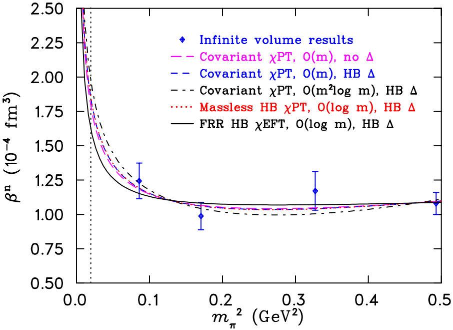


Figure: A comparison of a variety of massless regularization schemes, and a dipole FRR effective field theory with $\bar{\Lambda}^{\text{scale}} = 0.99$ GeV.

Chiral extrapolation

- The final prediction of the magnetic polarisability is $\beta_n = 1.62(11)^{\mathrm{stat}}(13)^{\mathrm{sys}} \times 10^{-4}\ \mathrm{fm}^3$.

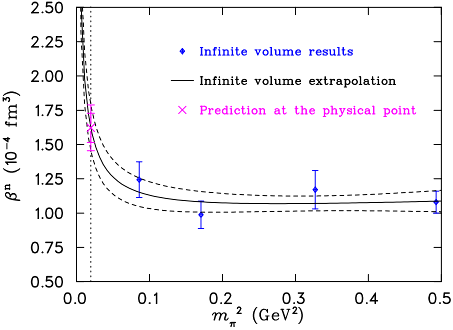


Figure: Prediction of the value of β_n at the physical point, at infinite volume. The inner error bar represents the statistical uncertainty, and the outer error bar adds the systematic uncertainty from $\bar{\Lambda}^{\mathrm{scale}}$ in quadrature.

Chiral extrapolation

- A comparison between the prediction and the experimental results.

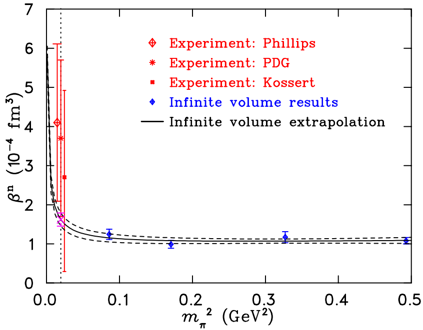


Figure: Prediction of the value of β_n at the physical point. To distinguish the different error bars easily, an offset is introduced among the experimental points along the m_π^2-axis.

Conclusion and outlook

Conclusion

- Lattice QCD has made significant progress in simulating uniform magnetic fields at finite volume to obtain moments and polarisabilities.
- The correct handling of finite-volume corrections is vital for a reliable chiral extrapolation.
- A range of finite box sizes were considered, providing future lattice QCD calculations with a benchmark in estimating the size of finite-volume effects.
- Using lattice results that extend outside the chiral power-counting regime, an extended effective field theory using the intrinsic scale must be used.
- By correcting to infinite volume, a prediction was made for the magnetic polarisability of the neutron at the physical point: $\beta_n = 1.62(11)^{\text{stat}}(13^{\text{sys}}) \times 10^{-4}$ fm^3.

Outlook for the future

- A more precise determination of the polarizabilities from experiment will assist in further testing the theoretical prediction.
- An analysis using partially-quenched QCD contributions is also a useful step for the future.

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