Review of Hadronic Structure in Lattice QCD

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Lattice 2014

lattice 14

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Thanks for the material received from:

- Constantia Alexandrou (ETMC)
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- Giannis Koutsou (ETMC)
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- Keh-Fei Liu (χQCD)
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- Shigami Ohta (RBC/UKQCD)
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- Thomas Rae (Mainz Group)
- Phiala Shanahan (CSSM)
- Carsten Urbach (ETMC)
- Yi-Bo Yang (χQCD)
- James Zanotti (QCDSF/UKQCD, CSSM)

OUTLINE

A Nucleon Sector

- Axial charge
- Electromagnetic form factors
- Dirac & Pauli radii
- Quark momentum fraction
- Nucleon Spin

B Hyperon Form Factors

- Hyperon EM form factors
- Axial form factors

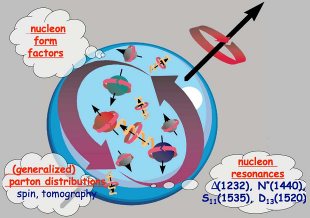
C Mesons

- Pion momentum fraction
- ρ-meson EM form factors

D Conclusions

LQCD meets Nature

LQCD:
- estimates for experimentally well known quantities
- input for not well known quantities



Rich experimental activities in major facilities: JLab, MAMI, MESA, etc

- Investigation of baryon and meson structure
- Origin of mass and spin
- New physics searches: $(g-2)_\mu$, dark photon searches
- proton radius puzzle
- the list is long...

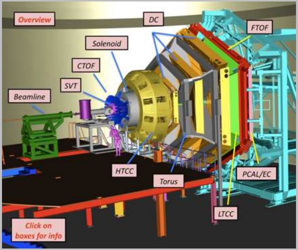
Proton Radius Puzzle

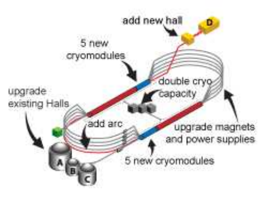
$< r_p^2 >$ from muonic hydrogen μp 7.7σ smaller than elastic $e - p$ scattering

[R. Pohl et al. Nature 466, 213-217 (2010)]

- measured energy difference between the 2P and 2S states of muonic hydrogen
- μp: 10 times more accurate than other measurements
- very sensitive to the proton size
- no obvious way to connect with other measurements (4$\%$ diff)

12GeV Upgrade at JLab

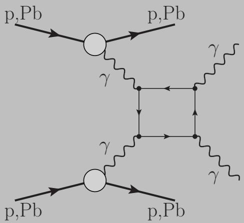




Physics Program for CLAS12 (Selected Hadron Experiments)

- The Longitudinal Spin Structure of the Nucleon
- Nucleon Resonance Studies with CLAS12
- Meson spectroscopy with low Q^2 electron scattering
- High Precision Measurement of the Proton Charge Radius
- and many more....

Light-by-Light scattering at LHC



[D. d' Enterria and G. G. Silveira, arXiv:1305.7142]

- Never observed directly
- Indirectly observed by its effects on anomalous magnetic moments of electrons and muons
- Photon-photon collisions in ultraperipheral collisions of proton have been detected
- arXiv:1305.7142: LCH could detect LbyL (5.5-14 TeV) due to:
- 'quasireal' photons fluxes in EM interactions of protons and lead ions

A

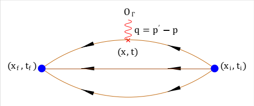
NUCLEON

SECTOR

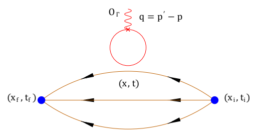
Nucleon on the Lattice in a nutshell

LQCD:
- estimates for experimentally well known quantities
- input for not well known quantities

- **Contributing diagrams:**



Connected



Disconnected

- **Computation of 2pt- and 3pt-functions:**

$$\text{2pt}: \quad G(\vec{q}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f \cdot \vec{q}}\, \mathbf{\Gamma}^{\mathbf{0}}_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) \overline{J}_\beta(0) \rangle$$

$$\Gamma^0 \equiv \tfrac{1}{4}(1 + \gamma_0)$$

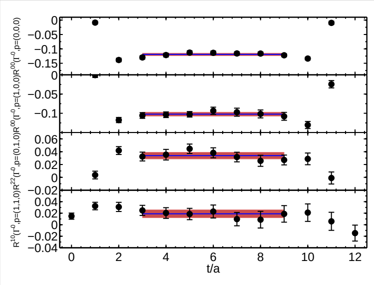
$$\Gamma^2 \equiv \Gamma^0 \cdot \gamma_5 \cdot \gamma_i$$

and other variations

$$\text{3pt}: \quad G_{\mathcal{O}}(\mathbf{\Gamma}^\kappa, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x} \cdot \vec{q}} e^{-i\vec{x}_f \cdot \vec{p}'}\, \mathbf{\Gamma}^\kappa_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) \mathcal{O}(\tilde{\mathbf{x}}, \mathbf{t}) \overline{J}_\beta(0) \rangle$$

★ Construction of optimized ratio:

$$R_{\mathcal{O}}(\Gamma, \vec{q}, t) = \frac{G_{\mathcal{O}}(\Gamma, \vec{q}, t)}{G(\vec{0}, t_f)} \times \sqrt{\frac{G(-\vec{q}, t_f - t) G(\vec{0}, t) G(\vec{0}, t_f)}{G(\vec{0}, t_f - t) G(-\vec{q}, t) G(-\vec{q}, t_f)}} \xrightarrow[t - t_i \to \infty]{t_f - t \to \infty} \Pi(\Gamma, \vec{q})$$



★ Plateau Method: Most common method

★ Renormalization: connection to experiments

$$\Pi^R(\Gamma, \vec{q}) = Z_{\mathcal{O}} \, \Pi(\Gamma, \vec{q})$$

★ Extraction of form factors

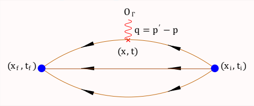
e.g. Axial current:

$$A_\mu^3 \equiv \bar{\psi} \gamma_\mu \gamma_5 \frac{\tau^3}{2} \psi \Rightarrow \bar{u}_N(p') \left[\mathbf{G_A(q^2)} \gamma_\mu \gamma_5 + \mathbf{G_p(q^2)} \frac{q_\mu \gamma_5}{2 m_N} \right] u_N(p)$$

Isovector Combination: (u-d)

★ disconnected contributions cancel out

★ Simpler renormalization

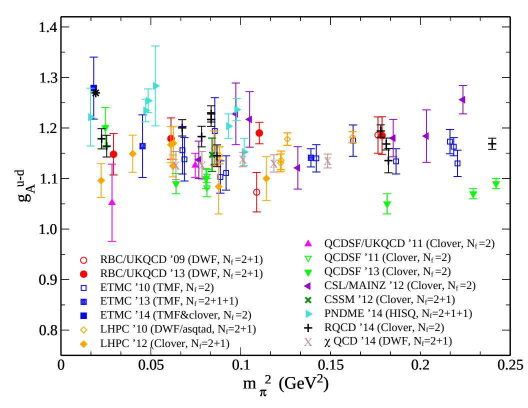


A1. NUCLEON AXIAL CHARGE

The chosen one

Axial current: $\bar{\psi}\,\gamma_\mu\,\gamma_5\,\frac{\tau^3}{2}\,\psi$ $\quad g_A \equiv G_A(0)$

- $g_A^{exp} = 1.2701(25)$ [PRD'12]
- governs the rate of β-decay
- determined directly from lattice data (no fit necessary)
- $m_\pi > 200$MeV: lattice results below exp.: $\sim$10-15%



★ Lattice data from 'plateau' methods

★ Latest achievement: lattice results at physical m_π

★ No necessity of chiral extrapolation

★ Different strategies for addressing systematic uncertainties

Selected Works:

- T. Yamazaki et al. (RBC/UKQCD,) [arXiv:0801.4016]
- T. Yamazaki(RBC/UKQCD), [arXiv:0904.2039]
- J.D. Bratt et at. (LHPC), [arXiv:1001.3620]
- C. Alexandrou et al. (ETMC), [arXiv:1012.0857]
- S. Collins et al. (QCDSF/UKQCD), [arXiv:1101.2326]
- B.B. Brandt et al. (CLS/MAINZ), [arXiv:1106.1554]
- G.S. Bali et al. (QCDSF), [arXiv:1112.3354]
- S. Capitani et al. (CLS/MAINZ), [arXiv:1205.0180]
- J.R. Green et al. (LHPC), [arXiv:1209.1687]
- J.R. Green et al. (LHPC), [arXiv:1211.0253]
- B.J. Owen et al. (CSSM), [arXiv:1212.4668]
- R. Horsley et al. (QCDSF), [arXiv:1302.2233]
- C. Alexandrou et al. (ETMC), [arXiv:1303.5979]
- T. Bhattacharya et al. (PNDME), [arXiv:1306.5435]
- S. Ohta et al. (RBC/UKQCD), [arXiv:1309.7942]
- G.S. Bali et al. (RQCD), [arXiv:1311.7041]
- A.J. Chambers et al. (QCDSF/UKQCD), [arXiv:1405.3019]

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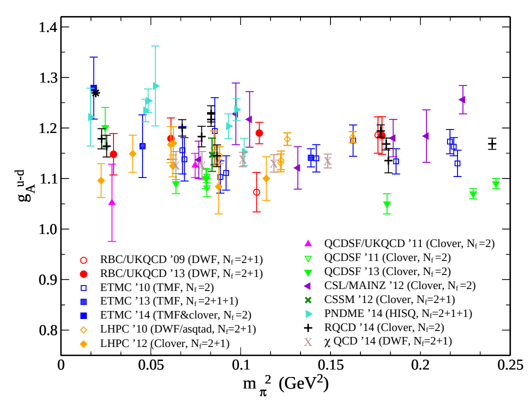


★ Lattice data from 'plateau' methods

★ Latest achievement: lattice results at physical m_π

★ No necessity of chiral extrapolation

★ Different strategies for addressing systematic uncertainties

Possible origin of systematics

→ **Cut-off Effects**

→ **Excited State Contamination**

- adjustment of source-sink separation
- 2-state fit
- summation method

→ **Finite Volume Effects**

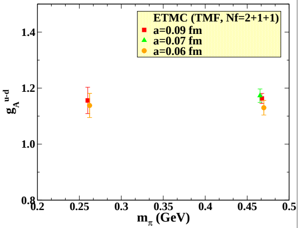
Investigation of volume effects as lattice box increases

→ **not being at the physical point**

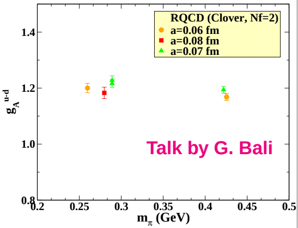
Cut-off effects

- Continuum extrapolation requires 3 lattice spacings

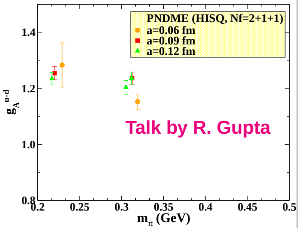
[C. Alexandrou et al. (ETMC), arXiv:1012.0857]



[G. Bali et al. (RQCD), 2014]



[R. Gupta et al. (PNDME), 2014]



1st Conclusion: $a < 0.1$ fm is sufficient

Excited State Contamination

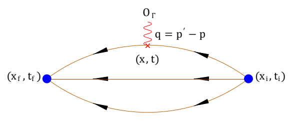
Plateau Method: single-state fit

$$R(t_i, t, t_f) \xrightarrow[(t-t_i)\,\Delta >> 1]{(t_f - t)\,\Delta >> 1} \mathcal{M}\left[1 + \alpha\, e^{-(t_f - t)\,\Delta(p'))} + \beta\, e^{-(t - t_i)\,\Delta(p))} + \dots\right]$$

$t_i\ :\ t_{\text{source}}$
$t\ :\ t_{\text{insersion}}$
$t\ :\ t_{\text{sink}}$
$T_{\text{sink}} \equiv t_f - t_i$

2-state fit on 3pt-functions

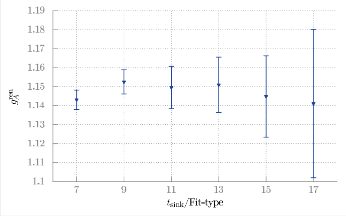
$$\begin{aligned}\mathcal{C}_\Gamma^{(3),T}(t_i, t, t_f; \vec{p}_i, \vec{p}_f) \approx\ & |\mathcal{A}_0|^2 \langle 0|O_\Gamma|0\rangle e^{-M_0\, T_{\text{sink}}} + \\ & |\mathcal{A}_1|^2 \langle 1|O_\Gamma|1\rangle e^{-M_1\, T_{\text{sink}}} + \\ & \mathcal{A}_0 \mathcal{A}_1^* \langle 0|O_\Gamma|1\rangle e^{-M_0(t-t_i)} e^{-M_1(t_f-t)} + \\ & \mathcal{A}_0^* \mathcal{A}_1 \langle 1|O_\Gamma|0\rangle e^{-M_1(t-t_i)} e^{-M_0(t_f-t)}\end{aligned}$$



Summation Method

$$\sum_{t=t_i}^{t_f} R(t_i, t, t_f) = \text{const.} + \mathcal{M}\, T_{\text{sink}} + \mathcal{O}\left(e^{-(T_{\text{sink}}\,\Delta(p'))}\right) + \mathcal{O}\left(e^{-(T_{\text{sink}}\,\Delta(p))}\right)$$

- suppressed excited states (exponentials decaying with T_{sink})
- Matrix element extracted from the slope
- Alternatively: sum over $t_i + 1 \leq t \leq t_f - 1$

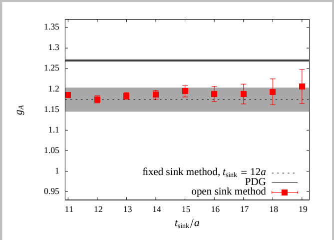


❶ Plateau Method: single-state

⇐ **RQCD (2014):**

[G.Bali et al. (RQCD), 2014]

- m_π=285MeV
- g_A not sensitive on T_{sink}: 0.49-1.19 fm



ETMC (2013): ⇒

[S.Dinter et al. (ETMC), arXiv:1108.1076]

- m_π=373MeV
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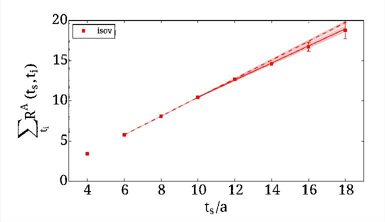
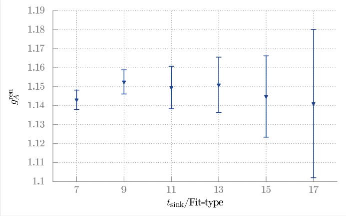


❷ Summation Method

⇐ **ETMC (2013):**

[S.Dinter et al. (ETMC), arXiv:1108.1076]

- m_π=373MeV
- T_{sink}: 0.3 fm-1.3 fm
- No curvature is seen in slope
- No detectable excited states



❶ Plateau Method: single-state

$\Leftarrow$ RQCD (2014):

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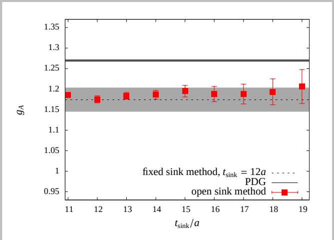

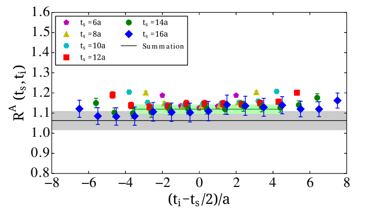

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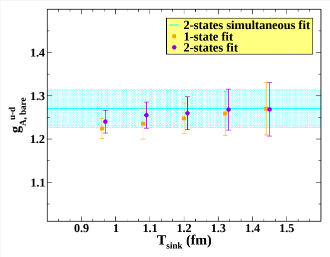
❸ Two-state fit on 3pt-functions



⇐ **PNDME (2013):**

[T. Bhattacharya (PNDME), arXiv:1306.5435]

- m_π=310MeV
- Largest difference for $T_{\text{sink}} < 1$ fm
- All fits in agreement

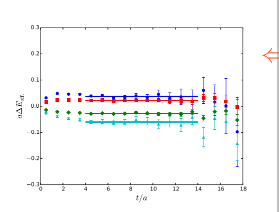
2nd Conclusion: $T_{\text{sink}} > 1$ fm safe*

* **based on $m_\pi\rangle$300MeV**

❹ Feynman-Hellmann Approach:

$$S \to S(\lambda) = S + \lambda \sum_x \overline{q}(x) i\gamma_5\gamma_3 q(x)$$

$$\Delta q = \left.\frac{\partial E(\lambda)}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2M}\langle N|\overline{q} i\gamma_5\gamma_3 q|N\rangle$$



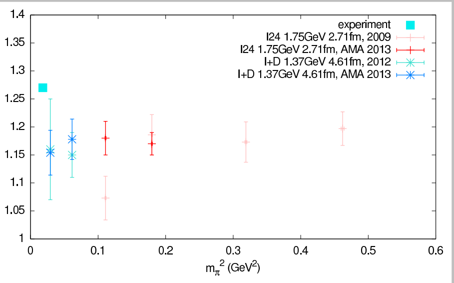
⇐ **CSSM/QCDSF/UKQCD (2014):**

[A.J.Chambers et al., arXiv:1405.3019]

- m_π=470MeV

Talk by J. Zanotti

- External spin operator in S_{fermion}
- Δq: linear response of nucleon energies
- Statistical Precision



RBC/UKQCD (2014): DWF $N_f = 2+1$

- A factor of 20 improvement in computational efficiency
- A sloppy calculation costs $\sim$1/65 of an exact calculation
- the speedup with AMA: $\sim$15-29 times

Talk by S.Ohta

Improvement Technique: All-Mode-Avaraging (AMA)

[E.Shintani et. al. arXiv:1402.0244]

signal/noise $\sim \sqrt{(N_{\text{meas}})} \times e^{-(m_N + 3\, m_\pi/2)}$

- ★ Reduction of statistical error for a given number of gauge configurations
- ★ Significant increase of N_{meas} at low computational cost
- ★ Improved operator:

$$\langle \mathcal{O}^{\text{impr}} \rangle = \langle \mathcal{O}^{\text{approx}} \rangle + \langle \mathcal{O}^{\text{rest}} \rangle$$

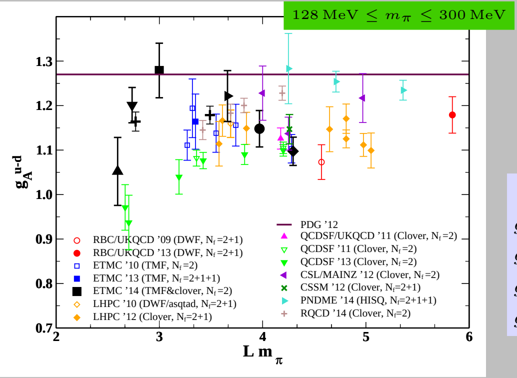
$\mathcal{O}^{\text{approx}}$: not precise but cheap
$\mathcal{O}^{\text{rest}}$: correction term

$$\mathcal{O}^{\text{rest}} = \mathcal{O}^{\text{exact}} - \mathcal{O}^{\text{approx}}$$

- ★ AMA result:

$$O_{\text{AMA}} = \frac{1}{N_{\text{apprx}}} \sum_{i=1}^{N_{\text{apprx}}} O^i_{\text{apprx}} + \frac{1}{N_{\text{exact}}} \sum_{j=1}^{N_{\text{exact}}} \left(O^j_{\text{exact}} - O^j_{\text{apprx}} \right)$$

Finite Volume Effects



Lattice data for plateau method

No volume corrections

Systematics not fully understood:

$g_A^{PNDME}(L\, m_\pi \sim 3.7)$ agrees with g_A^{exp}

$g_A^{ETMC}(L\, m_\pi \sim 3)$ agrees with g_A^{exp}

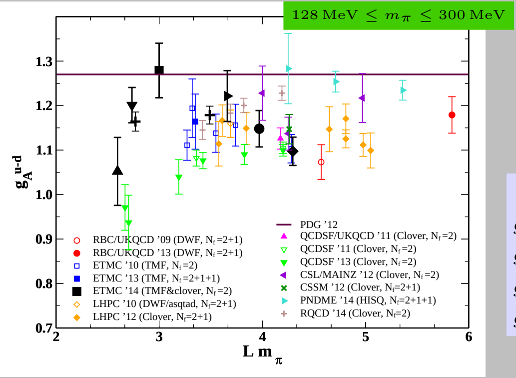
$g_A^{QCDSF}(L\, m_\pi \sim 2.7)$ close to g_A^{exp}

$g_A^{LHPC}(L\, m_\pi \sim 4)$ lower than g_A^{exp}

- **PNDME** (m_π=128MeV) : L_s=5.76 fm, a=0.09 fm
- **ETMC** (m_π=135MeV) : L_s=4.37 fm, a=0.091 fm
- **LHPC** (m_π=149MeV) : L_s=5.57 fm, a=0.116 fm
- **RQCD** (m_π=150/157MeV) : L_s=4.48/3.36 fm, a=0.07 fm
- **QCDSF** (m_π=158MeV) : L_s=3.41 fm, a=0.071 fm
- **QCDSF/UKQCD** (m_π=170MeV) : L_s=3.36 fm, a=0.07 fm
- **RBC** (m_π=170MeV) : L_s=4.6 fm, a=0.141 fm

[S. Collins et al. (QCDSF/UKQCD), arXiv:1101.2326]:
'Simulations in the region $L\, m_\pi > 3$ are expected to have sufficiently small finite size effects'

Finite Volume Effects



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Volume effects still unclear

[S. Collins et al. (QCDSF/UKQCD), arXiv:1101.2326]:
'Simulations in the region $L\,m_\pi > 3$ are expected to have sufficiently small finite size effects'

Axial Charge: Summary

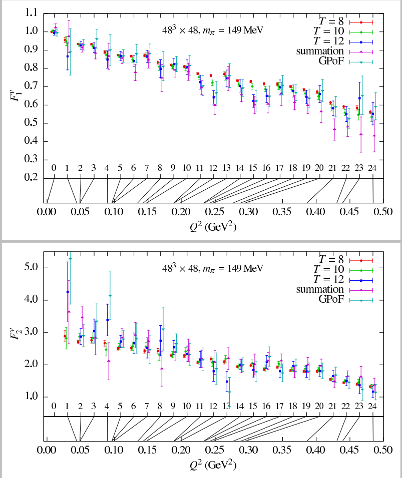
High statistical analyses to date reveal:

- **Cutoff effects small for:** $a < 0.1\,\mathrm{fm}$
- **No excited states for:** $T_{\mathrm{sink}} > 1\,\mathrm{fm}$
- **Finite Volume effects:** $L\,m_\pi > 3$

A2. Nucleon EM form factors

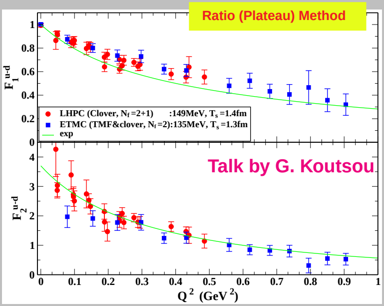
$$\langle N(p',s')|\gamma_\mu|N(p,s)\rangle \sim \bar{u}_N(p',s')\left[\mathbf{F_1(q^2)}\gamma_\mu + \mathbf{F_2(q^2)}\frac{i\,\sigma^{\mu\rho}\,q_\rho}{2m_N}\right]u_N(p,s)$$

LHPC: $m_\pi = 149$MeV, $a = 0.116$fm, $\mathcal{O}(7\,800)$ stat.

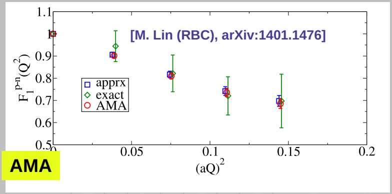


[J.R.Green et al. (LHPC), arXiv:1211.0253]

- Summation method goes either direction
- errors are large

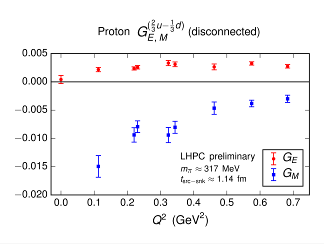


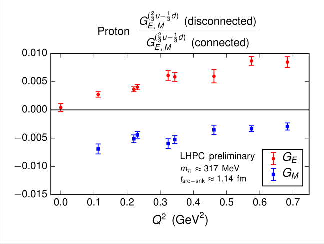
- **ETMC:** $a = 0.091$ fm, $L = 4.4$ fm, $L\,m_\pi = 3$
- **LHPC:** $a = 0.116$ fm, $L = 5.6$ fm, $L\,m_\pi = 4.2$



Disconnected Insertion

Sachs FFs: $G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$





[S.Meinel et al. (LHPC) 2014]

- Clover ($N_f=2+1$), $L=3.58$ fm
- $\mathcal{O}(100\,000)$ statistics
- $G_E^{\rm dis}$ increases $G_E^{\rm tot}$
- $G_M^{\rm dis}$ decreases $G_M^{\rm tot}$

Talk by S.Meinel

Quark loops with hierarchical probing [A.Stathopoulos et al., arXiv:1302.4018]

- **Gain depends on observable: for EM significant improvement**
- **Allows to increase the level of spatial dilution at any stage while reusing existing data**

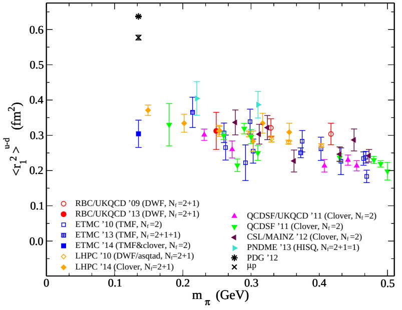
- **Improves the stochastic estimator** $\mathrm{Tr}[A^{-1}] = E\{z^\dagger A^{-1} z\}$ **(**z**: noise vector)**
- **deterministic orthonormal vectors (*Hadamard*)**
- **Optimal distance** k **for** $A_{i,j}^{-1} \approx 0$ **obtained using probing**
- **Recursive probing (results from level** $i-1$ **is used at level** i**)**
- **Multi coloring of sites is done hierarchically**
- **Bias is removed by using a random starting vector**
- **Up to factor of 10 speed up (**$32^3 \times 64$ **clover lattice)**

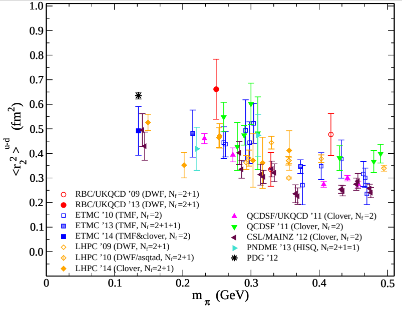
A3. Dirac & Pauli radii

$$F_i(Q^2) \sim F_i(0)\left(1 - \frac{1}{6}\,Q^2\langle \mathbf{r_i^2}\rangle + \mathcal{O}(Q^4)\right)$$

$$\langle \mathbf{r_i^2}\rangle = -\frac{6}{F_i(Q^2)}\,\frac{dF_i(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

$$F_i(Q^2) \sim \frac{F_i(0)}{\left(1+\frac{Q^2}{m_i^2}\right)^2} \quad\Rightarrow\quad \langle \mathbf{r_i^2}\rangle = \frac{12}{m_i^2}$$

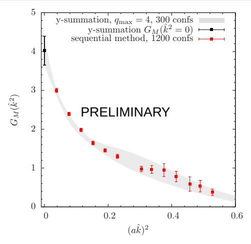




Lattice data for plateau method

- ★ **Estimation of radii strongly depends on small Q^2**
- ★ **Need access for momenta close to zero $\Rightarrow$**
- ★ **larger volumes**

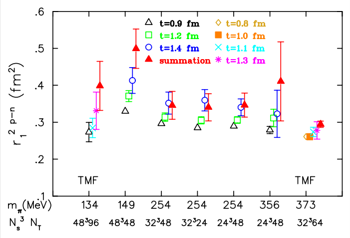
Avoid model dependence-fits:

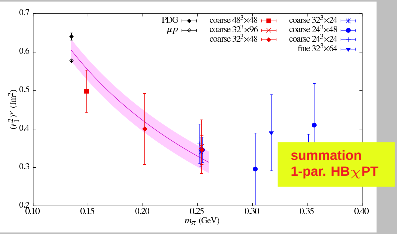


- Position space method

Poster by K.Ottnad (ETMC)

Systematic Effects





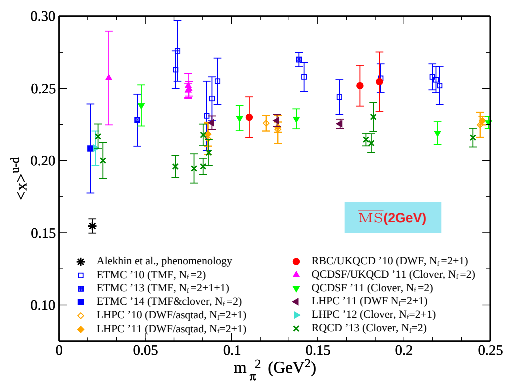
[J.R.Green et al. (LHPC), arXiv:1404.4029]

- ★ Upward tendency with increase of $T_{\rm sink}$
- ★ Summation agrees with larger $T_{\rm sink}$ value
- ★ Chiral extrapolation of summation method agrees with exp

A4. Quark Momentum Fraction

1-D Vector current: $\mathcal{O}^{\mu\nu} \equiv \bar{\psi}\,\gamma^{\{\mu}\,\overset{\leftrightarrow}{D}{}^{\nu\}}\psi \quad \Rightarrow \quad \mathbf{A_{20}(q^2),\ B_{20}(q^2),\ C_{20}(q^2)}$

$\langle x \rangle_q = A_{20}(0)$

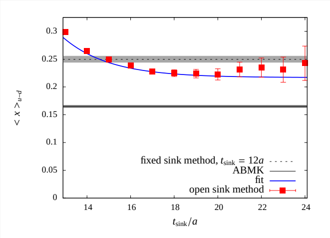


- Measured in DIS experiments. Value uses input from phen. models
- $\langle x \rangle^{\rm phen} = 0.1646(27)\ (\overline{\rm MS}(2{\rm GeV}))$ **[S. Alekhin et al., arXiv:0908.2766]**
- Scheme and scale dependence
- All lattice results overestimate phen. value
- Chiral behavior: $m_\pi^2\ \log(m_\pi^2)$

Excited States Investigation

TMF, $m_\pi = 373$MeV

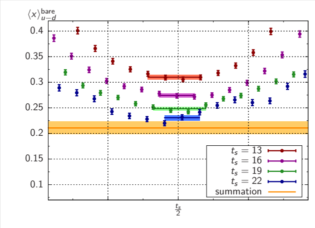
[S.Dinter et al. (ETMC), arXiv:1108.1076]



- $\mathcal{O}(23\,000)$ **measurements**
- $0.94\text{ fm} < T_{\text{sink}} < 1.87\text{ fm}$

Clover, $m_\pi = 340$MeV

[T.Rae et al.(Mainz Group), 2014]



- $\mathcal{O}(3\,800)$ **measurements**
- $0.6\text{ fm} < T_{\text{sink}} < 1.4\text{ fm}$

ALL WORKS AGREE:

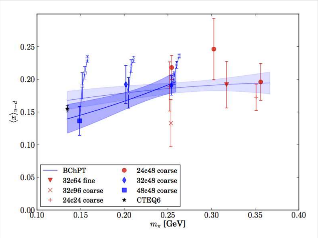
★ Contaminated by excited states

★ Convergence by varying T_{sink}

★ Downward shift

Clover, $m_\pi = 149$MeV

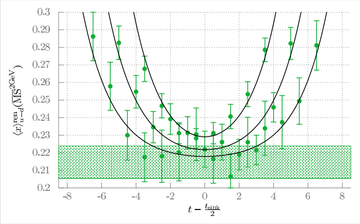
[J.R.Green et al. (LHPC), arXiv:1209.1687]



- $\mathcal{O}(7\,800)$ **measurements**
- $T_{\text{sink}} = 0.9,\ 1.2,\ 1.4$ **fm**

Clover, $m_\pi = 150$MeV

[G.S.Bali et al. (RQCD), 2014]



- $\mathcal{O}(2\,800)$ **measurements**
- $0.63\text{ fm} < T_{\text{sink}} < 1.05\text{ fm}$

Renormalization

RI' scheme:

$$Z_q = \frac{1}{12}\mathrm{Tr}\Big[\Big(S^L(p)\Big)^{-1}\ S^{\mathrm{Born}}(p)\Big]\Big|_{p^2=\bar{\mu}^2}$$

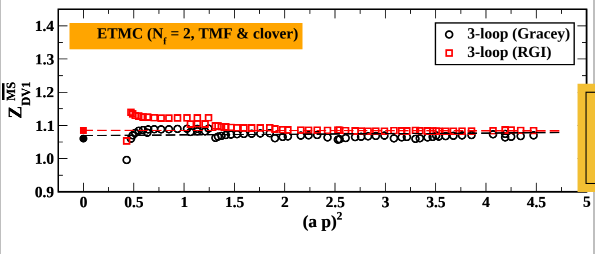
$$Z_q^{-1}\ Z_{\mathcal{O}}\ \frac{1}{12}\mathrm{Tr}\Big[\Gamma^L_{\mathcal{O}}(p)\ \Big(\Gamma^{\mathrm{Born}}_{\mathcal{O}}(p)\Big)^{-1}\Big]\Big|_{p^2=\bar{\mu}^2} = 1$$

★ **Tension between $Z_{\mathcal{O}}^{\mathrm{pert}}$ and $Z_{\mathcal{O}}^{\mathrm{non-pert}}$**

- up to 15%
- either direction

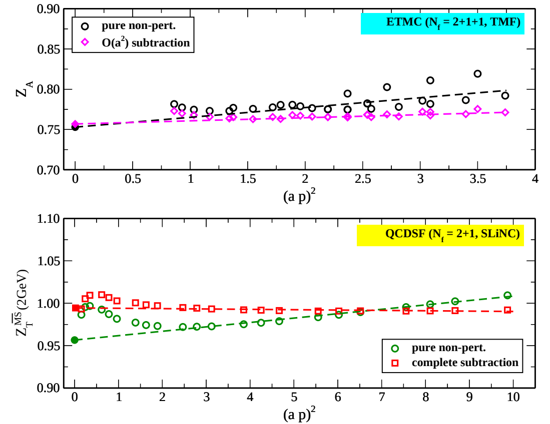
➥ **Non-perturbative renormalization**

★ **Conversion to $\overline{\mathrm{MS}}(\mu = 2\mathrm{GeV})$**



- Z_{DV1} : Z-factor of $\langle x\rangle_{u-d}$
- Systematic due to conversion insignificant

★ Lattice Artifacts



A. Subtraction of $\mathcal{O}(g^2\, a^2)$ perturbative terms

[C. Alexandrou et al. (ETMC), arXiv:1006.1920]

[M. Constantinou et al. (ETMC), arXiv:0907.0381]

B. Complete Subtraction of $\mathcal{O}(g^2)$ artifacts

[M. Constantinou et al. (QCDSF), 2014]

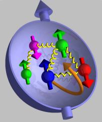
★ Usage of momentum-source method :

- Dirac equation solved with momentum source
- $\#$ of inversion depends on $\#$ of momenta considered
- Application of any operator
- High statistical accuracy

Control of lattice artifacts (non-Lorentz invariant):

$$\frac{\sum_\rho p_\rho^4}{\left(\sum_\rho p_\rho^2\right)^2} < 0.4$$

(empirically)



A5. Nucleon Spin

Quark orbital angular momentum

Spin Sum Rule:

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q \right) + J^G$$

Quark Spin

Extraction from LQCD:

$$J^q = \frac{1}{2}\left(A^q_{20} + B^q_{20}\right)\,, \quad L^q = J^q - \Sigma^q\,, \quad \Sigma^q = g^q_A$$

★ **Individual quark contributions $\Longrightarrow$ disconnected insertion contributes**

Renormalization of Disconnected Contributions

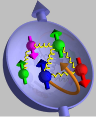
- Requirement of renormalization for the singlet operators
- $Z_{\mathcal{O}}^{\rm singlet}$ unknown non-perturbatively
- $Z_{\mathcal{O}}^{\rm s} - Z_{\mathcal{O}}^{\rm ns}$ first appears to 2 loops in perturbation theory
- Recent perturbative results for **[H.Panagopoulos et al. (Cyprus Group), 2014]**

  Axial: $Z_A^{\rm s} - Z_A^{\rm ns}$ **Scalar:** $Z_S^{\rm s} - Z_S^{\rm ns}$
- Applicable for various actions: **(Wilson, Clover, SLiNC, TM)**$_F$ & **(Wilson, t.l. Symanzik, Iwasaki, DBW2)**$_G$

tree-level Symanzik gluons:

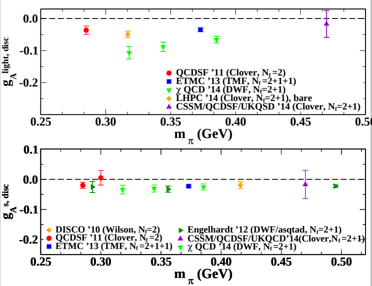
$$Z_A^{\rm s} - Z_A^{\rm ns} = \frac{g^4\, C_{\rm f}\, N_{\rm f}}{\left(16\,\pi^2\right)^2}\left(-2.0982 + 12.851 c_{\rm sw} + 3.3621 c_{\rm sw}^2 - 1.7260 c_{\rm sw}^3 - 0.0164 c_{\rm sw}^4 - 6\log(a^2\mu^2)\right)$$

Talk by H. Panagopoulos



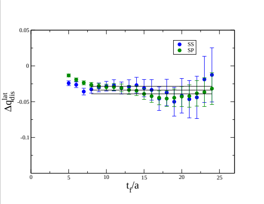
Nucleon Spin

Disconnected Contributions



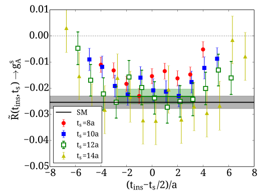
$$g_A^s \; : \; \langle N(p')|\bar{s}\,\gamma_\mu\,\gamma_5\,s|N(p)\rangle\Big|_{q^2=0}$$

g_A^{light}

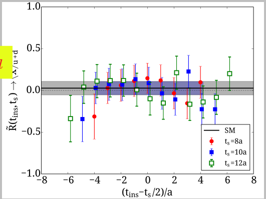


[G.S.Bali et al. (QCDSF), arXiv:1112.3354]

g_A^s



$\langle x \rangle_{u+d}$



[A.Abdel-Rehim et al. (ETMC), arXiv:1310.6339]

- **Agreement between different discretizations:**

  [R.Babich et al. (DISCO), arXiv:1012.0562]

  [G.S.Bali et al. (QCDSF), arXiv:1112.3354]

  [M.Engelhardt, arXiv:1210.0025]

  [A.Abdel-Rehim et al. (ETMC), arXiv:1310.6339]

  [S.Meinel et al. (LHPC), 2014], **bare results**

  [J.Zanotti et al. (CSSM/QCDSF/UCKCD), 2014]

  [M.Gong et al. (χ QCD), 2014]

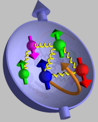
Talks by:

M.Gong (χ QCD)

A.Vaquero (ETMC)

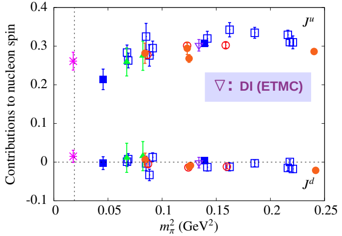
J.Zanotti (CSSM/QCDSF/UKQCD)

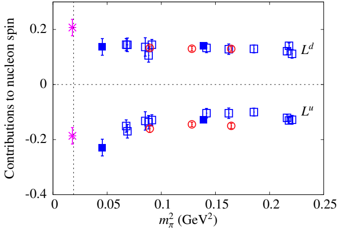
- **DI for g_A lower the total value**

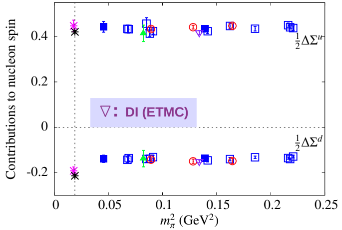


Nucleon Spin

Results







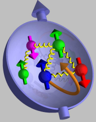
Lattice results:

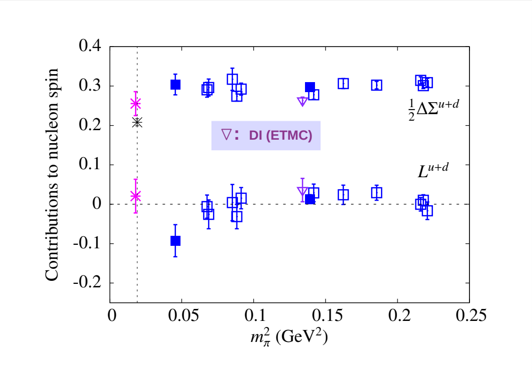
- Most results only CI
- TMF: include $Z_A^s - Z_A^{ns}$
- $m_\pi = 135$ MeV: $J^u \sim 0.25\,,\quad J^d \sim 0$
- $L^{u+d} \sim 0$ ($L^u,\ L^d$ cancel out)
- $m_\pi = 135$ MeV: $\Delta\Sigma^u,\ \Delta\Sigma^d$ agrees with exp.

○ LHPC '11 (DWF/asqtad, $N_f = 2+1$) ● LHPC '11 (DWF, $N_f = 2+1$) ▲ QCDSF '12 (Clover, $N_f = 2$)
□ ETMC '10 (TMF, $N_f = 2$) ■ ETMC '13 (TMF, $N_f = 2+1+1$) ★ ETMC '14 (TMF&c_{SW}, $N_f = 2$)

[S.N.Syritsyn et al. (LHPC), arXiv:1111.0718]
[A.Sternbeck et al. (QCDSF), arXiv:1203.6579]
[C.Alexandrou et al. (ETMC), arXiv:1303.5979]

Nucleon Spin





□ ETMC '10 (TMF, $N_f=2$) ■ ETMC '13 (TMF, $N_f=2+1+1$) ★ ETMC '14 (TMF&c_{SW}, $N_f=2$)

★ $m_\pi=135$MeV: Agreement with exp

★ DI: lowers the total value

B

HYPERON

FORM

FACTORS

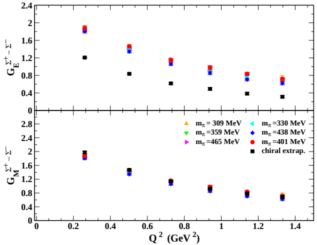
Hyperon EM form factors

$$\langle B(p',s')|j_\mu(q)|B(p,s)\rangle = \overline{u}(p',s')\left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_B}F_2(Q^2)\right]u(p,s)$$

$$\text{Sachs FFs}: \; G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

D1. G_E, G_M of Hyperons

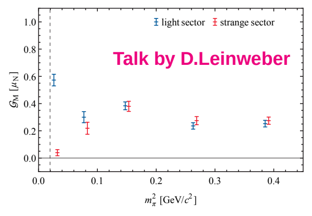
- 'Connected χPT':
  - valence and sea quarks are treated separately
  - disconnected contractions may be omitted
- extrapolation on each Q^2 separately
- N_f=2+1 Clover, a =0.074 fm



[P.E. Shanahan et al. (CSSM & QCDSF/UKQCD), arXiv:1401.5862, 1403.1965]

D2. G_M^s of Λ(1405)

- contains strange quark, but lighter than other excited spin-1/2 baryons
- superposition of molecular meson-baryon states ($\pi\Sigma$ & $\overline{K}N$)?
- 1st lattice computation of the EM FFs of Λ(1405) (variational approach)
- in $\overline{K}N$: s-quark does not contribute in G_M



★ Approaching the physical point: $G_M^s \to 0$

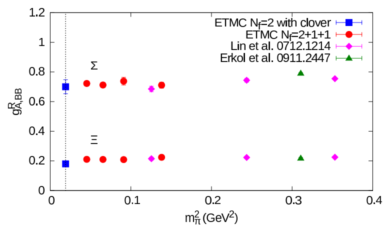
[D.Leinweber et al. (CSSM), 2014]

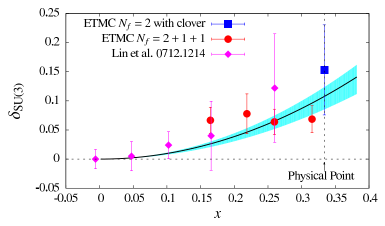
[B.J.Menadue et al., arXiv:1109.6716]

Axial charges of hyperons

Axial matrix element: $\langle B(p')|\bar{\psi}(x)\,\gamma_\mu\,\gamma_5\,\psi(x)|B(p)\rangle\Big|_{q^2=0}$

- **Connected part**





[C.Alexandrou et al. (ETMC), preliminary]

- **First promising results at the physical point**
- **SU(3) breaking** $\delta_{SU(3)} = g_A^N - g_A^\Sigma + g_A^\Xi$ **versus** $x = \left(m_K^2 - m_\pi^2\right)/(4\pi^2 f_\pi^2)$

Talk by C. Alexandrou

C

MESONS

E1. Pion Quark distribution function

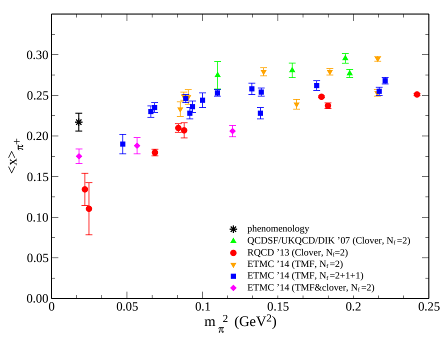
[C.Urbach et al. (ETMC), 2014]: $N_f=2,\ 2+1+1$ TMF, $N_f=2$ TMF & clover

Lowest moment with H(4)-operator:

$$\mathcal{O}_{44}(x) = \frac{1}{2}\bar{u}(x)[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\sum_{k=1}^{3}\gamma_k \overleftrightarrow{D}_k]u(x)$$

$$\langle x\rangle^{\text{bare}}_{\pi^+} = \frac{1}{2\,m_\pi^2}\langle\pi,\vec{0}|\mathcal{O}_{44}|\pi,\vec{0}\rangle$$

- No external momentum is needed in the calculation
- Stochastic time slice sources:
  - less inversions
  - statistical accuracy
- disconnected contributions not included



phenomenology: $\langle x\rangle_{\pi^+} = 0.0217(11)$

[K. Wijesooriya et al., nucl-ex/0509012]

[R. Baron et al. (ETMC), arXiv:0710.1580]

[D. Brommel (QCDSF/UKQCD) Pos(LATTICE) 2007, 140]

[G. Bali et al. (RQCD), arXiv:1311.7639]

[C. Urbach et al. (ETMC), 2014]

E2. ρ-meson EM form factors

[B.J.Owen et al. (CSSM), 2014] $N_f=2+1$ Clover

$$\langle \rho(p', s')|j_\mu|\rho(p, s)\rangle: \mathbf{G_C(q^2)}, \mathbf{G_M(q^2)}, \mathbf{G_Q(q^2)}$$

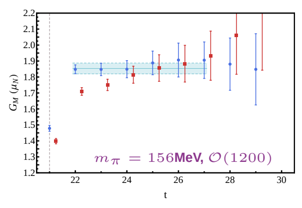
Variational approach

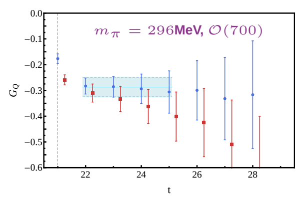
- automatic method for suppressing excited state effects
- separation of the correlators for individual energy eigenstates
  $\Rightarrow$ rapid ground state dominance
  $\Rightarrow$ access to excited states
- Set of operators: various source and sink smearings
  $$\chi^i_\rho(x) = \bar{d}(x)\gamma^i\, u(x)$$
- 4 levels of smearing $\Rightarrow$ 4$\times$4 correlation matrix
- substantial improvement for G_M and G_Q

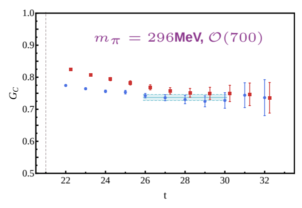
Blue points: variational method (VM)
Red points: standard method (SM)

★ G_M, G_Q (VM): plateau right after the current insertion
★ G_M (SM): plateau at later timeslices
★ G_Q (SM): No plateau identification
★ G_C: plateau of VM earlier that in the SM

first excitation of ρ-meson







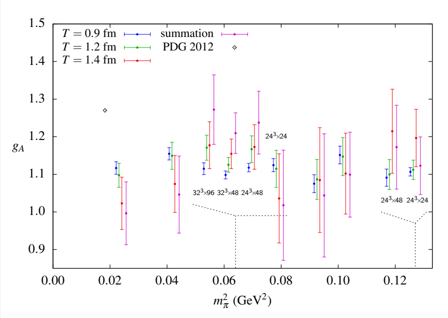
CONCLUSIONS

Breakthrought: Simulating the physical world!

- **Dedication of human force and computational resources on:**
  - Control of statistical uncertainties $\Rightarrow$ noise reduction techniques crucial
  - comprehensive study of systematic uncertainties
  - removal of excited states where necessary
  - cross-checks between methods
  - Simulations at different lattice spacings and volumes
  - study of DI at lower masses (Target: physical m_π!)
    - challenging task
    - exploid techniques: AMA, hierarchical probing, others
    - usage of GPUs
    - current computations of DI provide bounds
- **Nucleon spin: include dynamical simulations for gluon angular momentum**
  - Difficulties with renormalization and mixing
  - rely on perturbation theory
- **Exciting results emerging from other particles**

THANK YOU

BACKUP SLIDES

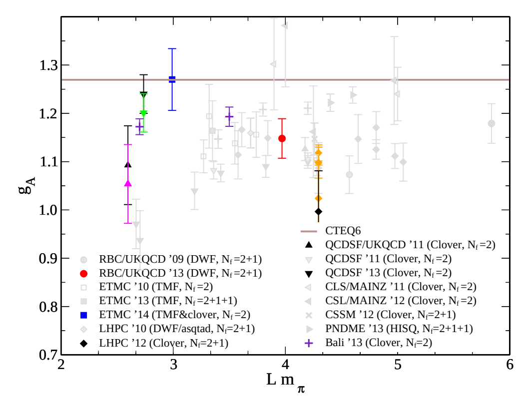


❶ Plateau Method: single-state fit

LHPC (2012): [J.R.Green et al. (LHPC), arXiv:1211.0253]

- $m_\pi \geq 149$MeV
- light m_π : g_A ↘ with T_s ↗
- $L_t/a \geq 48$: g_A ↗ with T_s ↗
- Indication of thermal pion states

Finite Volume Effects

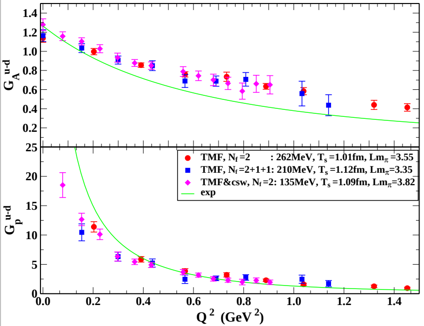


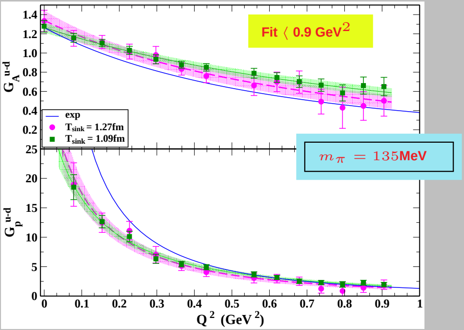
◆ **Black diamond: summation (LHPC)**

▲ **Black triangles: volume corrected (QCDSF)**

B2. Nucleon Axial form factors

TMF, $N_f = 2$, $N_f = 2+1+1$ and TMF & clover , $N_f = 2$





★ **Dipole fits:**

$$G_A(Q^2) = \frac{g_A}{\left(1 + Q^2/m_A^2\right)^2}$$

$$G_p(Q^2) = \frac{G_A(Q^2)\, G_p(0)}{\left(Q^2 + m_p^2\right)}$$

$m_A^{exp} = 1.069\text{GeV}$ †

$1.2\text{GeV} \langle\, m_A^{lattice} \,\langle\, 1.45\text{GeV}$ ⋆

$0.3\text{GeV} \langle\, m_p^{lattice} \,\langle\, 0.5\text{GeV}$ ⋆

† [V.Bernard et al., hep-ph/0607200]

⋆ TMF, m_π=135MeV (ETMC 2014)

- G_p strongly dependent on the lowest values of Q^2

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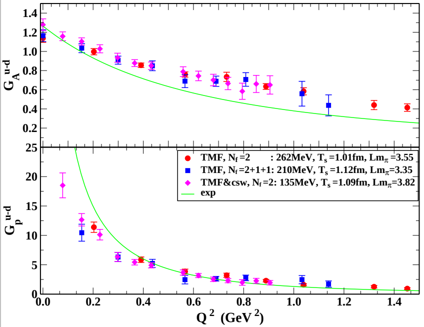

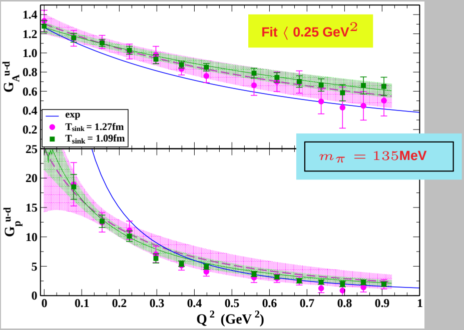

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Generalized pencil-of-function

- Better extraction of states contributing to a correlator
- Variational method using 3pt-functions with 3 equally spaced sink locations

$$\mathbf{C}^{3-\mathrm{pt}}(t_i, t, t_f) = \begin{pmatrix} C^{3-\mathrm{pt}}(t_i, t, t_f) & C^{3-\mathrm{pt}}(t_i, t, t_f+\tau) \\ C^{3-\mathrm{pt}}(t_i, t+\tau, t_f+\tau) & C^{3-\mathrm{pt}}(t_i, t+\tau, t_f+2\tau) \end{pmatrix}$$

- Computational cost $\times 3$, but better ground signal

B

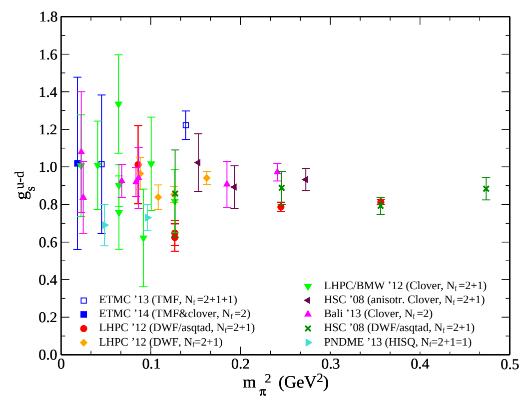
NUCLEON

CHARGES

B1. Scalar Charge

$$g_S \equiv \langle N|\bar{u}u - \bar{d}d|N\rangle$$

- $g_S,\ g_T$ provide constrains for scalar interactions at the TeV scale



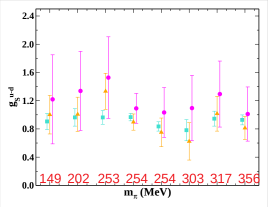
★ Severe contamination of excited states

★ Confidence in results requires:

Dedicated study with high statistics of plateau, 2-state fit, summation method

LHPC: m_π=149−356MeV

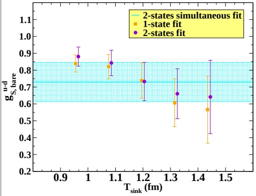
[J.R.Green et al. (LHPC), arXiv:1206.4527]



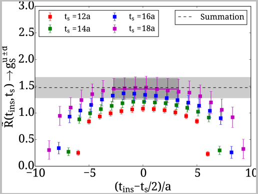
★ m_π=149MeV: 0.93 fm ⟨ $T_{\rm sink}$ ⟨ 1.39 fm

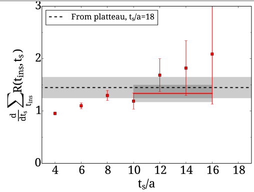
PNDME: m_π=310MeV

[T. Bhattacharya (PNDME), arXiv:1306.5435]



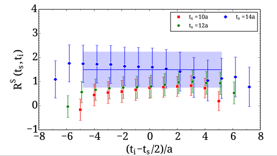
TMF: $N_f=2+1+1$, $m_\pi=373$MeV **[A.Abdel-Rehim et al. (ETMC), arXiv:1310.6339]**

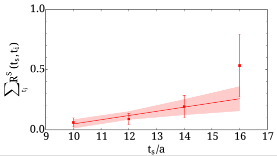




- 0.98 fm $\langle T_{\rm sink} \langle$ 1.48 fm
- $T_{\rm sink}$ $\lesssim$1.31 fm: agreement with SM

TMF & c_{SW}: $N_f=2$, $m_\pi=135$MeV **[C.Alexandrou et al. (ETMC), 2014]**

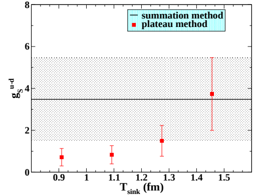




- 0.98 fm $\langle T_{\rm sink} \langle$ 1.48 fm
- $T_{\rm sink}$ $\lesssim$1.3 fm: agreement with SM

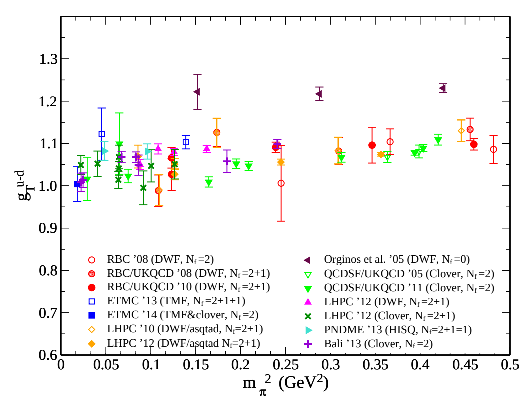
(similar to $m_\pi=373$MeV)

- Stat. errors must come down



B2. Tensor Charge

$$\langle N(p', s')|\, \sigma^{\{\mu\nu}\, |N(p, s)\rangle \quad \Rightarrow \quad \mathbf{A_{T10}(q^2)},\ \mathbf{B_{T10}(q^2)},\ \mathbf{C_{T10}(q^2)} \qquad \langle 1\rangle_{\delta q} = A_{T10}(0)$$



At scale $\mu^2 = 110$ GeV^2:

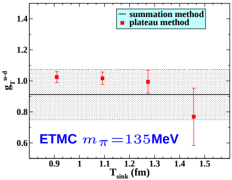
- $A_{T10}^{exp}(0.8\text{GeV}^2)=0.64^{+0.35}_{-0.16}$ (†)
- $A_{T10}^{exp}(0.8\text{GeV}^2)=0.77^{+0.13}_{-0.27}$ (⋆)

evolution to 4 GeV^2 (NNLO):

- $A_{T10}^{exp}(0.8\text{GeV}^2)=0.72^{+0.39}_{-0.18}$ (†)
- $A_{T10}^{exp}(0.8\text{GeV}^2)=0.87^{+0.15}_{-0.30}$ (⋆)

(⋆) [M.Anselmino et al., arXiv:0812.4366]

(†) [M.Anselmino et al., arXiv:1303.3822]



★ probes the transverse spin structure of the nucleon

★ Agreement among most lattice points

★ Mild m_π dependence

PNDME $m_\pi = 310$MeV ⇒