Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

Chiral Effective Field Theory Beyond the Power Counting Regime

Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

CSSM, University of Adelaide

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Overview

- Introduction
- Effective Field Theory for nucleons
  - Loop integrals
  - Renormalization
- Ideal ‘pseudodata’
- Intrinsic energy scale
  - Evidence
  - Statistical uncertainty
  - Higher chiral order
- Quenched ρ meson case
- Conclusion & future directions

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
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Aims

- Lattice QCD can rarely be evaluated at physical quark masses. We want to be able to extrapolate current results to this physical point.
- Chiral Perturbation Theory gives insight into this low energy region, but is limited to use over a very small range of quark masses.
- We will discover that using more of the available data often entails model-dependence. But the extent of the model-dependence can be quantified and thus removed.
- This will lead us to realizing the presence of an 'intrinsic energy scale', embedded in such lattice QCD data.

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Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
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Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- Chiral Effective Field Theory (χEFT) complements lattice QCD.
- It assists in understanding the consequences of dynamical chiral symmetry breaking.
- It provides a scheme-independent approach for investigating the properties of hadrons.
- In particular, it can be used in conjunction with lattice QCD data to extrapolate results:
    - to physical quark masses,
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
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- χPT provides a formal expansion in terms of low energy momenta and quark masses.
- The expansion is convergent if the quark mass is small so that higher order terms are negligible. This is called the Power Counting Regime (PCR).
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EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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- EFT can be used to search for the possible presence of an 'intrinsic energy scale' embedded in lattice QCD results.
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Overview
Introduction
EFT for Nucleons
Pseudodata
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EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
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Formal Background

- For an effective field theory, one writes out a low energy effective Lagrangian.
- The terms of the Lagrangian are ordered in powers of momenta and mass.
- For nucleons (fermions) written as an $\mathrm{SU}(2)$ doublet $\Psi = (p \;\; n)^{\mathrm{T}}$, the first order (lowest energy) Lagrangian takes the form:

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}\left(\not{\partial} - \overset{\circ}{M}_N + \frac{\overset{\circ}{g}_A}{2 f_\pi}\gamma^\mu \gamma_5 \vec{\tau}\cdot\partial_\mu\vec{\pi}\right)\Psi\,,$$

- The circle $\circ$ denotes a "bare" quantity: it gets renormalized by chiral loops from the field theory. Let's look at the nucleon mass M_N …

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- The nucleon mass M_N is renormalized by:
  - an analytic polynomial associated with the quark masses m_q.
  - chiral loop integrals Σ_{loops} .
- The low energy expansion formula about the chiral limit (small m_q) is expressed using the Gell-Mann–Oakes–Renner Relation $m_q \propto m_\pi^2$:

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\begin{aligned}
M_N &= \{\text{terms analytic in } m_\pi^2\} + \{\text{chiral loop corrections}\} \\
&= \{a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \mathcal{O}(m_\pi^6)\} + \{\Sigma_{\text{loops}}\}\,.
\end{aligned}
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- The analytic terms will be collectively called the 'residual series', and their coefficients a_i will be determined by fitting to lattice QCD data.
- The chiral loops have known, scheme-independent coefficients, but given rise to non-analytic behaviour.

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
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Conclusion

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EFT for Nucleons
Pseudodata
Intrinsic Scale
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Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
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Chiral Loops

- The integral form of the chiral loops are obtained using the Feynman Rules for χPT, and can then be solved.
- Each loop, when evaluated from its integral form, produces a non-analytic term.
- To finite chiral order ($\mathcal{O}(m_\pi^4 \log m_\pi)$), the leading order chiral loops are:
  - the 1−pion loop ($\Sigma_N \sim m_\pi^3$),
  - the pion loop decuplet transition ($\Sigma_\Delta \sim m_\pi^4 \log m_\pi$),
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Overview
Introduction
EFT for Nucleons
Pseudodata
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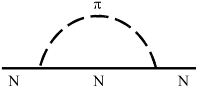

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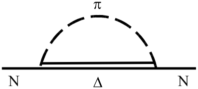
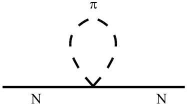


$$= \Sigma_\Delta = b_0^\Delta + b_2^\Delta m_\pi^2 + b_4^\Delta m_\pi^4 + \frac{3}{4\pi\Delta}\chi_\Delta m_\pi^4 \log\frac{m_\pi}{\mu} + \mathcal{O}(m_\pi^5)$$



$$= \Sigma_{tad} = b_2^{t'} m_\pi^2 + b_4^{t'} m_\pi^4 + \chi_t' m_\pi^4 \log\frac{m_\pi}{\mu} + \mathcal{O}(m_\pi^5)$$

- Note: Δ is the mass splitting $M_\Delta - M_N$, and μ is the mass scale associated with renormalization.

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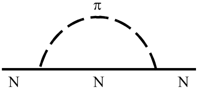

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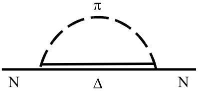
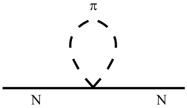


$$= \Sigma_\Delta = b_0^\Delta + b_2^\Delta m_\pi^2 + b_4^\Delta m_\pi^4 + \frac{3}{4\pi\Delta}\chi_\Delta m_\pi^4 \log\frac{m_\pi}{\mu} + \mathcal{O}(m_\pi^5)$$



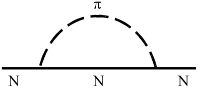
$$= \Sigma_{tad} = b_2^{t'} m_\pi^2 + b_4^{t'} m_\pi^4 + \chi_t' m_\pi^4 \log\frac{m_\pi}{\mu} + \mathcal{O}(m_\pi^5)$$

- Note: Δ is the mass splitting $M_\Delta - M_N$, and μ is the mass scale associated with renormalization.

Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

Chiral Loops



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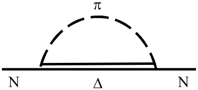

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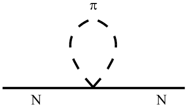

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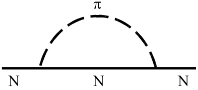

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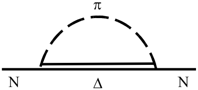

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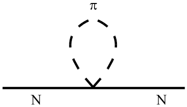

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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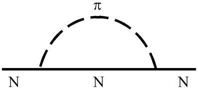

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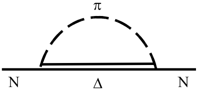

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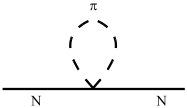

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Jonathan Hall
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

Expansion Formulae

- When the integrals are evaluated, the expansion becomes an expansion in increasing powers of m_π, with renormalized analytic and non-analytic terms occuring at their respective orders:

$$\begin{aligned} M_N &= c_0 + c_2 m_\pi^2 + \chi_N m_\pi^3 + c_4 m_\pi^4 \\ &+ \left(-\frac{3}{4\pi\Delta}\chi_\Delta + \chi_t'\right) m_\pi^4 \log\frac{m_\pi}{\mu} + \mathcal{O}(m_\pi^5)\,. \end{aligned}$$

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- The coefficients χ_N , χ_Δ & χ'_t are known, scheme-independent parameters (related to $\overset{\circ}{g}_A$, f_π, etc).
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Renormalization Issues

Jonathan Hall
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

Renormalization Issues

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Renormalization Issues

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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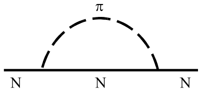
- How does the renormalization take place? Consider the 1−pion loop integral as a test example:



$$\Sigma_N = \frac{2\chi_N}{\pi}\int_0^\infty \mathrm{d}k \frac{k^4}{k^2+m_\pi^2}$$

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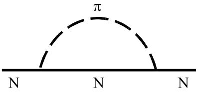

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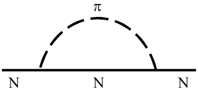

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Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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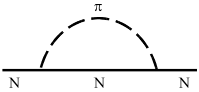

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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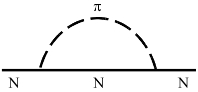

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Jonathan Hall
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Leinweber &
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Jonathan Hall
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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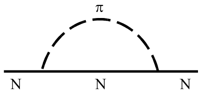

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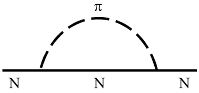

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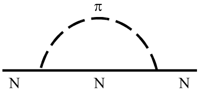

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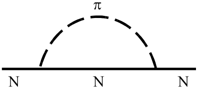

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- In Finite Range Regularization (FRR), a momentum cutoff Λ is introduced (via a regulator function), and the chiral expansion is resummed.
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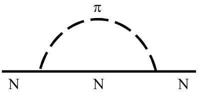

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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- The massless renormalization scheme result is recovered as $\Lambda \to \infty$.

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c_0 &= a_0 + \frac{2\chi_N}{3}\Lambda^3\,, \\
c_2 &= a_2 - \frac{2\chi_N}{\pi}\Lambda\,, \\
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Loop Integrals

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- This simply means that, by convention, relevant b_0 and b_2 terms will be subtracted from each integral.

- Taking the heavy-baryon limit and performing the k_0 integration, the loop integrals take the following forms:

$$\tilde{\Sigma}_N = \frac{\chi_N}{2\pi^2}\int \mathrm{d}^3k \frac{k^2 u^2(k;\Lambda)}{\omega^2(k)} - b_0^{\Lambda,N} - b_2^{\Lambda,N} m_\pi^2 ,$$

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- The pion energy is $\omega(k) \equiv \sqrt{k^2 + m_\pi^2}$ and the FRR regulator function is denoted by $u(k;\Lambda)$.

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- Note that the tadpole integral has a coefficient $\chi'_t = c_2 \chi_t$, which involves c_2 (obtained from the Lagrangian $\mathcal{L}_{\pi N}^{(2),tad} = c_2 \mathrm{Tr}_f[\mathcal{M}_q]\bar{\Psi}\Psi$).
- Thus the nucleon mass expansion formula can be conveniently factorized:

$$\begin{aligned} M_N &= \{a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \mathcal{O}(m_\pi^6)\} + \{\Sigma_N + \Sigma_\Delta + \Sigma_{tad}\} \\ &= c_0 + c_2 m_\pi^2 (1 + \tilde{\sigma}_{tad}) + a_4^\Lambda m_\pi^4 + \tilde{\Sigma}_N + \tilde{\Sigma}_\Delta \,. \end{aligned}$$

- This formula can be used for extrapolations, with fit coefficients c_0, c_2 and a_4^Λ.

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Finite Volume Corrections

- Lattice QCD is done on a finite volume box.
- Our ideal infinite volume expansion formula should be modified to include finite volume corrections.
- Each integral can be converted into a discrete summation, and then the difference is taken to achieve the correction:

$$\delta_i^{\text{FVC}} = \frac{\chi_i}{2\pi^2}\left[\frac{(2\pi)^3}{L_x L_y L_z}\sum_{k_x,k_y,k_z} - \int d^3k\right].$$

- The tadpole finite volume corrections are subtle and will not be dealt with in this talk.

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Jonathan Hall
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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- We are almost ready to try an extrapolation from lattice QCD data. But what form ought the regulator $u(k;\Lambda)$ to take?

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Finite-Range Regulators

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- All forms of $u(k;\Lambda)$ are equivalent within the PCR, as long as they are normalized to 1, and are suppressed to 0 for large momenta k. Dimensional Regularization (DR) corresponds to $\Lambda \to \infty$.
- The step function $\theta(\Lambda - k)$ is acceptable, but is unfavorable for use with the finite volume of the lattice.
- Consider the family of smooth $n-$tuple dipole attenuators:

$$u_n(k;\Lambda) = \left(1 + \frac{k^{2n}}{\Lambda^{2n}}\right)^{-2}.$$

- The dipole corresponds to $n = 1$. We shall also consider the cases $n = 2, 3$, the double and triple dipole forms, respectively.
- We shall analyze data using these three different regulators to demonstrate the model-independence of this approach.

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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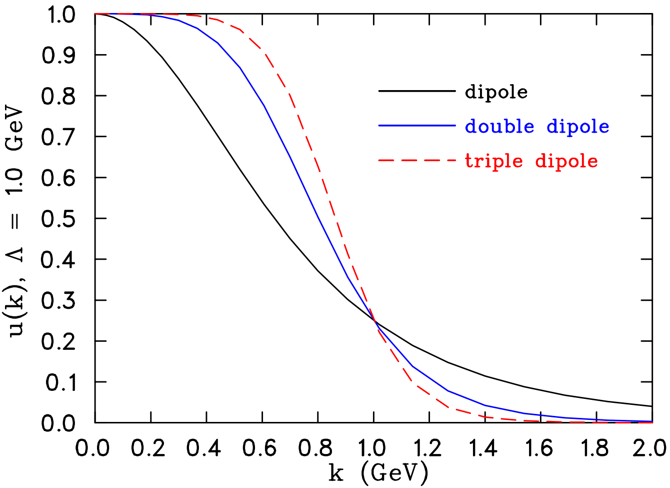
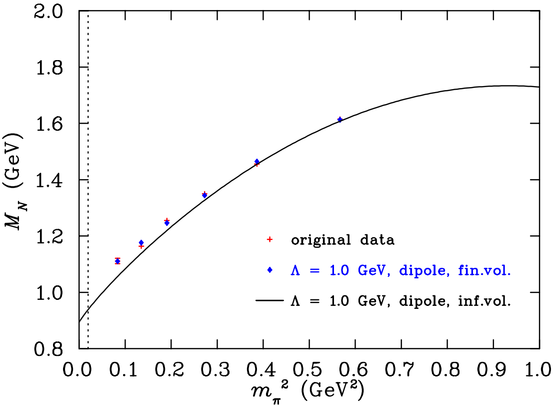
Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- Here are the three dipole-like forms plotted for $\Lambda = 1.0$ GeV:



Trial Extrapolations

- Consider the behaviour of M_N as a function of m_π^2.
- Here is an extrapolation of data from JLQCD, using a dipole regulator with $\Lambda_{\text{dip}} = 1.0$ GeV.
- The JLQCD data uses $N_f = 2$ overlap fermions at $L = 1.9$ fm.



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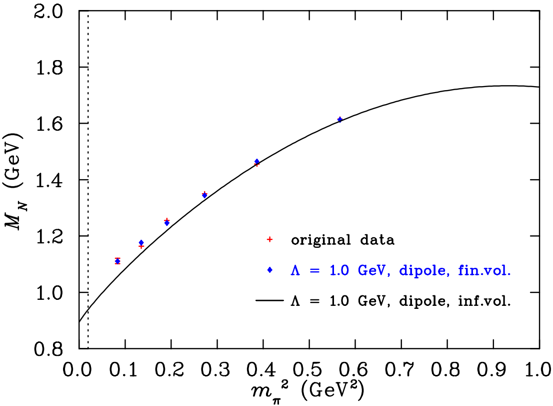

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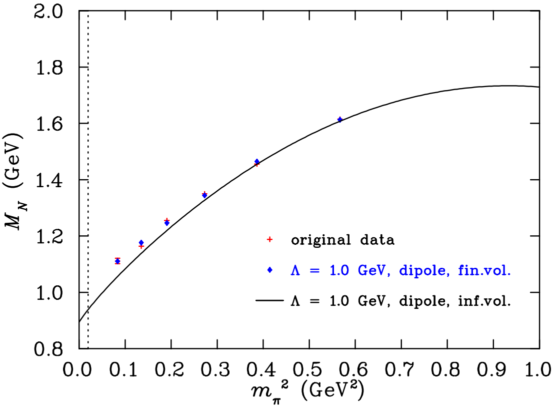
- Consider the behaviour of M_N as a function of m_π^2.
- Here is an extrapolation of data from JLQCD, using a dipole regulator with $\Lambda_{\mathrm{dip}} = 1.0$ GeV.
- The JLQCD data uses $N_f = 2$ overlap fermions at $L = 1.9$ fm.



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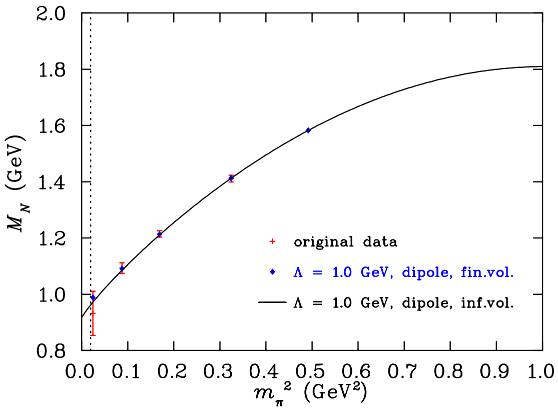

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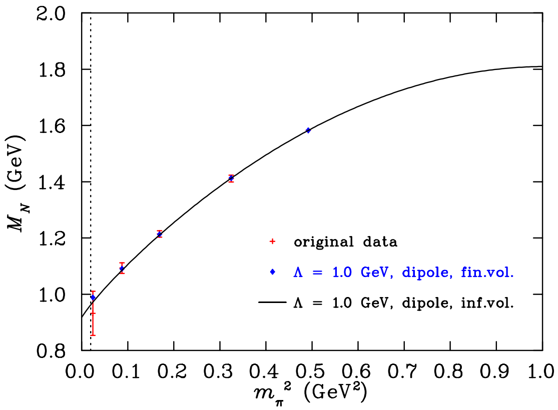

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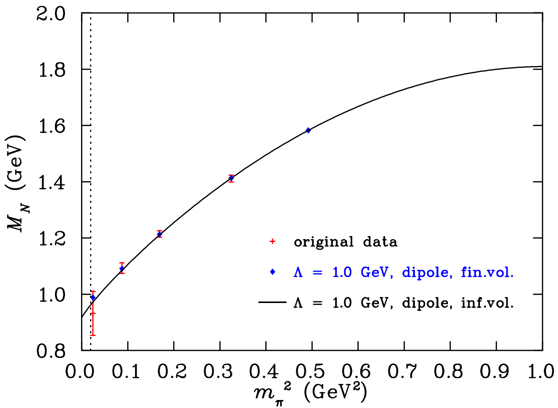

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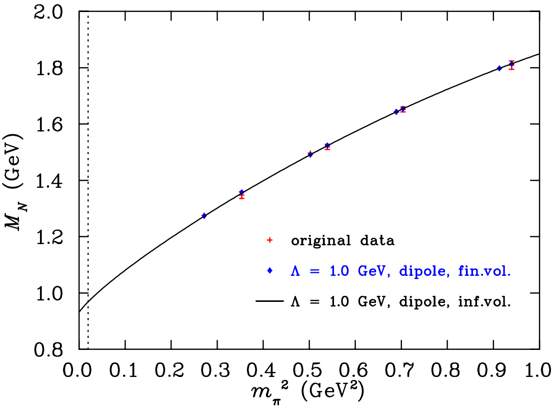

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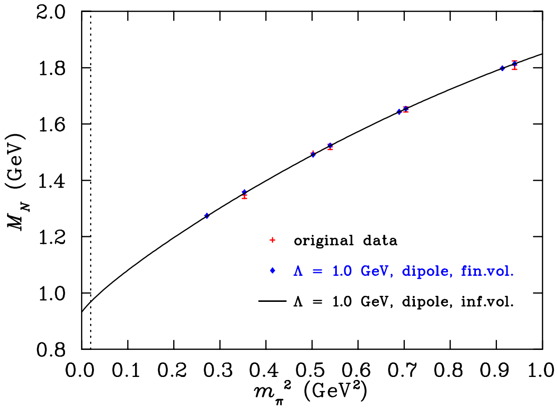

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Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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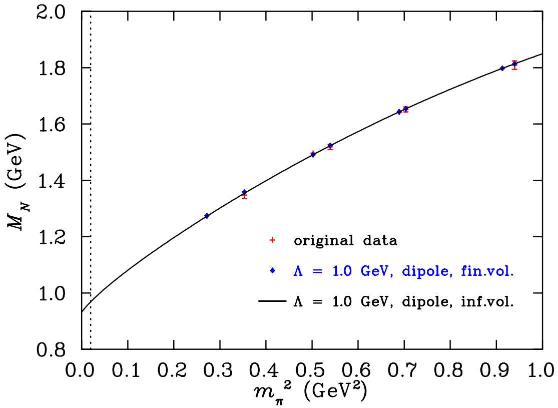

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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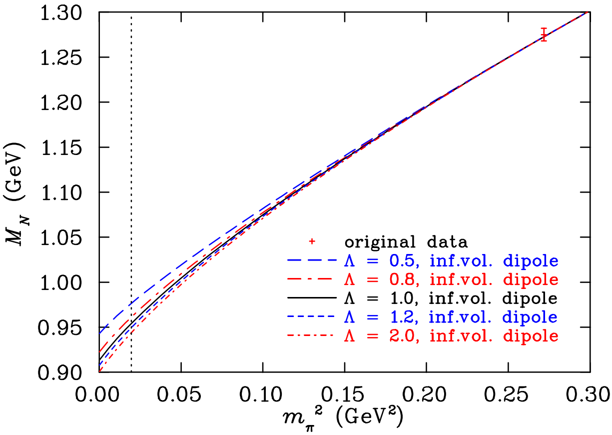

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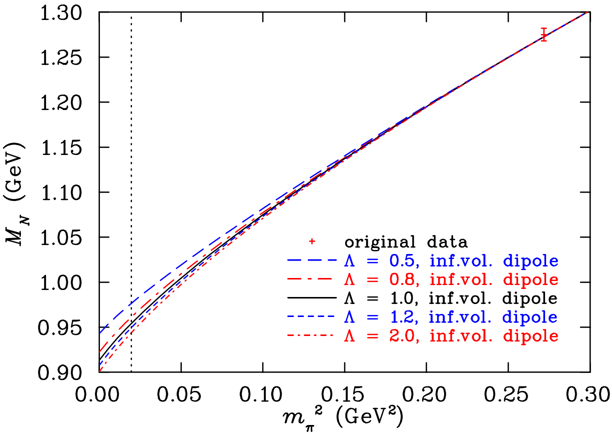

Pseudodata

- Different choices of regulator give different results! But is there an optimal choice?
- Also, if we want to stay close to the PCR, how many data points should we use? Does it matter?
- Let's do a test: Using the extrapolation formula for M_N, generate some ideal 'pseudodata'.

  - Generate one set of 100 closely spaced, low energy pseudodata points entirely within the PCR, created at $\Lambda^c_{\text{dip}} = 1.0$ GeV.
  - Generate two more sets, at different upper values $m^2_{\pi,\text{max}}$, thus progressing outside the PCR.
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Jonathan Hall
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- We can use these pseudodata sets for our analysis of regulator dependence.
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- Let's plot our fit coefficients c_0 and c_2 over a range of Λ_{dip} values, for each of the three data sets. We have chosen $m^2_{\pi,\text{max}} = 0.04,\ 0.25,\ 0.5$ GeV^2.

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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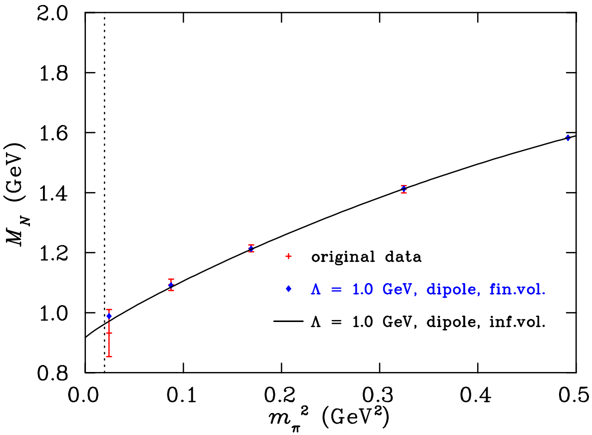
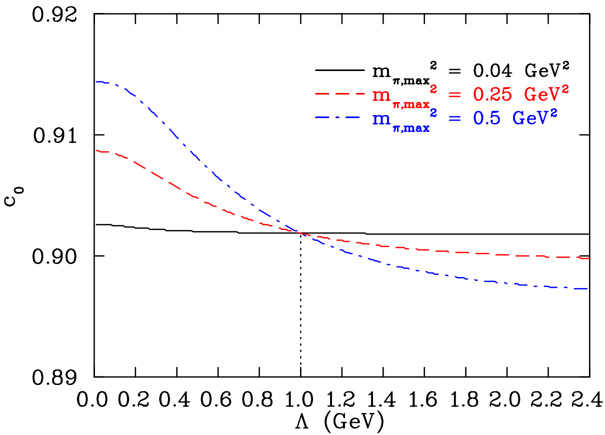
- The PACS-CS data, on which the pseudodata is based, is shown below:



Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- Here is the result for c_0 .
- Notice that the correct value of c_0 is recovered exactly when $\Lambda_{\text{dip}} = \Lambda^c_{\text{dip}}$.



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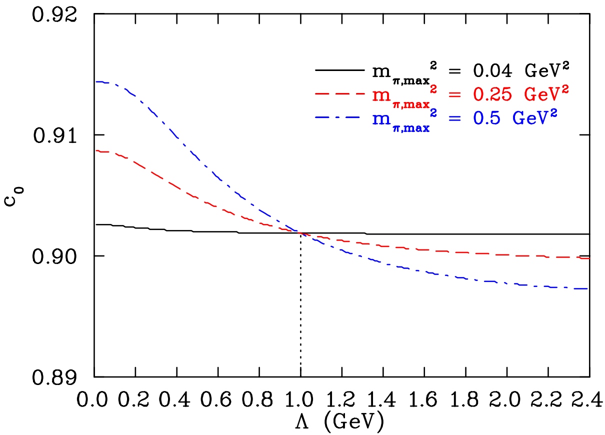
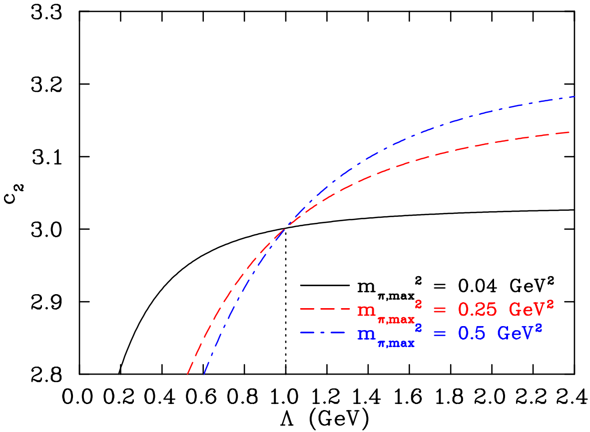


- Here is the result for c_2 .
- Though it is tempting to read off the value of any c_i as $\Lambda \to \infty$, recovering the DR result, it is wrong!



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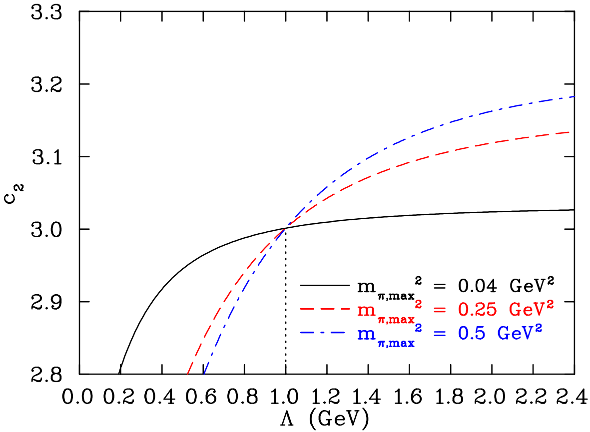

Jonathan Hall
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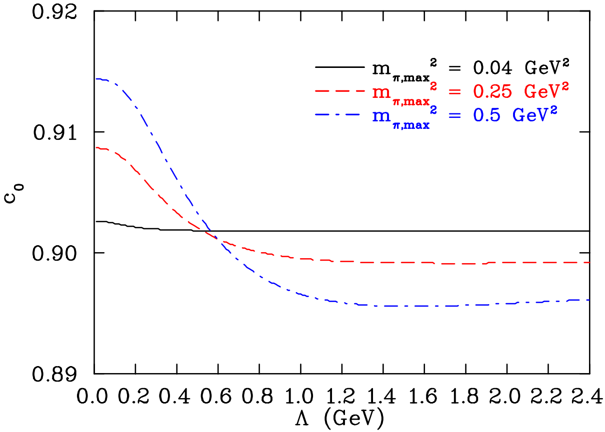
Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

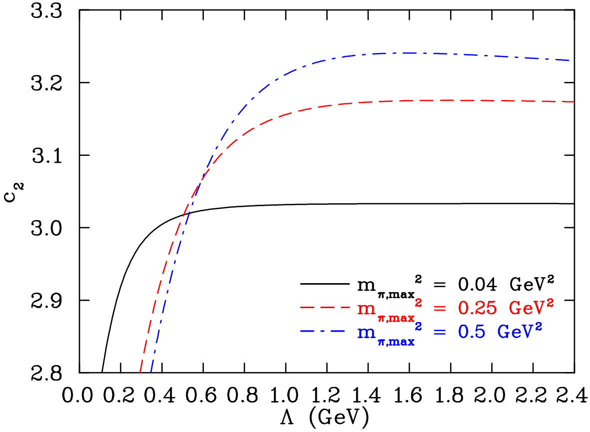
Pseudodata- Renormalization Flow

- This intersection point is not trivial. To demonstrate this, we can analyze the pseudodata using a triple dipole.
- Here is the result for c_0:



Pseudodata- Renormalization Flow

- Here is the result for c_2:
- This intersection is no longer a clear point, but a cluster at $\Lambda_{\text{dip}} \approx 0.5 - 0.6$ GeV. This is the preferred value of Λ_{trip}.



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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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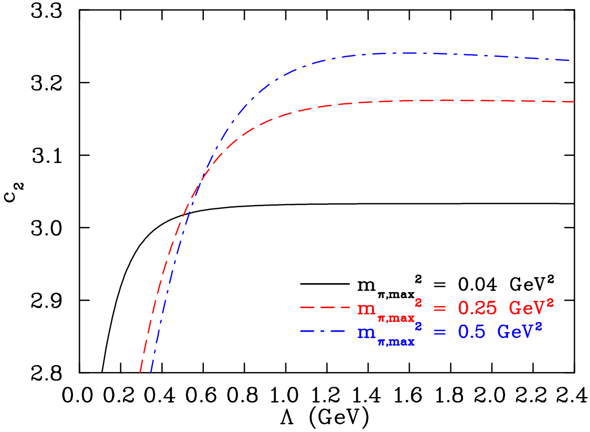

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Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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- We also see that FRR breaks down if Λ is too small.
- This makes sense mathematically, as $b_i^\Lambda \propto \Lambda^{3-i}$, and so for $i = 4, 6, ..$ these higher order coefficients blow up for small Λ.
- This also makes sense physically, as any ultraviolet regulator Λ must be large enough to allow inclusion of the chiral physics being studied. Otherwise we essentially destroy the non-analytic behaviour by making the integrals ≈ 0.
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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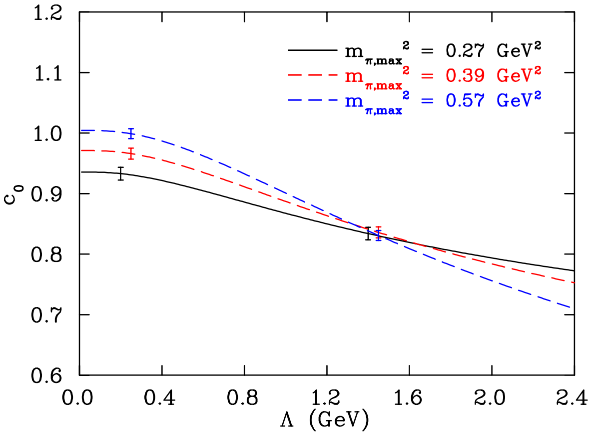

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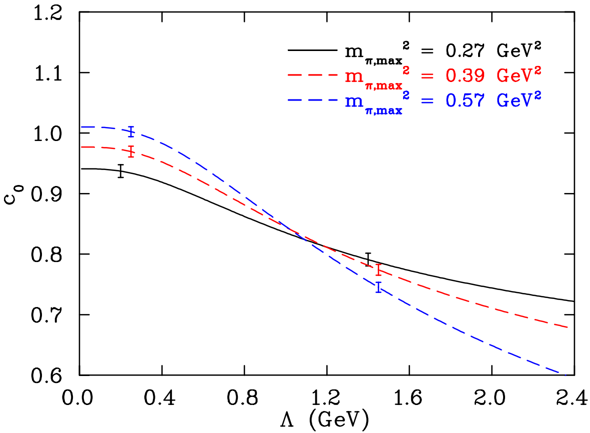

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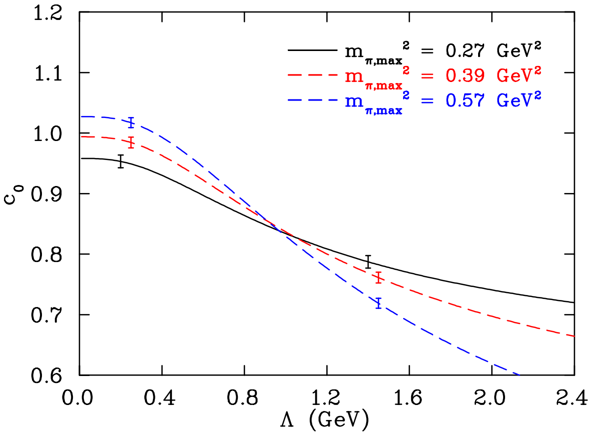

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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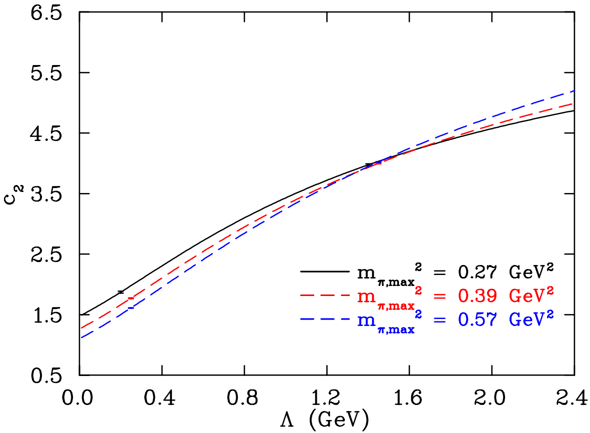

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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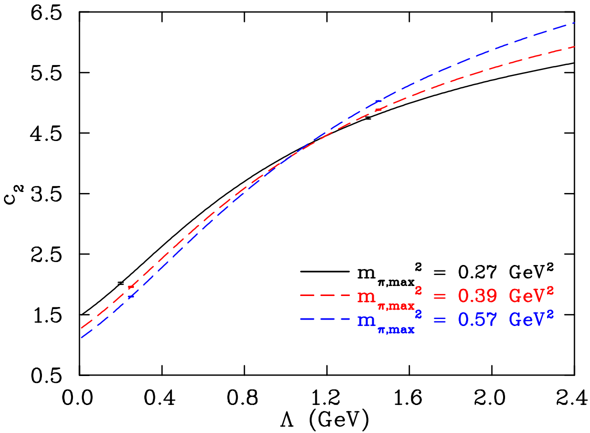

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
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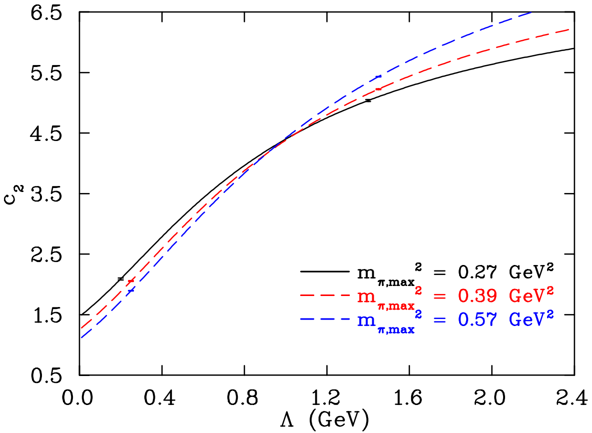
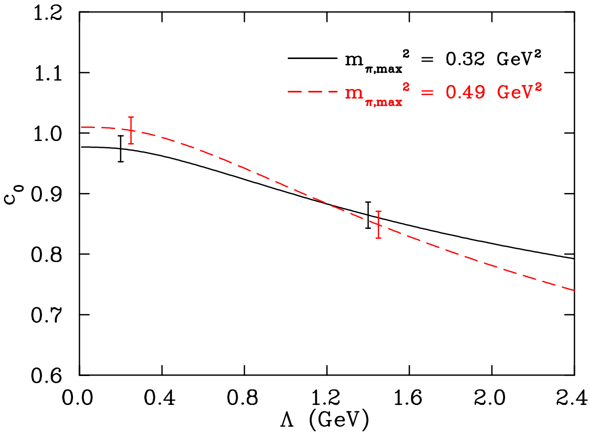


Evidence for an Intrinsic Scale

Jonathan Hall
Supervisors: Derek Leinweber & Ross Young

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- Here is the result for c_0 using PACS-CS data, working to chiral order $\mathcal{O}(m_\pi^3)$ and using a dipole regulator:



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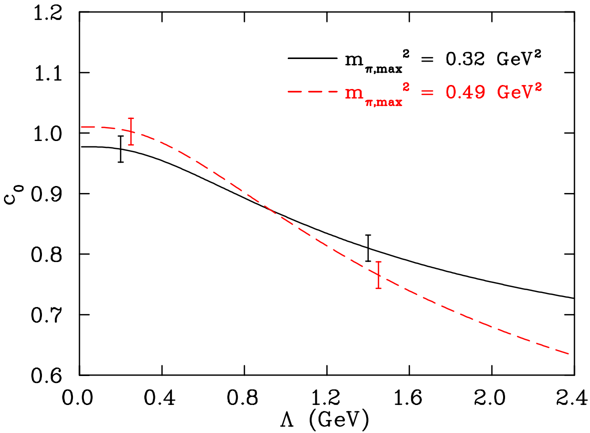

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

Evidence for an Intrinsic Scale

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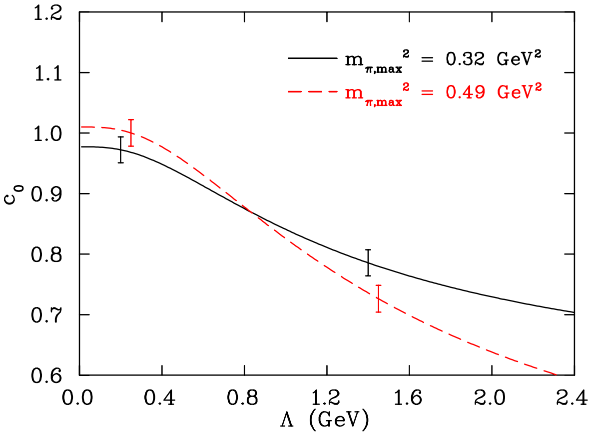

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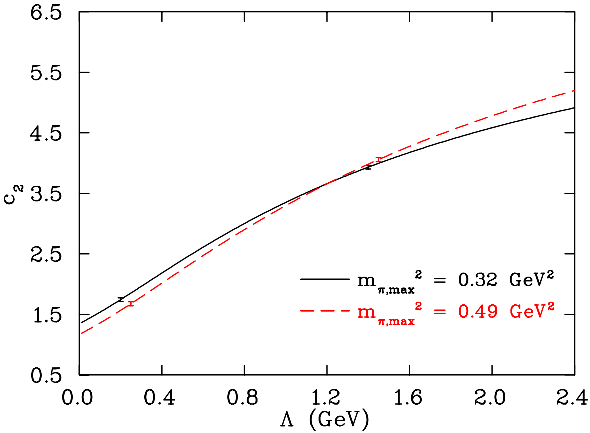

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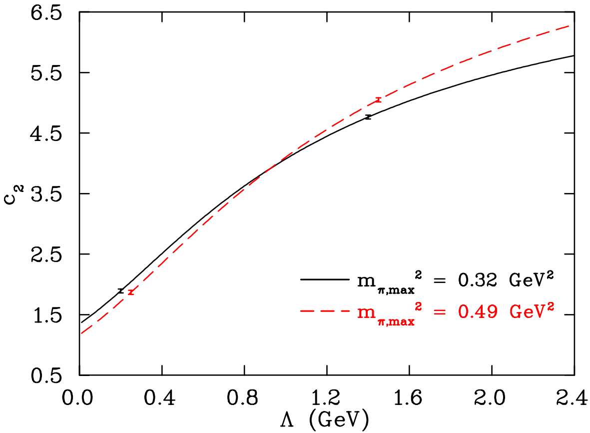

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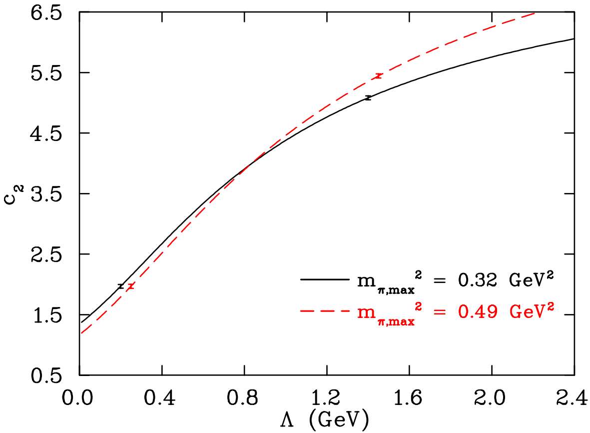

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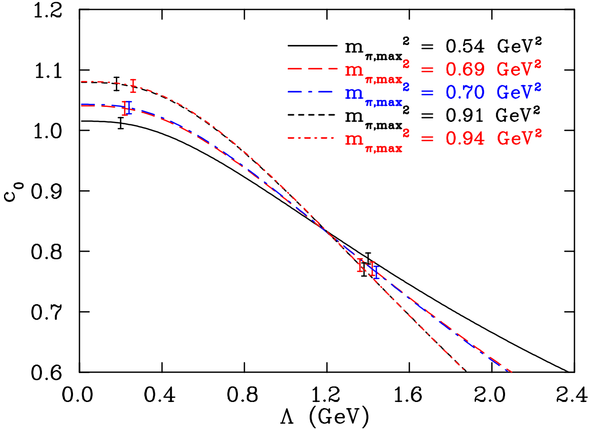

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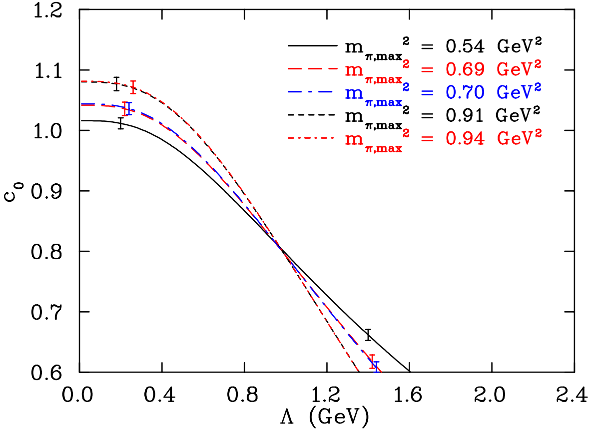

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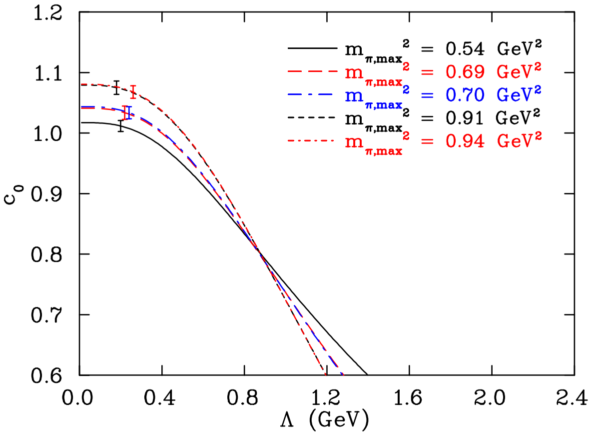

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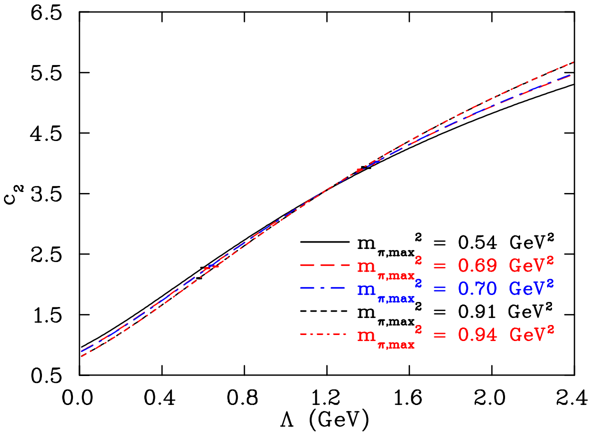

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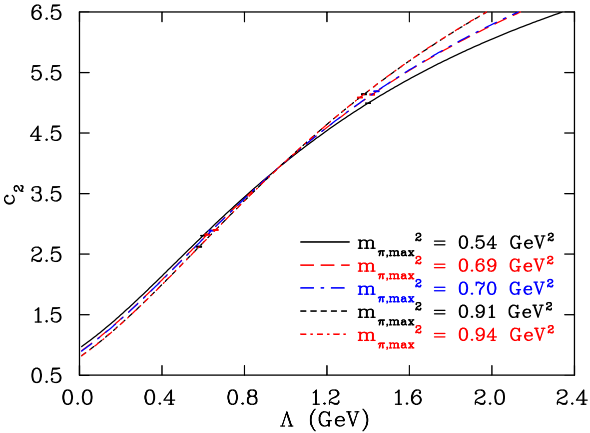

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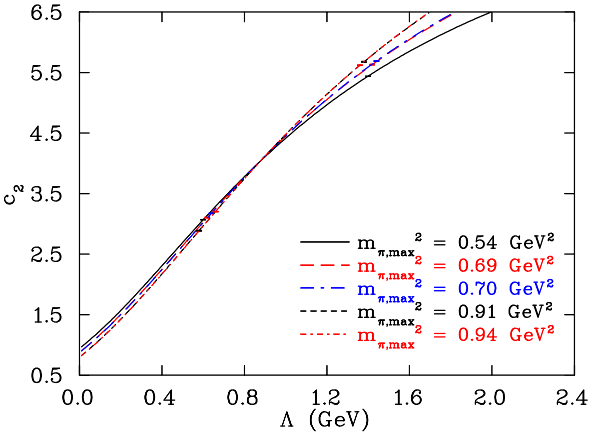

Evidence for an Intrinsic Scale

- There is a reasonably well-defined intersection point indicating the intrinsic scale.
- For each regulator, the intersection occurs at the same value of Λ for both c_0 and c_2. This is a highly significant result.
- The value of the intrinsic scale differs between regulator types. The regulators are different shapes and a different cutoff is required to achieve a similar suppression of the large loop momenta.
- To obtain a systematic uncertainty in the intrinsic scale, apply a kind of χ^2_{dof} analysis...

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- On each of these renormalization flow plots, different curves correspond to different values of $m^2_{\pi,\max}$.
- To what extent do the curves match?
- Construct χ^2_{dof}, where dof equals the number of $m^2_{\pi,\max}$ values:

$$\chi^2_{dof} = \frac{1}{n-1}\sum_{i=1}^{n}\frac{(c_i(\Lambda) - c^{\text{av}}(\Lambda))^2}{(\delta c_i(\Lambda))^2},$$

$$\text{where } c^{\text{av}}(\Lambda) = \frac{\sum_{i=1}^{n} c_i(\Lambda)/(\delta c_i(\Lambda))^2}{\sum_{j=1}^{n} 1/(\delta c_j(\Lambda))^2}.$$

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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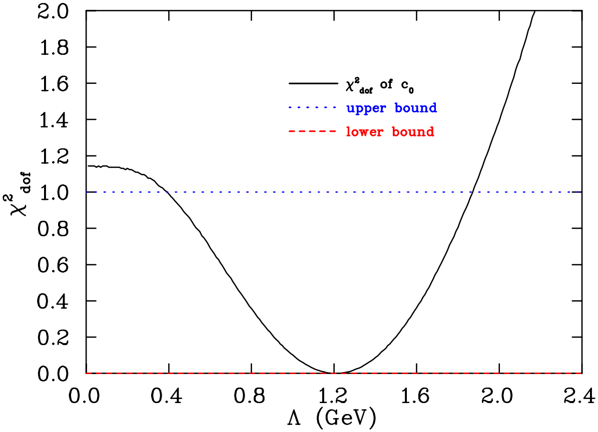
Statistical Uncertainty

- Example plot: here is the result for χ^2_{dof} obtained from c_0 using PACS-CS data, working to chiral order $\mathcal{O}(m_\pi^3)$ and using a dipole regulator:



Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

Results

- The central values of Λ (GeV) are tabulated below:

	regulator form		
optimal scale	dipole	double	triple
$\Lambda^{\text{scale}}_{c_0,\text{JLQCD}}$	1.44	1.08	0.96
$\Lambda^{\text{scale}}_{c_2,\text{JLQCD}}$	1.40	1.05	0.94
$\Lambda^{\text{scale}}_{c_0,\text{PACS-CS}}$	1.21	0.93	0.83
$\Lambda^{\text{scale}}_{c_2,\text{PACS-CS}}$	1.21	0.93	0.83
$\Lambda^{\text{scale}}_{c_0,\text{CP-PACS}}$	1.20	0.98	0.88
$\Lambda^{\text{scale}}_{c_2,\text{CP-PACS}}$	1.19	0.97	0.87

Higher Chiral Order

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- We found strong scheme-dependence when working to chiral order $\mathcal{O}(m_\pi^3)$ outside the PCR.
- What happens if we try the higher chiral order $\mathcal{O}(m_\pi^4 \log m_\pi)$?

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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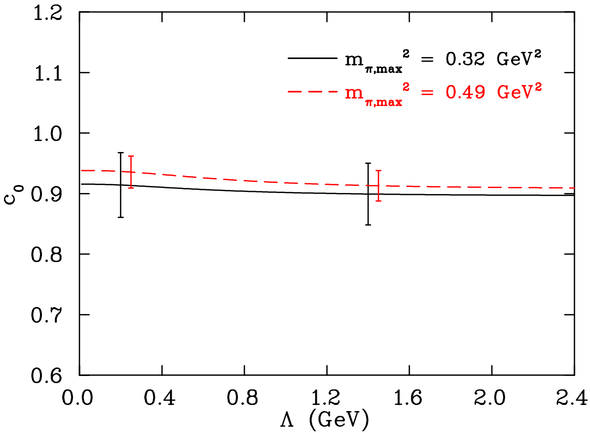

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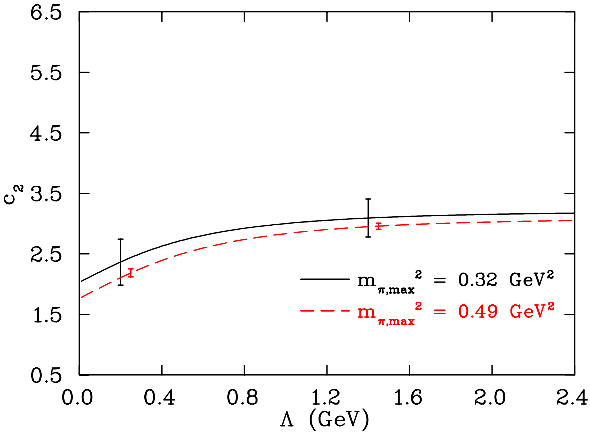

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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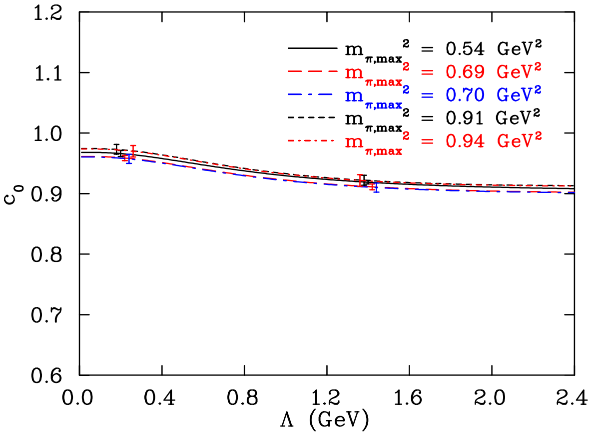

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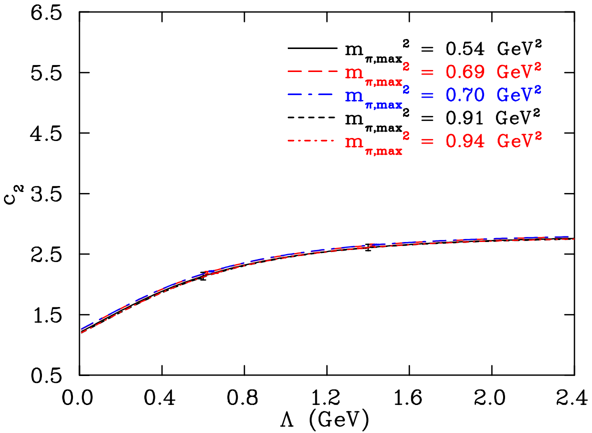

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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- At higher chiral order, there are no clear intersection points. We are unable to identify an intrinsic scale.
- This means that the scheme-dependence is weakened by working to higher chiral order.
- This systematic error in c_0 and c_2 is larger than their statistical errors, thus indicating that the data is outside the PCR.
- There are now at least two ways of assessing the systematic uncertainty in Λ:
  - from the χ^2_{dof} analysis at $\mathcal{O}(m_\pi^3)$,
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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Summary for the Nucleon

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- We are now able to extrapolate $M_{N,\text{phys}}$ and obtain c_0 and c_2 by using FRR χEFT and selecting the intrinsic scale.
- We are also able to provide a realistic systematic error in the result.
- Examples using the dipole regulator, with uncertainties (stat)(sys- # of points)(sys- Λ):
  - $c_0^{\text{PACS-CS}} = 0.900(51)(15)(6)$ (GeV),
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- re: PACS-CS data uses non-perturbatively $\mathcal{O}(a)$-improved Wilson quark action at $L = 2.9$ fm.

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- We are now able to extrapolate $M_{N,\text{phys}}$ and obtain c_0 and c_2 by using FRR χEFT and selecting the intrinsic scale.
- We are also able to provide a realistic systematic error in the result.
- Examples using the dipole regulator, with uncertainties (stat)(sys- $\#$ of points)(sys- Λ):
  - $c_0^{\text{PACS-CS}} = 0.900(51)(15)(6)$ (GeV),
  - $c_2^{\text{PACS-CS}} = 3.06(32)(15)(25)$ (GeV^{-1}),
  - $M_{N,\text{phys}}^{\text{PACS-CS}} = 0.967(45)(43)(3)$ (GeV).
- re: PACS-CS data uses non-perturbatively $\mathcal{O}(a)$-improved Wilson quark action at $L = 2.9$ fm.

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

'The Challenge'

- Consider the quenched ρ meson.
- We want to predict the mass of the quenched ρ meson at physical pion mass ($m_{\pi,\text{phys}} = 140$ MeV)
- We have quenched lattice QCD (QQCD) results from the Kentucky Group, but we are blinded to the lowest energy data.
- QQCD observables are an important testing ground, since there are no experimentally known values that can introduce a prejudice in the final result.

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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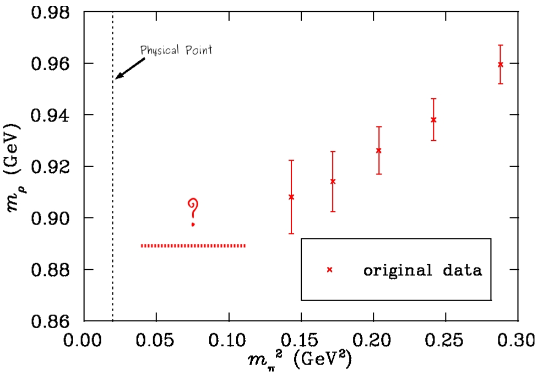
QQCD Data from the Lattice

- The following data from Kentucky Group ($L = 3.06$ fm) are missing points close to the chiral limit ($m_q = 0$).
- The available data lie in the range $380 < m_\pi < 720$ MeV,
- The unavailable data lie in the range $200 < m_\pi < 380$ MeV.



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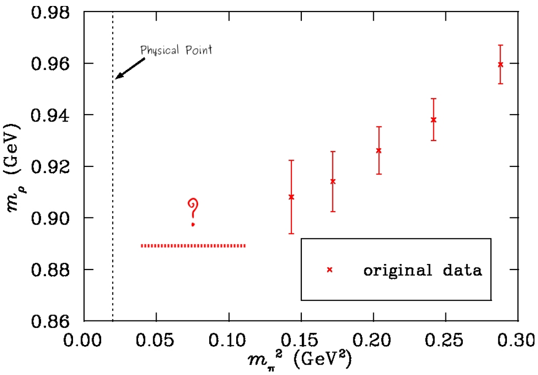

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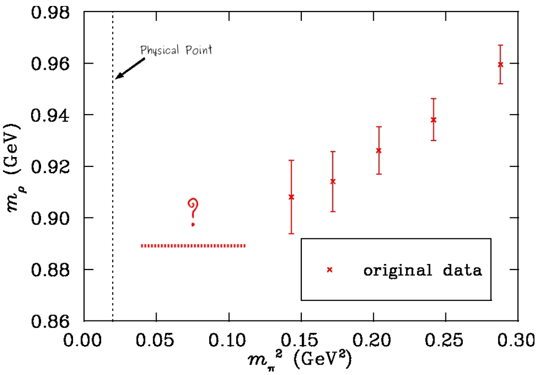

Chiral Extrapolation Formulae

- The quenched ρ meson mass $m_{\rho,Q}$ has a similar chiral expansion to the nucleon.
- The expansion similarly contains a residual series and loop integrals. We will work to chiral order $\mathcal{O}(m_\pi^4)$.
- The renormalization of the low energy constants takes place just as before. The fit parameters are c_0, c_2 and c_4.
- We can generate some pseudodata as before, and plot some renormalization flow curves.

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Chiral Extrapolation Formulae

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Finite-Range Regulators

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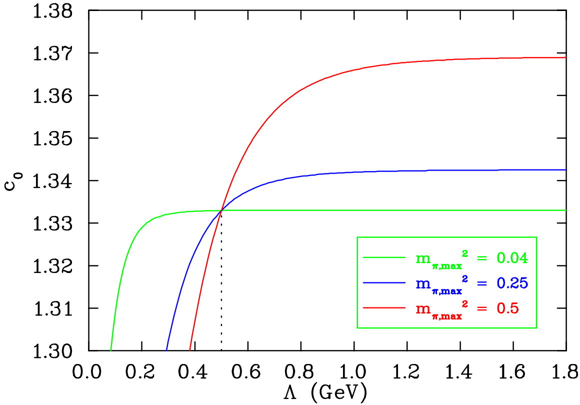
Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- Firstly, try pseudodata created at $\Lambda_\theta^c = 0.5$ GeV using a step function regulator ($u^2(k;\Lambda) = \theta(\Lambda - k)$).
- Analyze c_0:



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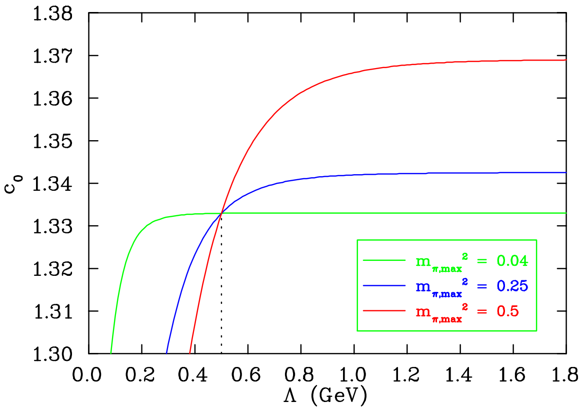
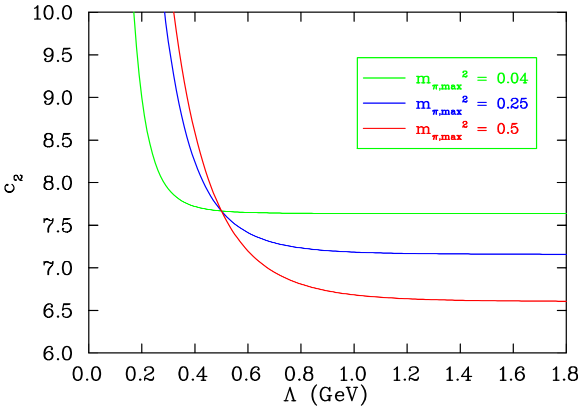
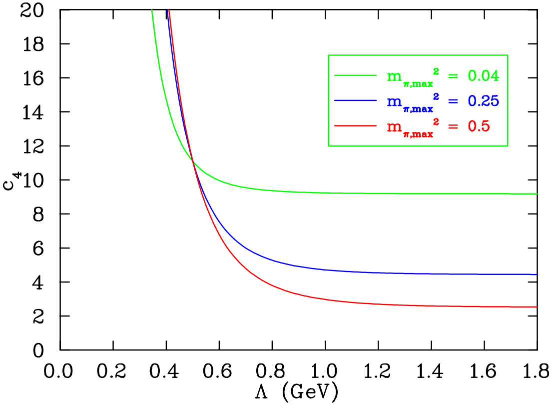


- Analyze c_2:

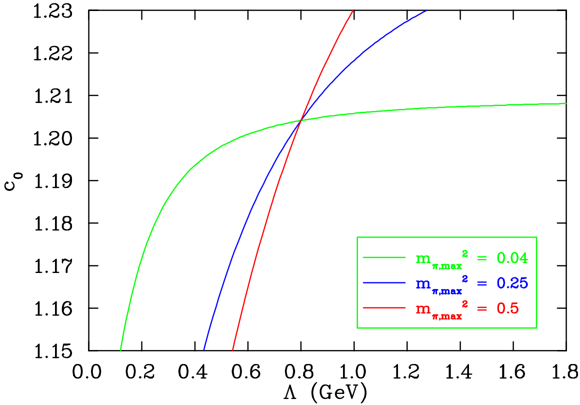

10.0
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
c2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
Λ (GeV)
m π,max 2 = 0.04
m π,max 2 = 0.25
m π,max 2 = 0.5

- Analyze c_4. Notice the chiral series truncation effect.

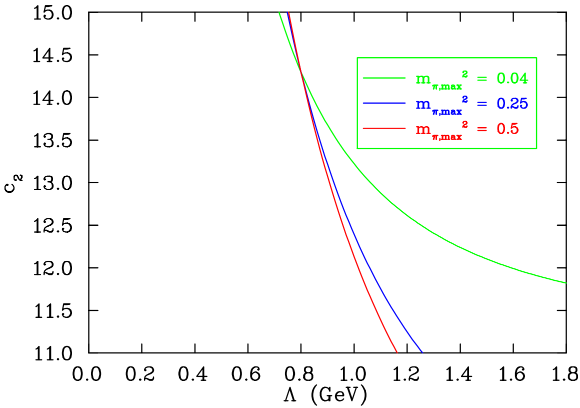


Finite-Range Regulators

- Now let's check to see if results are regulator independent.
- Consider pseudodata created using the dipole regulator, with $\Lambda_c = 0.8$ GeV. Analyze c_0:



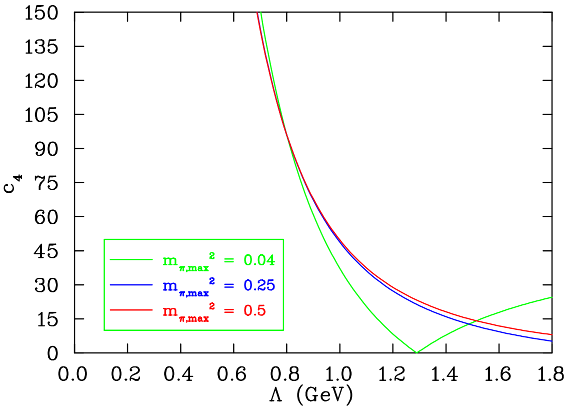
- Analyze c_2:



15.0
14.5
14.0
13.5
13.0
12.5
12.0
11.5
11.0
c_2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
Λ (GeV)
$m_{\pi,max}^2$ = 0.04
$m_{\pi,max}^2$ = 0.25
$m_{\pi,max}^2$ = 0.5

Finite-Range Regulators

- c_4 is also problematic.



Finite-Range Regulators

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- The dipole regulator renormalization procedure was unsuccessful.
- There are scheme-dependent extra non-analytic terms in the chiral expansion that have not been provided for in the fit. Pulling out the explicit Λ-dependence:

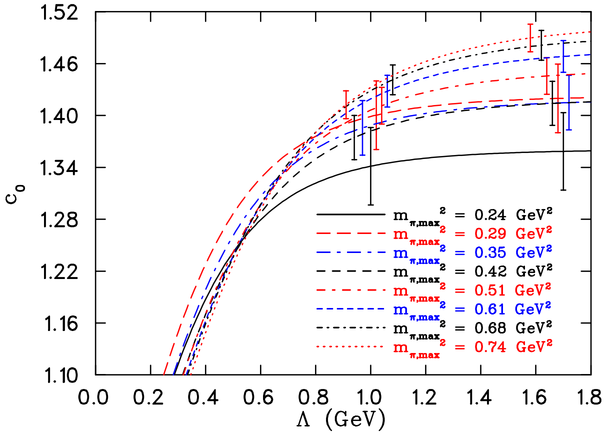
$$\tilde{\Sigma}^{Q}_{\eta'\eta'} = \chi_{\eta'!\eta'} m_\pi + \frac{b_3^{\eta'\eta'}}{\Lambda^2} m_\pi^3 + \frac{b_5^{\eta'\eta'}}{\Lambda^4} m_\pi^5 + \mathcal{O}(m_\pi^6)\,,$$

$$\tilde{\Sigma}^{Q}_{\eta'} = \chi_{\eta'} m_\pi^3 + \frac{b_5^{\eta'}}{\Lambda^2} m_\pi^5 + \mathcal{O}(m_\pi^6)\,.$$

Test for an Intrinsic Scale

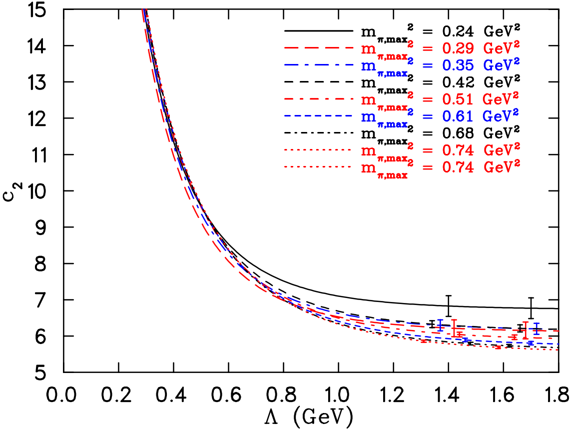
- Choice: We could use an a_3 and an a_5 parameter to contain the contribution from these terms, or:
- Better: choose a regulator which eliminates these extra terms to finite order.
- The triple dipole regulator is sufficient to suppress the $m_\pi^{3,5}$ terms.
- We shall use it exclusively from now on.

- Here is the result for c_0 using Kentucky Group data, working to chiral order $\mathcal{O}(m_\pi^4)$ and using a triple dipole regulator:



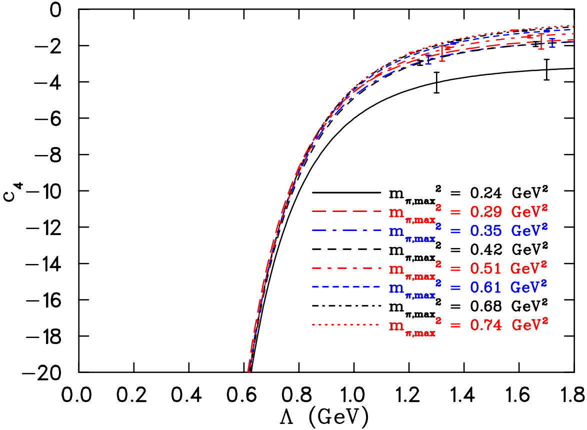
Test for an Intrinsic Scale

- Here is the result for c_2 using Kentucky Group data, working to chiral order $\mathcal{O}(m_\pi^4)$ and using a triple dipole regulator:



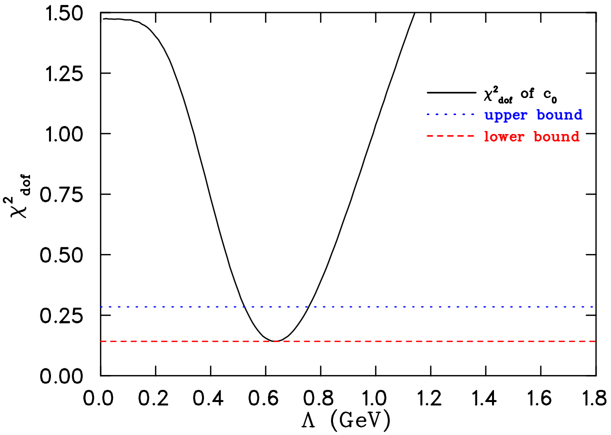
Test for an Intrinsic Scale

- Here is the result for c_4 using Kentucky Group data, working to chiral order $\mathcal{O}(m_\pi^4)$ and using a triple dipole regulator:



Test for an Intrinsic Scale

- The crossings are much harder to identify, so we will rely on our χ^2_{dof} method.
- Here is the result for χ^2_{dof} obtained from the same c_0:



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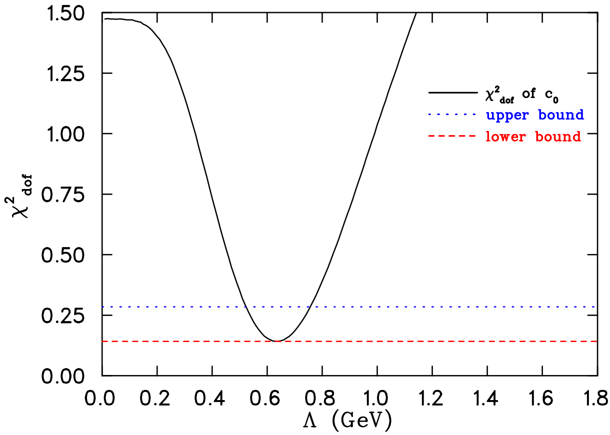

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- The central, upper and lower values of Λ (GeV) are tabulated below:

scale (GeV)	for c_0	for c_2	for c_4
Λ_{central}	0.64	0.64	0.64
Λ_{upper}	0.76	0.70	0.68
Λ_{lower}	0.52	0.58	0.59

The Intrinsic Scale

- By averaging the result for the central value, the upper and the lower limits among c_0, c_2, and c_4, the optimal regulator scale $\Lambda^{\text{scale}}_{\text{trip}}$ for the quenched ρ meson mass can be calculated for this data set.
- Using the triple dipole regulator, $\Lambda^{\text{scale}}_{\text{trip}} = 0.64$ GeV $(+0.08 - 0.07)$ GeV.

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- The extrapolation of the quenched ρ meson mass can now be completed.
- Treating the various coupling constants and $\Lambda^{\text{scale}}_{\text{trip}}$ as independent, their errors can be added in quadrature.
- We shall plot an inner error bar corresponding to the systematic error coming from the choice in parameters only.
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Completing 'The Challenge'

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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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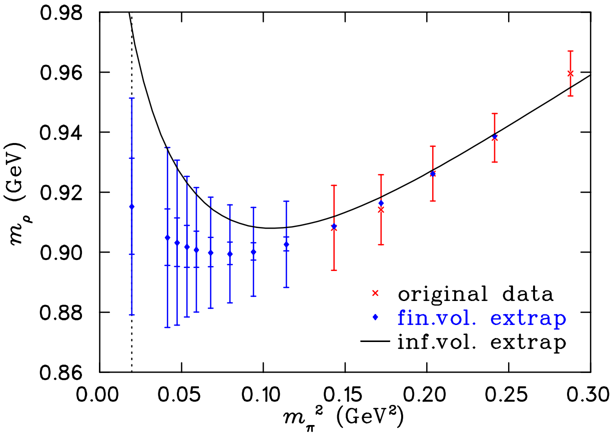
- Here is the result of the extrapolation, filling in for the missing Kentucky Group data points.
- At the physical point, we find $m_{\rho,Q}(m^2_{\pi,\text{phys}}) = 0.915$ GeV (± 0.036) GeV, an error just under 4%.



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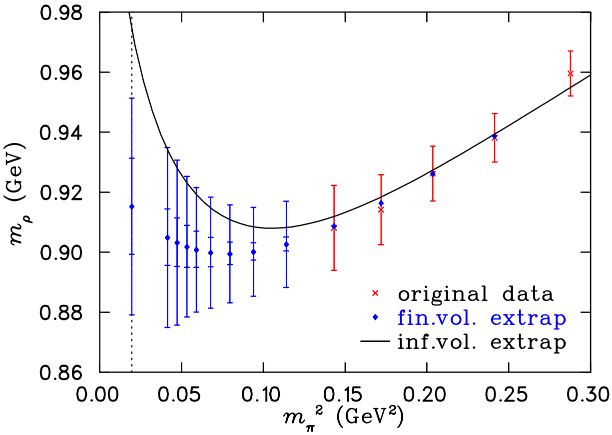
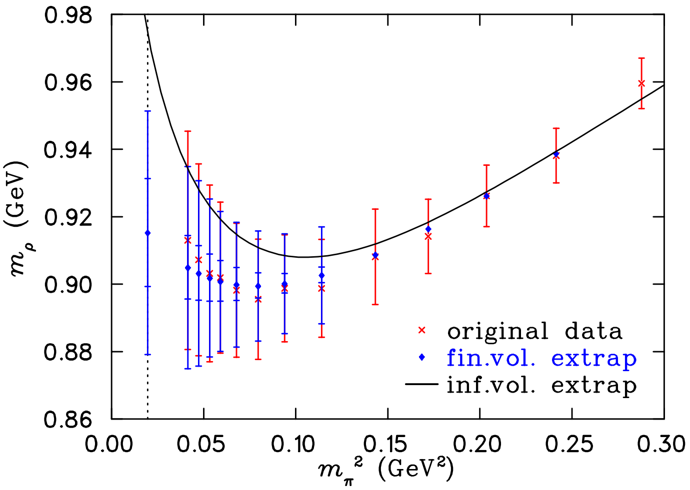
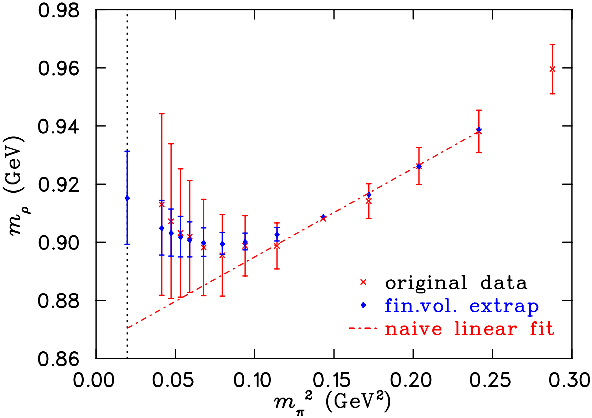


Completing 'The Challenge'

- Now, the lattice results are added to the plot:



- Here, the error bars are correlated relative to the lightest data point in the original set, $m_\pi^2 = 0.143$ GeV^2.
- Our extrapolation error bars are smaller than for the numerically evaluated data.



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Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

Completing 'The Challenge'

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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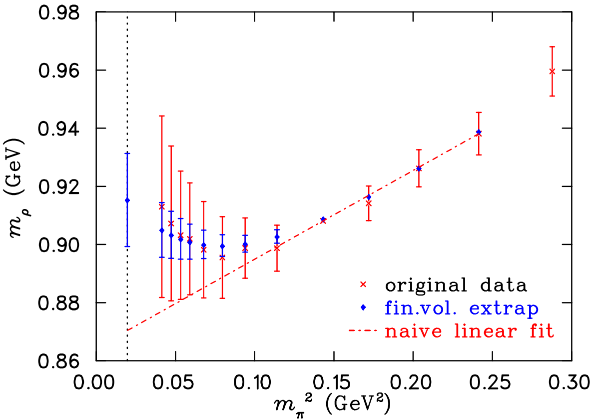

Conclusion

- We have been able to extrapolate current lattice QCD results to the physical point, using Chiral Effective Field Theory.
- We have discovered that Finite-Range Regularization is instrumental for the analysis of data extending outside the chiral Power Counting Regime.
- We have developed a robust procedure for quantifying the degree of scheme-dependence, through the search for an intrinsic scale Λ^{scale}.
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Conclusion

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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Future Directions

Overview
Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

- An alternative technique for propagation of uncertainty in the scale-dependence would be to consider marginalization of the scale.
- The extrapolation scheme can be applied to other observables such as magnetic moment and charge radii of octet baryons, which have large chiral curvature.
- Finite volume corrections are of particular interest when considering such observables.
- The extrapolation scheme will also be useful for calculating the Roper resonance, which is difficult to evaluate in lattice QCD.

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Introduction
EFT for Nucleons
Pseudodata
Intrinsic Scale
Quenched ρ Meson
Conclusion

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