THE UNIVERSITY OF ADELAIDE AUSTRALIA

THE CHIRAL EXTRAPOLATION GAME

CHIRAL EFFECTIVE FIELD THEORY

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

-:Jonathan Hall:-

1. The Quenched Rho Meson Mass

-Motivation and Aim:

-***Lattice Quantum Chromodynamics (QCD)*** is an *ab initio* method for simulating computationally intensive subatomic physics on a ***finite-sized periodic box,*** which produces discrete momenta values.

-***The Kentucky Group*** have produced precision Lattice QCD results for the ***mass of the quenched rho meson***, at low and high energies/quark masses.

-Our Goal: to ***predict*** the missing low energy (chiral) data points **(Fig. 1)** by extrapolating from the higher energy data points shown. We have ***no prior knowledge*** of the behaviour of the low energy data points.

-Why do this? The quenched rho meson's mass is ***completely unknown,*** thus provides an unbiased test of the ***chiral extrapolation methods*** developed at the University of Adelaide. If proved robust, the scheme can be used to extrapolate observables where ***no low energy data exists***.

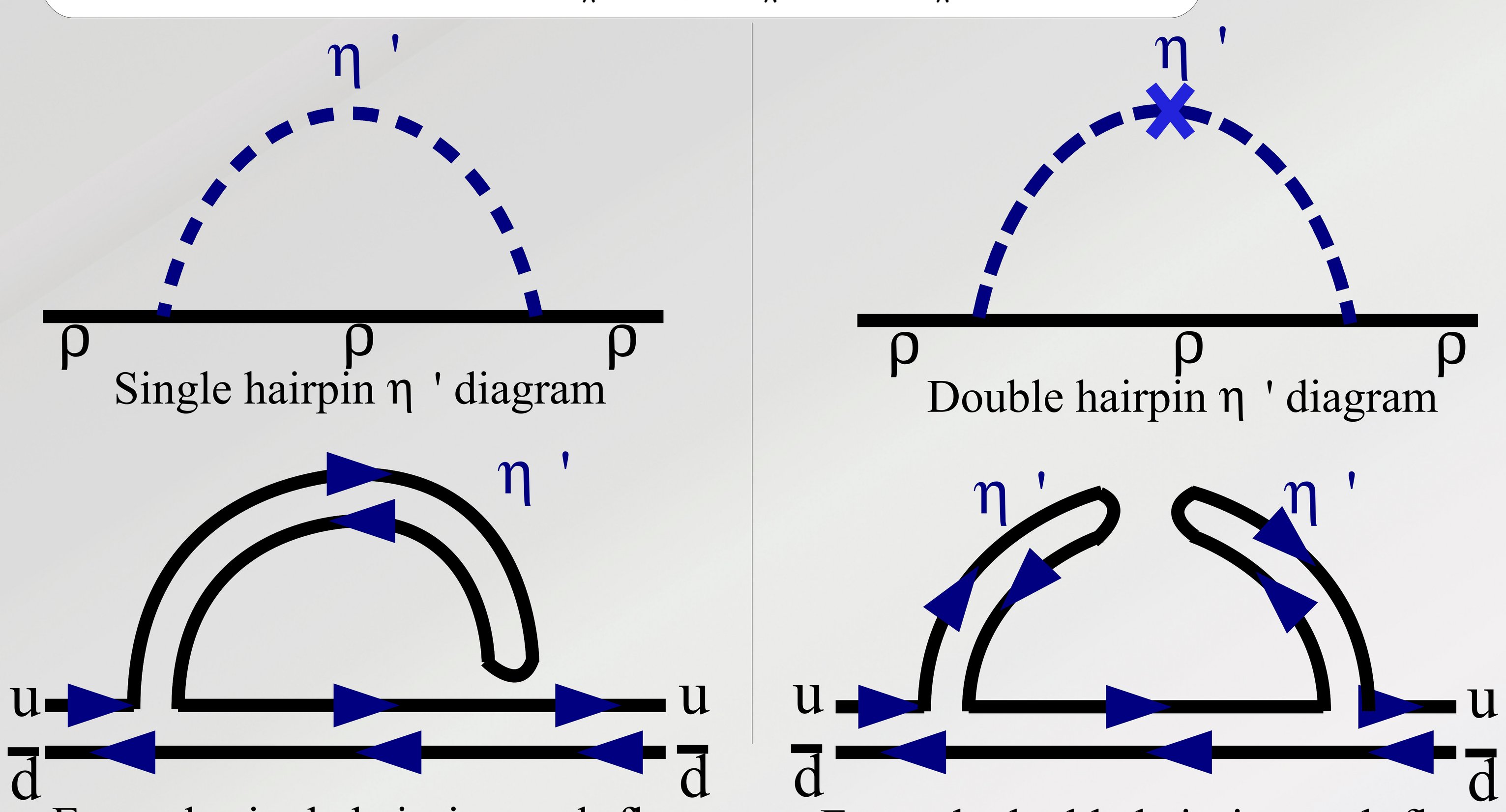
2. Chiral Effective Field Theory

-State of the Art:

-Should we do a simple linear fit? No, that would ignore the ***chiral physics***:

-***Chiral Effective Field Theory (χ EFT)*** contributions come into play at low energies. The quenched rho meson's mass is described by a ***polynomial function*** of quark mass ($\sim m_\pi^2$) plus the most important contributions from χ EFT: the ***leading-order loop diagrams*** shown below. They contribute to the ***non-analytic*** behaviour, which is ***significant*** at low (chiral) energies:

$$m_\rho^2 = \{a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \cdots\} + \Sigma_{\eta'} + \Sigma_{\eta'\eta'}$$
$$= c_0 + \chi_1 m_\pi + c_2 m_\pi^2 + \chi_3 m_\pi^3 + c_4 m_\pi^4 + \cdots$$



Single hairpin η ' diagram

Double hairpin η ' diagram

Example single hairpin quark-flow

Example double hairpin quark-flow

$$\Sigma_{\eta'}(m_\pi;\Lambda) = \chi_3 \frac{2}{\pi}\int_0^\infty dk\, \frac{k^4 u^2(k^2)}{k^2+m_\pi^2}, \quad \Sigma_{\eta'\eta'}(m_\pi;\Lambda) = \chi_1 \frac{-4}{3\pi}\int_0^\infty dk\, \frac{k^4 u^2(k^2)}{(k^2+m_\pi^2)^2}$$

3. Results

(see Graphical Results)

-Research and Critical Discussion:

-We have discovered that for any choice of cut-off function $u(k^2)$ in the integrals, an ***intrinsic energy scale Λ*** can be calculated from the data itself! The intrinsic scale depends on the ***quality*** and ***amount*** of low energy data, and ***to what order we calculate*** the χ EFT expansion of m_ρ^2.

-By taking the ***non-trivial chiral curvature*** from the loop diagrams into account, and calculating systematic errors in the integrals' coefficients and in the intrinsic scale (**Λ = 0.65 + 0.26 - 0.14 GeV** (2sf)), we produce **Fig. 2**.

-Now, ***the Kentucky Group data is revealed*** for comparison in **Fig. 3**! Important non-trivial features in the low energy curvature include a ***plateau*** at $m_\pi^2 \approx$ 0.08 GeV^2 followed by an ***'upturn'***, both of which agree with our extrapolation. Note that an upturn occurs ***even for the lower standard deviation*** of the extrapolated data. The naïve linear extrapolation is non-trivially wrong.

-In **Fig. 4**, each data point is subtracted about a particular data point, which helps to clarify the plot by ***removing much of the correlated statistical error*** in the Kentucky Group data. Almost all data points lie within the corresponding extrapolations' ***systematic*** error bars.

4. Conclusion

- Future Direction and Updated Plan:

The results clearly indicate a ***successful procedure*** for using ***high energy Lattice QCD data*** to extrapolate an observable to the ***low energy region***. In future research, this scheme will be applied to other observables such as magnetic moments of nucleons, which have large chiral curvature, and the mass-energy of the Roper resonance of nucleons, which are difficult to calculate on the Lattice. Now that the scheme has been proved predictive, the plan is to predict the ***finite volume corrections*** for such other lattice QCD observables which are of current experimental interest.

Graphical Results

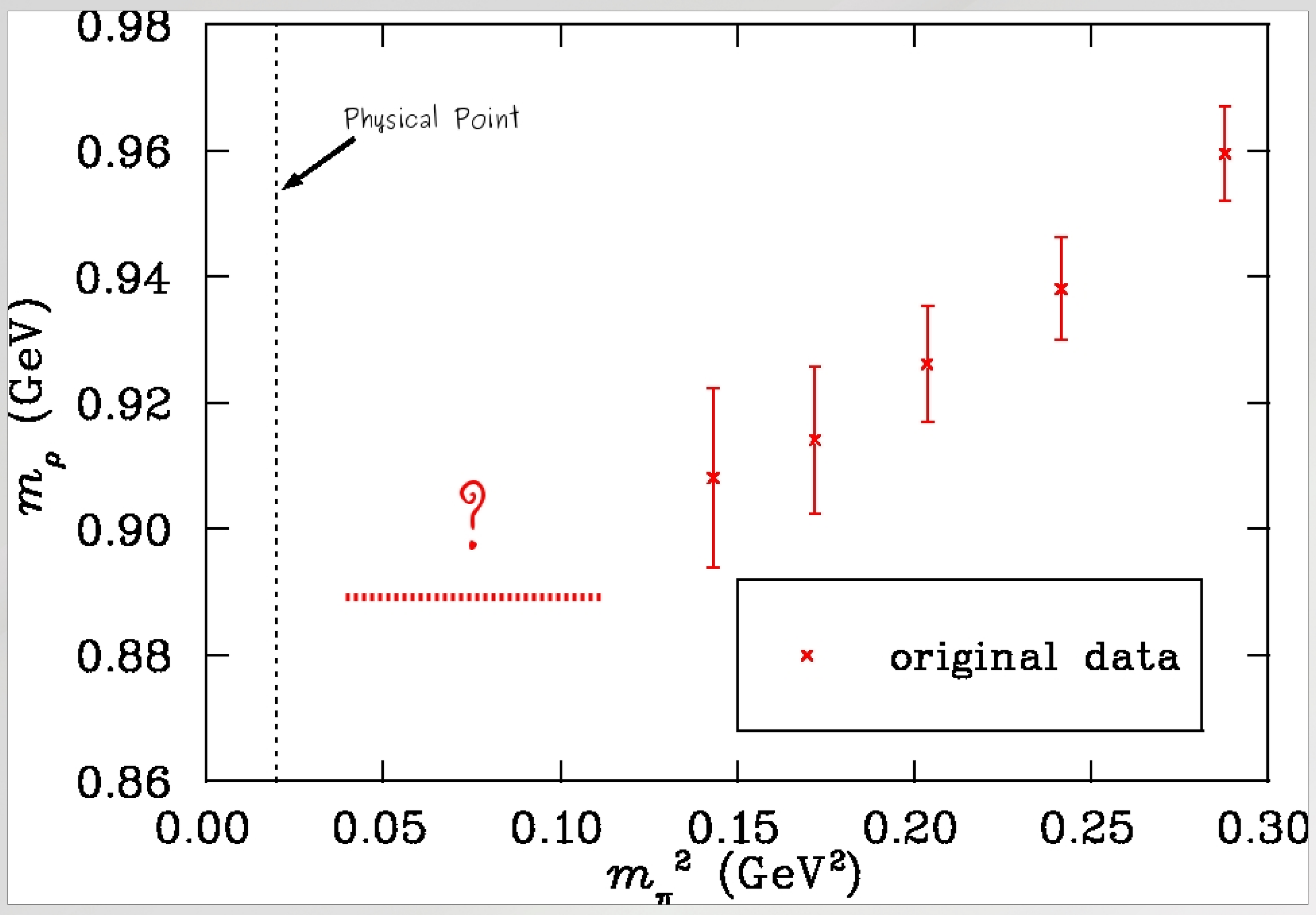


Fig. 1. Precision quenched rho meson mass Lattice QCD data from Kentucky Group with the 'omitted data' low energy region shown.

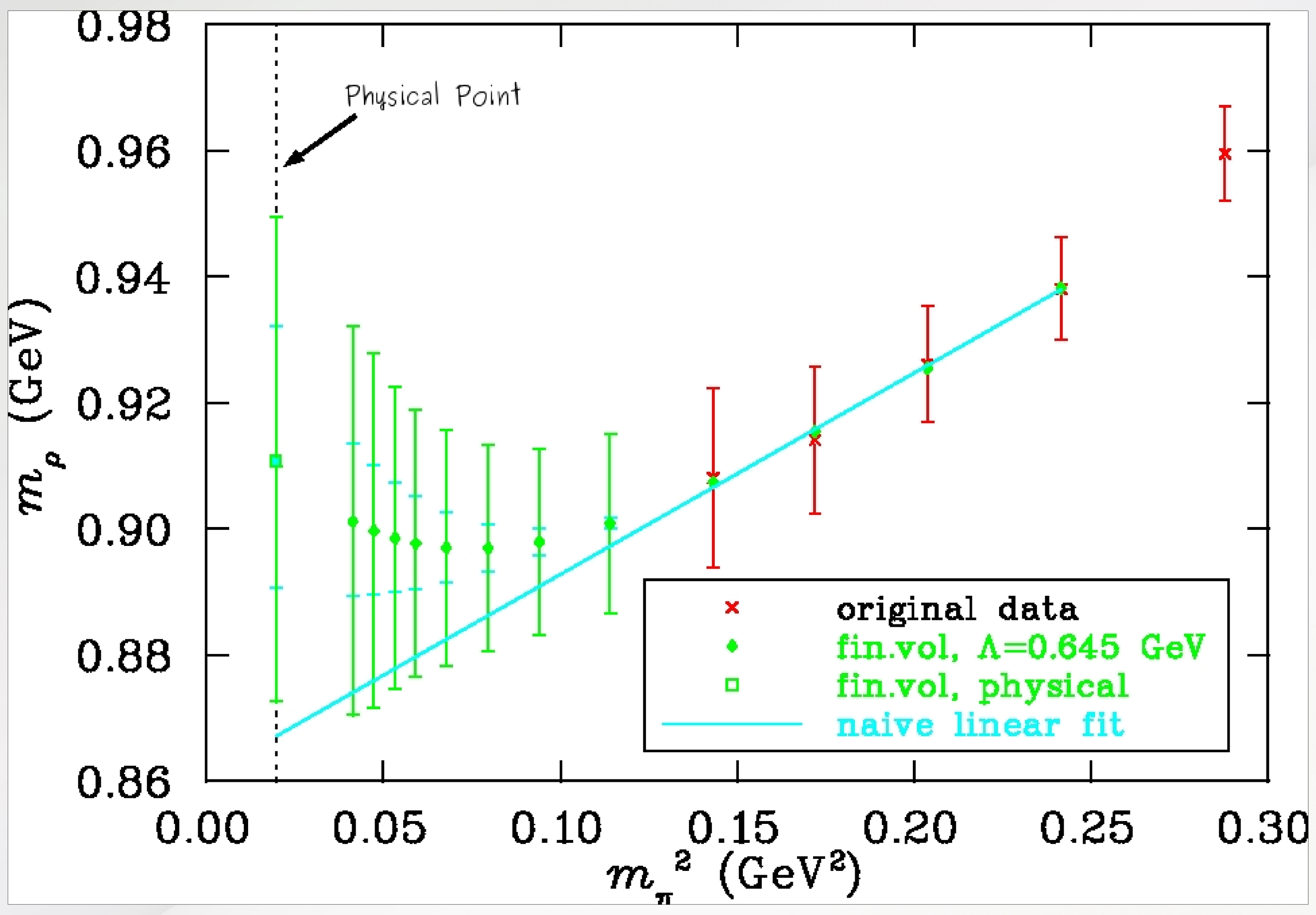


Fig. 2. Extrapolation of the rho meson mass performed linearly (in blue), and taking into account loop integrals at intrinsic scale (in green).
-Inner error bar: systematic error.
-Outer error bar: statistical error estimate.

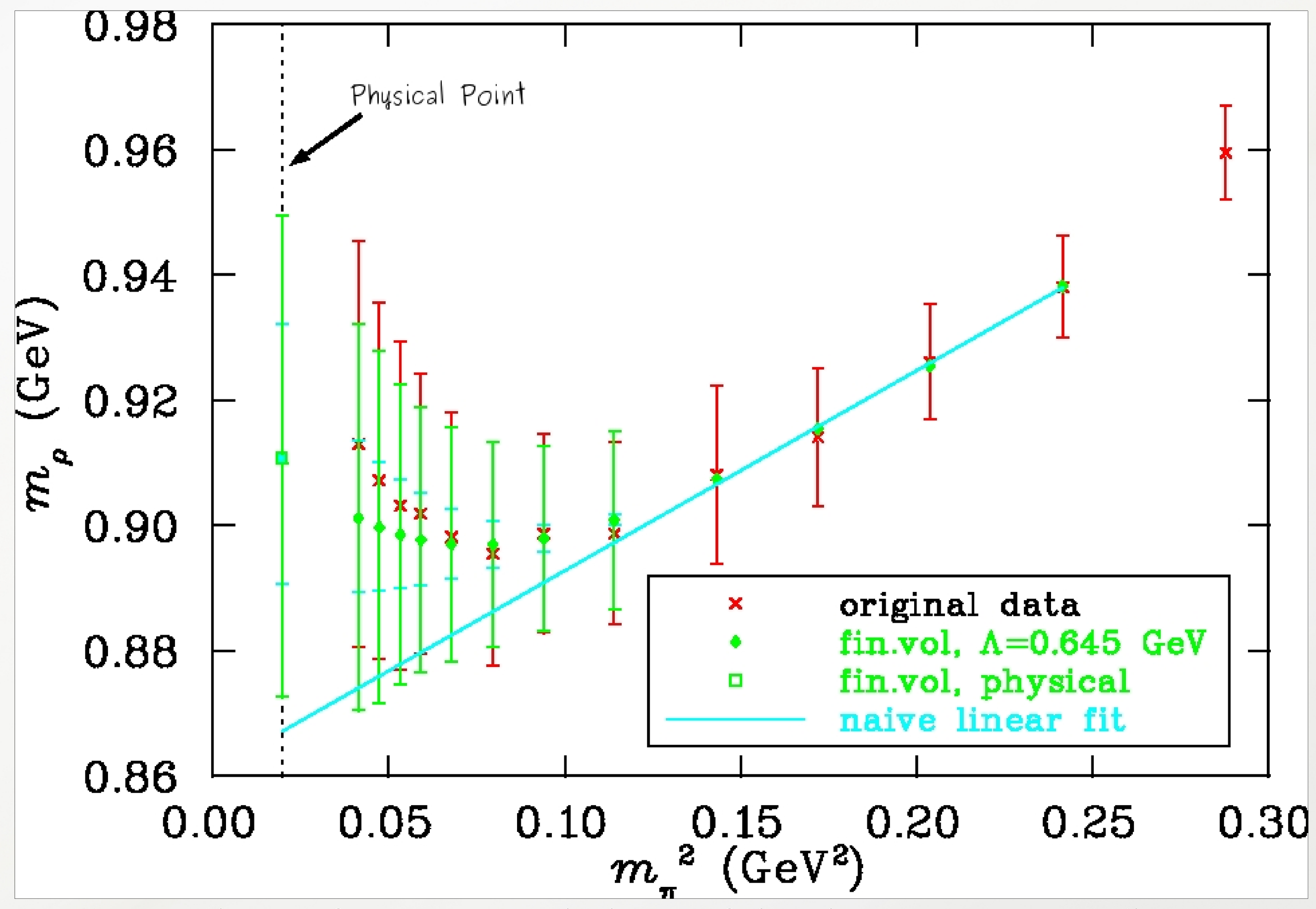


Fig. 3. Comparison of our extrapolations of the rho meson mass (in green) to the newly revealed low energy data from Kentucky Group (in red).
-Inner error bar: systematic error.
-Outer error bar: statistical error estimate.

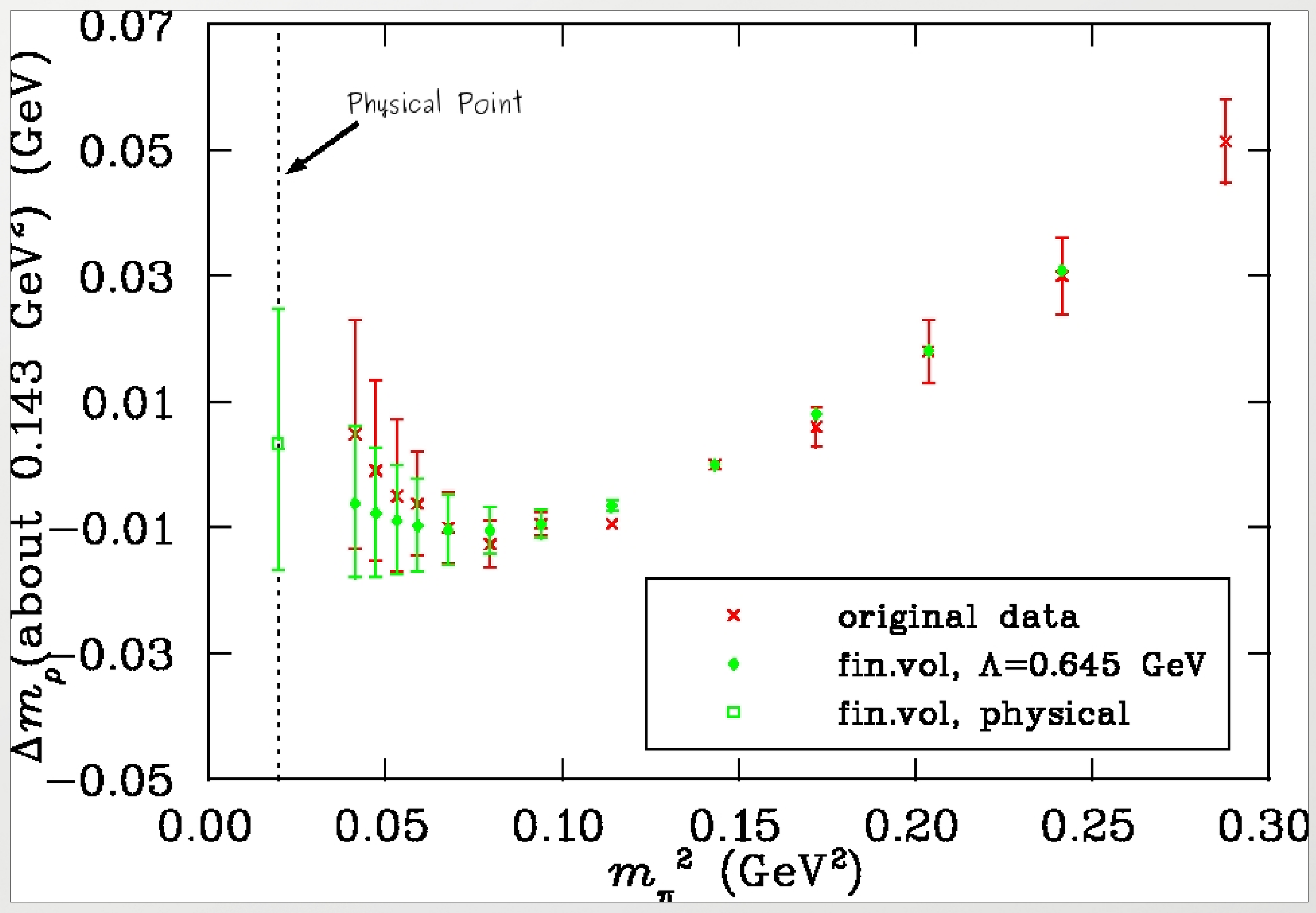


Fig. 4. The rho meson mass is subtracted about a central point (m_π^2 = 0.143 GeV^2), thus reducing the correlated statistical error on the Kentucky Group data (in red). Our extrapolations (in green) have systematic error shown only.

Special thanks to-
Principal Supervisor: ***Derek Leinweber,*** *Co-Supervisor:* ***Rod Crewther*** *and* ***Ross Young*** *(Argonne National Lab)*