Search for an Intrinsic Scale in Chiral Effective Field Theory

Jonathan Hall, Derek Leinweber, Ross Young

CSSM, University of Adelaide

4 December 2008

Overview

- Introduction: 'The Challenge'
  - Chiral extrapolation of the quenched ρ meson mass
- Quenched Chiral Effective Field Theory
  - Quenched approximation lattice technique- a computational simplification
  - Finite-Range Regularisation
  - Power Counting Regime
- Extraploations
  - Pseudodata analysis
  - Properties of the renormalised expansion coefficients c_i
  - Reveals a test for an intrinsic scale
- Conclusions
  - Created a procedure to determine a preferred range of FRR scale based on nonperturbative lattice results — without phenomenological predjudice.

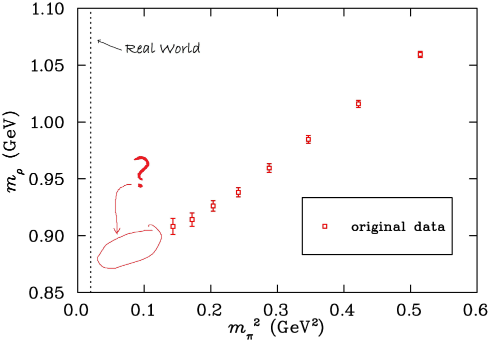
'The Challenge'

Jonathan Hall, Derek Leinweber, Ross Young

- We want to predict the mass of the quenched ρ meson, m_ρ, at physical pion mass.
- We have quenched lattice QCD results from the Kentucky Group, excluding the results for lighter pion masses.
- We cannot rely on the experimental value of m_ρ because we are doing Quenched QCD (QQCD).
- What is the best way to use Chiral Effective Field Theory (χEFT) to make a prediction?

QQCD Data from the Lattice

- The following data from Kentucky Group is missing points close to the chiral limit $(m_q = 0)$.
- Data illustrated lies in the range $380 < m_\pi < 720$ MeV, but the missing data are as low as 200 MeV.
- Using the standard relation: $m_q \sim m_\pi^2$:



Chiral Effective Field Theory

- A perturbative quantum field theory using low-energy degrees of freedom. (mesons, baryons etc)
- General low-energy expansion formula, about chiral limit:

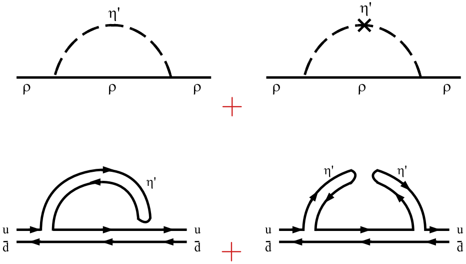
$$\begin{aligned} m_\rho^2 &= \{\text{Terms analytic in } m_q\} + \{\text{Chiral loop corrections}\} \\ &= \{a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \cdots\} + \{\Sigma_{\text{total}}\}\,. \end{aligned}$$

- Analytic Terms :
  - To be determined via analysis of Lattice QCD results.
  - Related to the low energy constants of Chiral Perturbation Theory (χPT).
- Chiral loops :
  - Predict nonanalytic behaviour in the quark mass.
  - Coefficients are known and are model independent.

ρ Meson Mass

- In meson sector of QQCD, we consider the single and the double hairpin diagrams:

$$\Sigma_{\text{total}} = \Sigma_{\eta'} + \Sigma_{\eta'\eta'}$$



- The quenched approximation in lattice QCD means no 'disconnected loops' allowed in diagrams.
- Therefore these flavour singlet η' loops are the only contributors to the quenched m_ρ(to this order).

Jonathan Hall, Derek Leinweber, Ross Young

- Each diagram represents a loop integral that can be evaluated using standard dimensional regularisation:

$$\begin{aligned}\Sigma_{\eta'} &= \chi_3 \frac{2}{\pi} \int_0^\infty \mathrm{d}k \, \frac{k^4}{k^2 + m_\pi^2} \\ &= \infty + \infty m_\pi^2 + \chi_3 m_\pi^3 + 0 \, m_\pi^4 + \cdots \, , \\ \Sigma_{\eta'\eta'} &= \chi_1 \frac{-4}{3\pi} \int_0^\infty \mathrm{d}k \, \frac{k^4}{(k^2 + m_\pi^2)^2} \\ &= \infty + \chi_1 m_\pi + 0 m_\pi^2 + \cdots \, .\end{aligned}$$

ρ Meson Mass

Jonathan Hall, Derek Leinweber, Ross Young

- The a_i coefficients undergo infinite renormalisation:

$$\begin{aligned} m_\rho^2 &= \{a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \cdots\} + \Sigma_{\eta'} + \Sigma_{\eta'\eta'} \\ &= c_0 + \chi_1 m_\pi + c_2 m_\pi^2 + \chi_3 m_\pi^3 + c_4 m_\pi^4 + \cdots . \end{aligned}$$

- We can use Finite-Range Regularisation (FRR) to cutoff momenta at the finite scale Λ.
- Idea is to prevent large momenta from flowing through low energy effective degrees of freedom (Ultraviolet suppression for $k > \Lambda$).
- This can be done in a chiral symmetry-preserving way if desired.

Finite-Range Regularisation

- Loop integrals are modified to include a regulator $u^2(k)$, eg:
  - Dipole form: $u(k) = \left(\frac{\Lambda^2}{\Lambda^2+k^2}\right)^2$
  - Sharp cutoff form: $u(k) = \theta(\Lambda - k)$

$$\Sigma_{\eta'} = \chi_3 \frac{2}{\pi} \int_0^\infty \mathrm{d}k \, \frac{k^4 u^2(k^2)}{k^2 + m_\pi^2}$$

$$\Sigma_{\eta'\eta'} = \chi_1 \frac{-4}{3\pi} \int_0^\infty \mathrm{d}k \, \frac{k^4 u^2(k^2)}{(k^2 + m_\pi^2)^2}$$

Finite-Range Renormalisation

Jonathan Hall, Derek Leinweber, Ross Young

- Integrals are be evaluated for finite Λ.
- Taylor expand the integral about the chiral limit ($m_\pi^2 = 0$):

$$\Sigma_{\eta'}(k;\Lambda) = b_0^{(3)}\Lambda^3 + b_2^{(3)}\Lambda m_\pi^2 + \chi_3 m_\pi^3 + \cdots ,$$

$$\Sigma_{\eta'\eta'}(k;\Lambda) = b_0^{(1)}\Lambda + \chi_1 m_\pi + \frac{b_2^{(1)}}{\Lambda} m_\pi^2 + \cdots$$

- Terms involving b_i coefficients are dependent on scale Λ.
- Note that terms $\chi_3 m_\pi^3$ and $\chi_1 m_\pi$ are independent of Λ.
- The renormalised expansion coefficients c_i are also independent of Λ:

$$\begin{aligned} m_\rho^2 &= (a_0^\Lambda + b_0^\Lambda) + \chi_1 m_\pi + (a_2^\Lambda + b_2^\Lambda) m_\pi^2 + \chi_3 m_\pi^3 + \cdots \\ &= c_0 + \chi_1 m_\pi + c_2 m_\pi^2 + \chi_3 m_\pi^3 + \cdots \end{aligned}$$

Finite-Range Regularisation

Jonathan Hall, Derek Leinweber, Ross Young

- Within the power-counting regime (PCR) of χPT ($m_\pi < 200$ MeV), higher-order terms are negligible by definition.
- Within the finite order of the expansion:
    - Terms are independent of Λ.
    - Results are independent of the regularisation scheme used.
    - FRR is mathematically equivalent to Dimensional Regularisation.
- Beyond the finite order of the expansion:
    - Terms are dependent on Λ.
    - This region lies outside the PCR.

Jonathan Hall, Derek Leinweber, Ross Young

- Recall the low energy expansion:

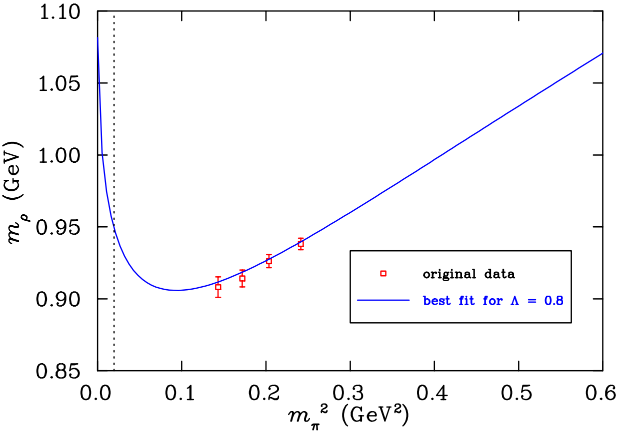
$$m_\rho^2 = \{a_0^\Lambda + a_2^\Lambda\, m_\pi^2 + a_4^\Lambda\, m_\pi^4 + \cdots\} + \Sigma_{\eta'}^\Lambda + \Sigma_{\eta'\eta'}^\Lambda$$

- Use Chiral Effective Field Theory: calculate the self energies to see the correct curvature near chiral regime.
- Example: consider dipole form factor with $\Lambda = 0.8$ GeV .
- Choose the lightest four points to determine the a_i^Λ parameters.

Extrapolation of the ρ Meson Mass

Jonathan Hall, Derek Leinweber, Ross Young

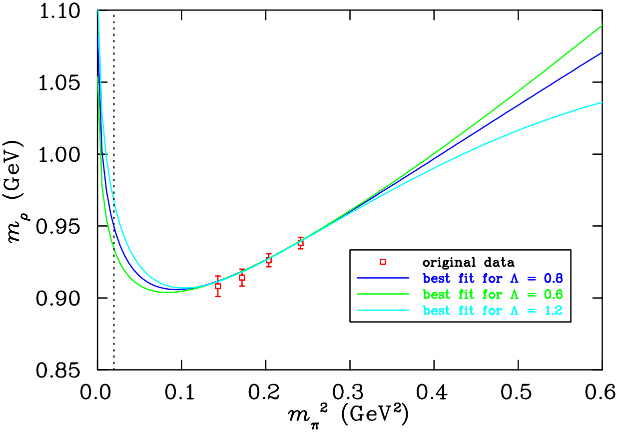
- Infinite volume extrapolation curve for dipole regulator shown.
- Non-trivial low-energy curvature improves result over a naive linear fit (eg: full QCD $m_\rho = 770$ MeV).



Extrapolation of the ρ Meson Mass

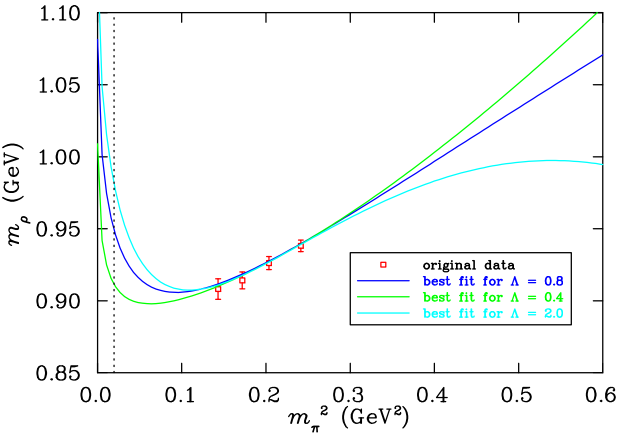
Jonathan Hall, Derek Leinweber, Ross Young

- What if we were to choose $\Lambda = 0.6$ or 1.2 GeV?



Jonathan Hall, Derek Leinweber, Ross Young

- What if we were to choose $\Lambda = 0.4$ or 2.0 GeV?



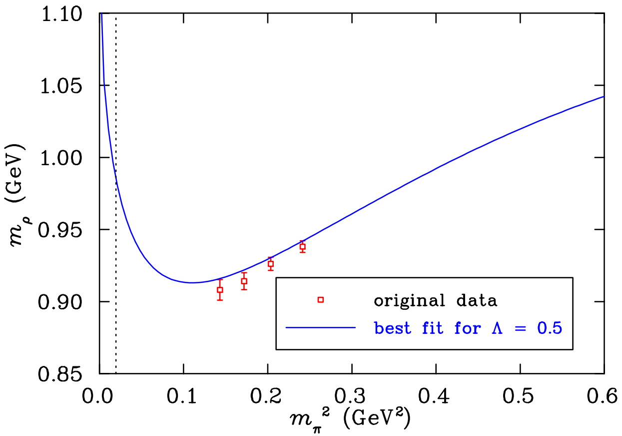
Test for an Intrinsic Scale

- results are dependent on Λ.
- Is there an intrinsic scale for Λ embedded in the data?
  - To explore this idea will will create some 'pseudodata'.
  - We know this artificial data has an intrinsic scale governed by the Λ_c used to create it.
- We want to graph the renormalised coefficients c_0, c_2, c_4 against arbitrary Λ. If the renormalisation is successful they should be constant.

$\theta(\Lambda - k)$ Regulator

Jonathan Hall, Derek Leinweber, Ross Young

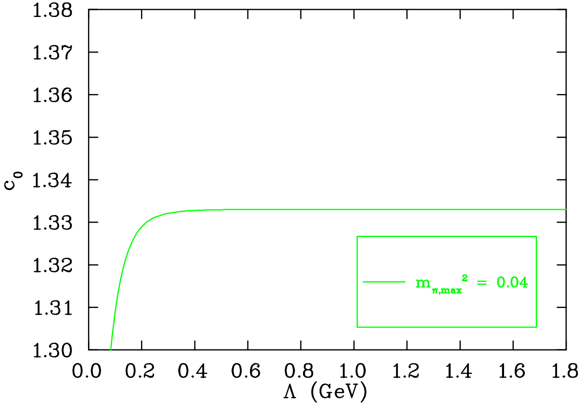
- Pseudodata: Infinite volume extrapolation curve for sharp cutoff regulator at $\Lambda_c = 0.5$ GeV.



$\theta(\Lambda - k)$ Regulator

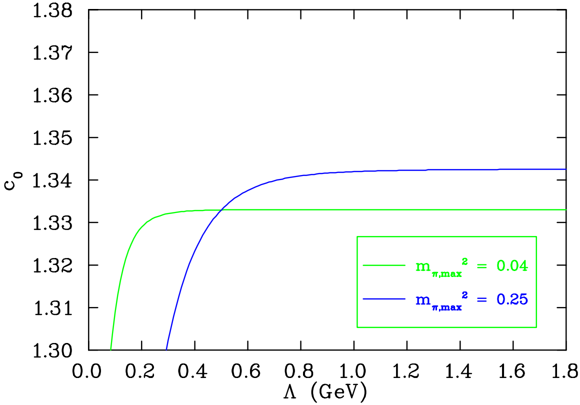
Jonathan Hall, Derek Leinweber, Ross Young

- Let us look at pseudodata created using the sharp cutoff regulator at $\Lambda_c = 0.5$ GeV for $m^2_{\pi,\max} = 0.04$ GeV^2.
- Pseudodata is just inside the PCR.



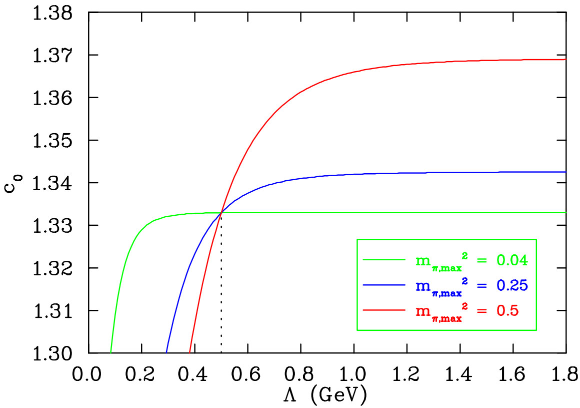
$\theta(\Lambda - k)$ Regulator

- Now add more data points so that $m^2_{\pi,\text{max}} = 0.25$ GeV^2: outside the PCR.



$\theta(\Lambda - k)$ Regulator

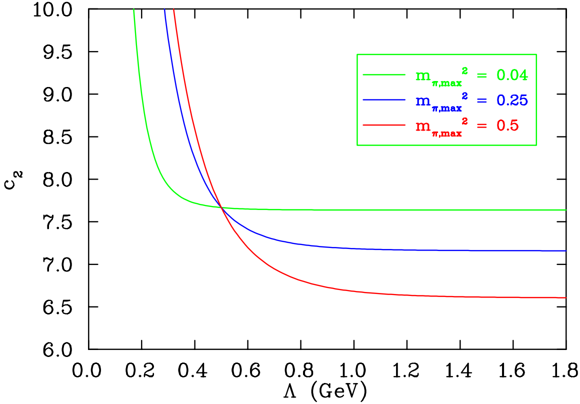
- Now add more data points so that $m^2_{\pi,\max} = 0.5$ GeV^2: further outside the PCR.



$\theta(\Lambda - k)$ Regulator

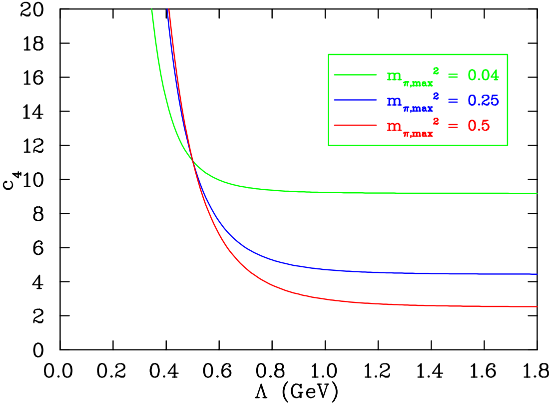
Jonathan Hall, Derek Leinweber, Ross Young

- c_2 can be analyzed in the same way:



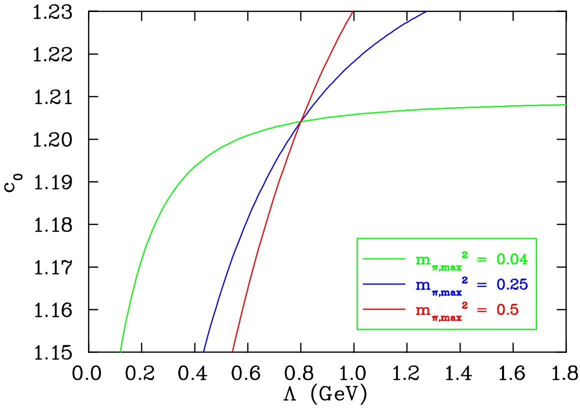
$\theta(\Lambda - k)$ Regulator

- c_4 can be analyzed in the same way:



Dipole Regulator

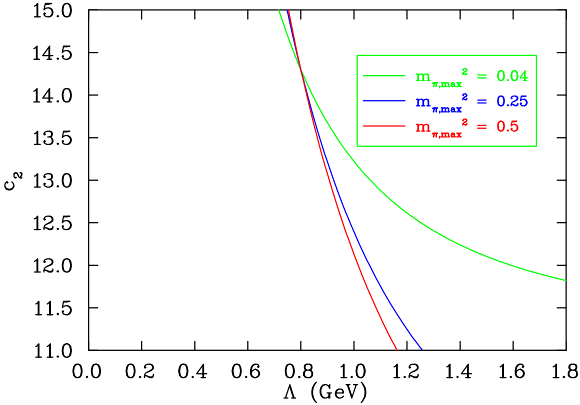
- Now let's check to see if results are regulator independent.
- Consider pseudodata created using the dipole regulator, with $\Lambda_c = 0.8$ GeV. For c_0:



Jonathan Hall, Derek Leinweber, Ross Young

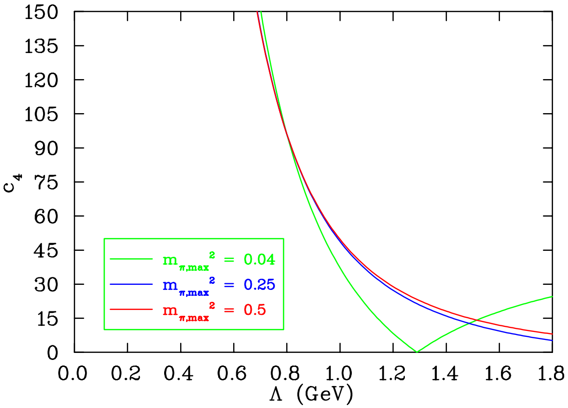
Dipole Regulator

- For c_2: (scale matched to the sharp cutoff analysis)



Dipole Regulator

- c_4 is also problematic



Dipole Regulator

- The dipole regulator does not preserve chiral symmetry.
- There are extra Λ-dependent nonanalytic terms in the chiral expansion that have not been catered for in the fit.

$$\begin{aligned}\Sigma_{\eta'}(k;\Lambda) &= \Lambda^3 b_0^{(3)} + \Lambda b_2^{(3)} m_\pi^2 + \chi_3 m_\pi^3 \\ &+ \frac{b_4^{(3)}}{\Lambda} m_\pi^4 + \frac{b_5^{(3)}}{\Lambda^2} m_\pi^5 + \mathcal{O}(m_\pi^6)\,,\end{aligned}$$

$$\begin{aligned}\Sigma_{\eta'\eta'}(k;\Lambda) &= \Lambda b_0^{(1)} + \chi_1 m_\pi + \frac{b_2^{(1)}}{\Lambda} m_\pi^2 + \frac{b_3^{(1)}}{\Lambda^2} m_\pi^3 \\ &+ \frac{b_4^{(1)}}{\Lambda^3} m_\pi^4 + \frac{b_5^{(1)}}{\Lambda^4} m_\pi^5 + \mathcal{O}(m_\pi^6)\,.\end{aligned}$$

Test for an Intrinsic Scale

Jonathan Hall, Derek Leinweber, Ross Young

- Choice: We could use an a_3 and an a_5 parameter to contain the contribution from these terms, or:
- Better: choose a regulator which respects chiral symmetry to finite order.
- The sharp cutoff $\theta(\Lambda - k)$ preserves chiral symmetry but finite volume is awkward.
- Let us choose something like a dipole, of the form:

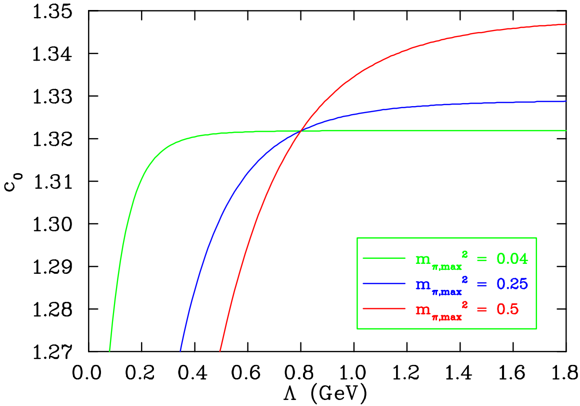
$$u(k) = \left\{ 1 + \left(\frac{k^2}{\Lambda^2} \right)^n \right\}^{-m}$$

- Choosing $n = 1$ & $m = 2$ gives us a dipole form.
- Choosing $n = 3$ is sufficient to suppress the $m_\pi^{3,5}$ terms.
- We will call this the 'triple dipole' regulator.

Jonathan Hall, Derek Leinweber, Ross Young

Triple Dipole Regulator

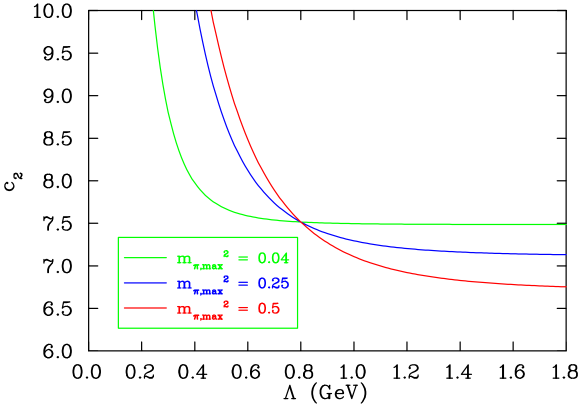
- Now consider pseudodata created using our triple dipole regulator. For c_0:



Jonathan Hall, Derek Leinweber, Ross Young

Triple Dipole Regulator

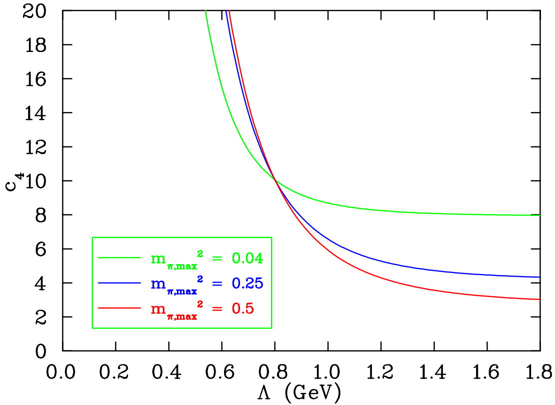
- For c_2:



Jonathan Hall, Derek Leinweber, Ross Young

Triple Dipole Regulator

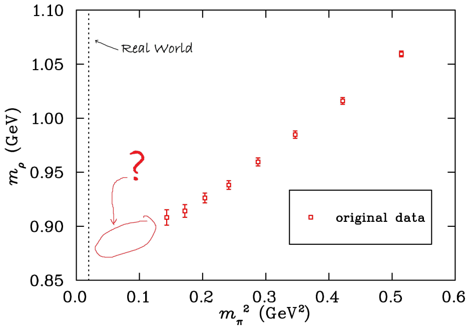
- For c_4:



Test for an Intrinsic Scale

Jonathan Hall, Derek Leinweber, Ross Young

- We are now in a position to analyze the actual data using this triple dipole regulator.



- We will take the lightest four data points we have, then increase $m_{\pi,\max}$ by adding another data point each time.

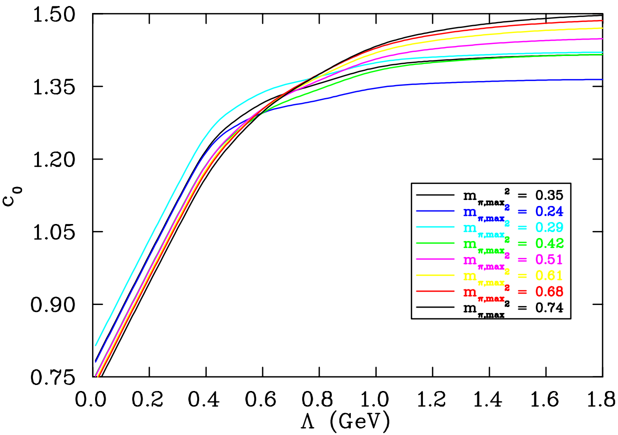
Test for an Intrinsic Scale

Jonathan Hall, Derek Leinweber, Ross Young

- The analysis is done on a finite volume box the same size as the lattice data.
- An intrinsic scale will reveal itself in the intersection points.
- Since these data are not 'perfect' pseudodata but rather have some scatter due to statistical uncertainties, the intersection point will not be unique.
- This will give us a range of Λ values.

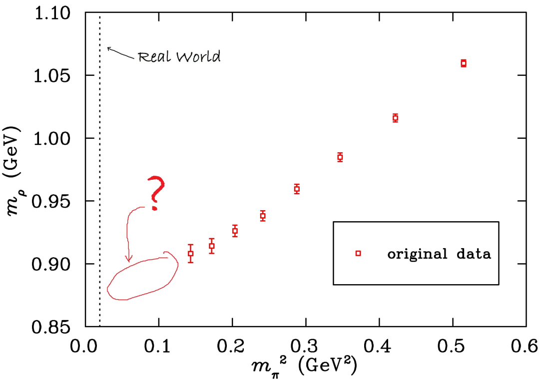
Jonathan Hall, Derek Leinweber, Ross Young

- Intersection points for c_0:



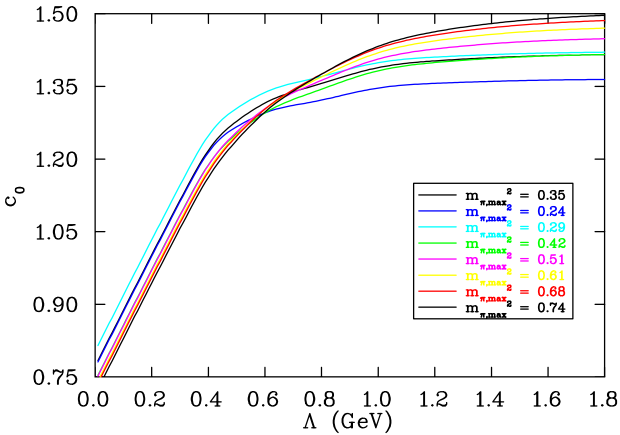
Jonathan Hall, Derek Leinweber, Ross Young

- To make these points clearer, we subtract all curves from the centre line corresponding to $m^2_{\pi,\mathrm{max}} = 0.35$ GeV^2.

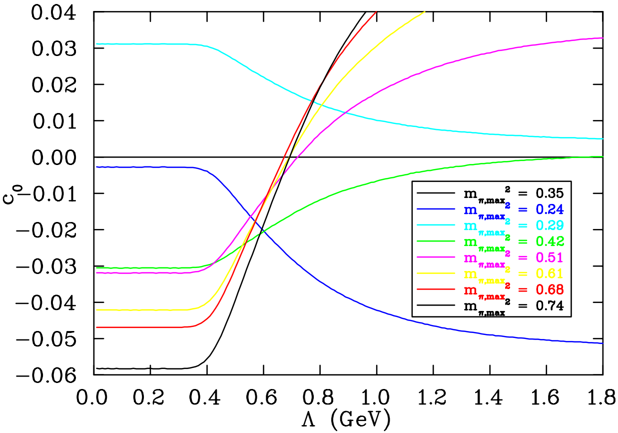


Jonathan Hall, Derek Leinweber, Ross Young

- Intersection points for c_0:

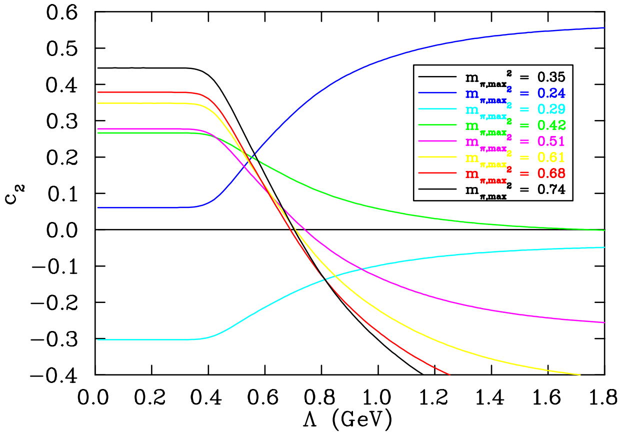


- For 'modified' c_0 about the central number of data points:



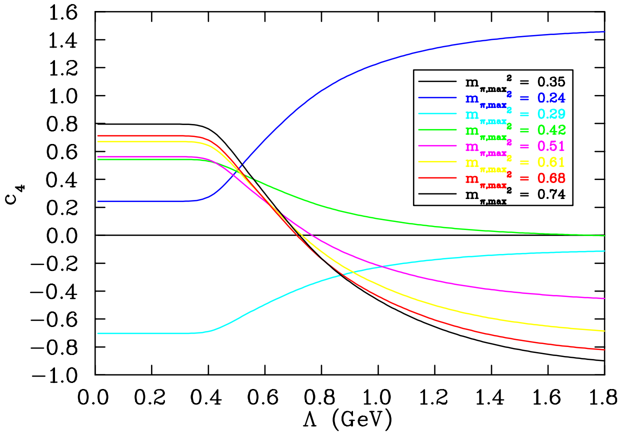
Jonathan Hall, Derek Leinweber, Ross Young

- For 'modified' c_2 about the central number of data points:

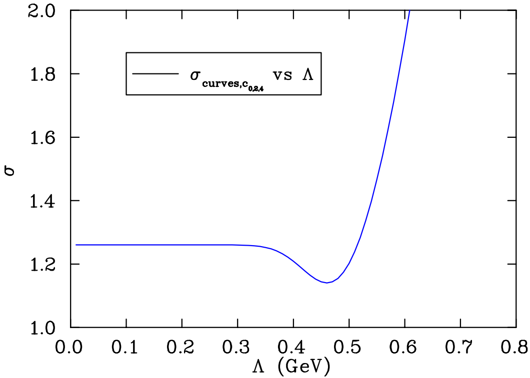


Jonathan Hall, Derek Leinweber, Ross Young

- For 'modified' c_4 about the central number of data points:



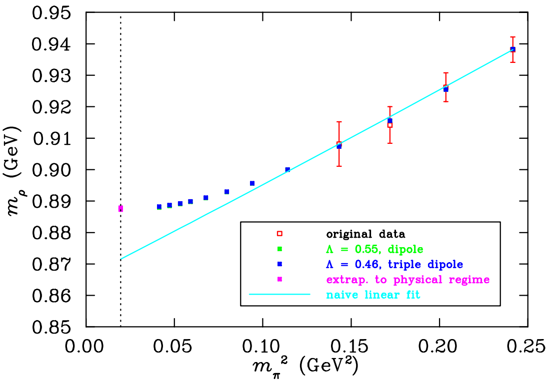
- The intrinsic scale Λ_0 is realised by the lattice data.
- Minimise the (weighted) variance with respect to the most constrained curve of $m^2_{\pi,\mathrm{max}} = 0.24$ GeV^2.



- $\Lambda_0^{\mathrm{trip}} = 0.46$ GeV

Conclusions

- Finite volume extrapolation of m_ρ using triple dipole at $\Lambda_0^{\text{trip}} = 0.46$ GeV.
- Try comparing the value of $\langle k^2 \rangle$ to the dipole regulator.
- Λ_0^{trip} corresponds to the dipole value of 0.55 GeV.



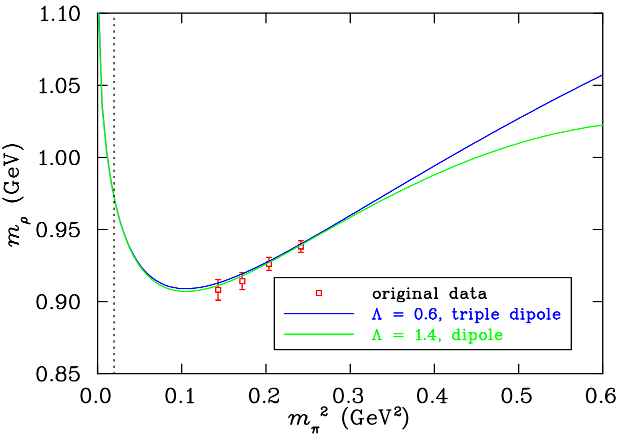
Conclusions

Jonathan Hall, Derek Leinweber, Ross Young

- Within the power-counting regime (PCR), a mathematical correspondence to dim. reg. is known.
- Outside the PCR, results of observables are Λ dependent. Does the data indicate a 'best value'?
- Created a procedure to determine a preferred range of FRR scale based on nonperturbative lattice results — without phenomenological predjudice.
- Promises a robust scale determination procedure that will facilitate *ab initio* studies.

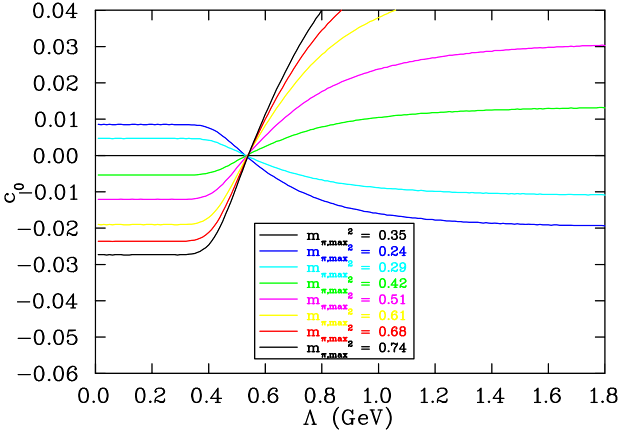
Appendix

- Choose a value of $\bar{\Lambda}_{\rm dip}$ such that both curves match at the physical pion value:



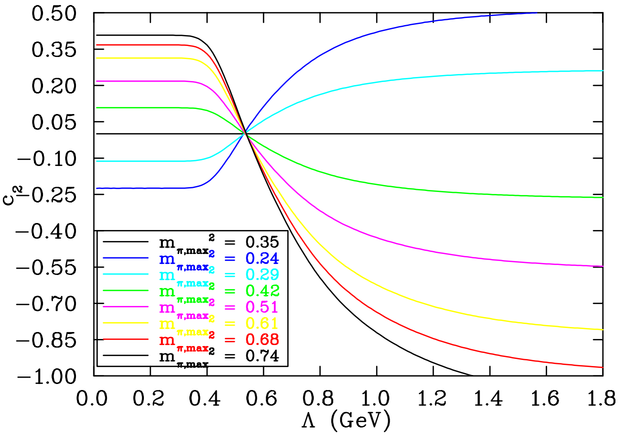
Jonathan Hall, Derek Leinweber, Ross Young

- Triple dipole analysis of pseudodata based on finite volume dipole created at $\Lambda = 0.8$ GeV. For c_0:



Jonathan Hall, Derek Leinweber, Ross Young

- For c_2:



Jonathan Hall, Derek Leinweber, Ross Young

Appendix

- For c_4: