Few-body physics

The burden of having 3 particles or more in a box

Raúl Briceño

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Lattice 2014, NYC June 2014

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The burden of having ~~3 particles or more~~ in a box

3-1 particles or more

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Jefferson Lab

lattice 14

Lattice 2014, NYC June 2014

Why few-body physics?

"Few-body problems are present in many branches of physics..."

all references are hyperlinked!

- particle physics

  e.g., $B^0 \to K^{*0}\ell^+\ell^-$

LHCb collaboration (2013)

First unquenched LQCD calculation:
Horgan, Liu, Meinel & Wingate (2013)

See poster by M. Wingate this afternoon

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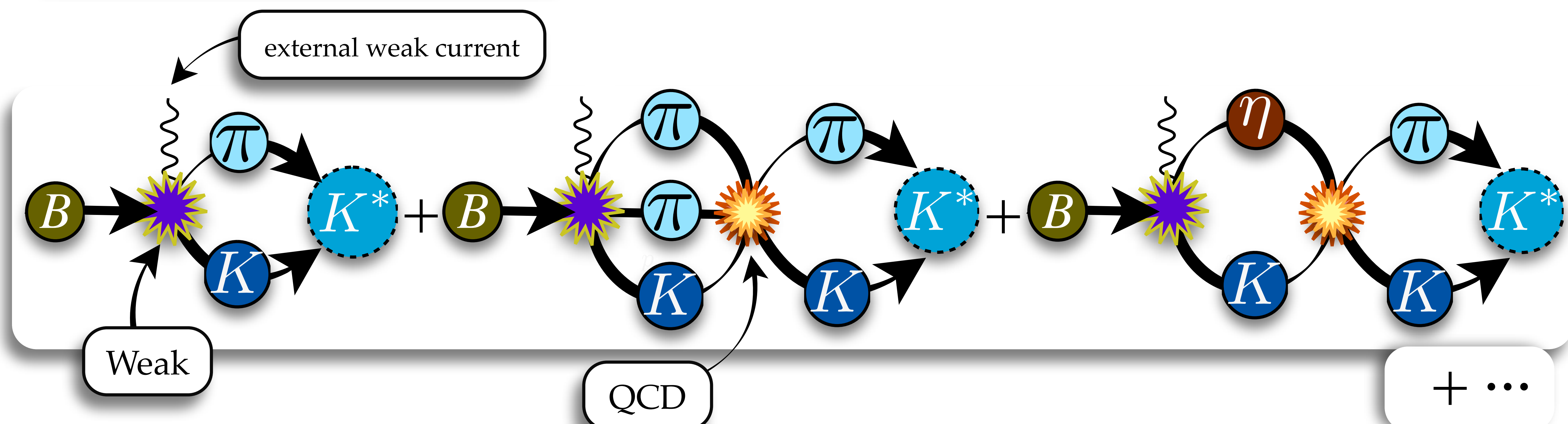
LHCb collaboration (2013)

First unquenched LQCD calculation:
Horgan, Liu, Meinel & Wingate (2013)

$K^*(892)$:
- $I\,(J^P) = ½\,(1^-)$ resonance
- above πK and ππK thresholds
- just below Kη~Kπππ threshold

See poster by M. Wingate this afternoon



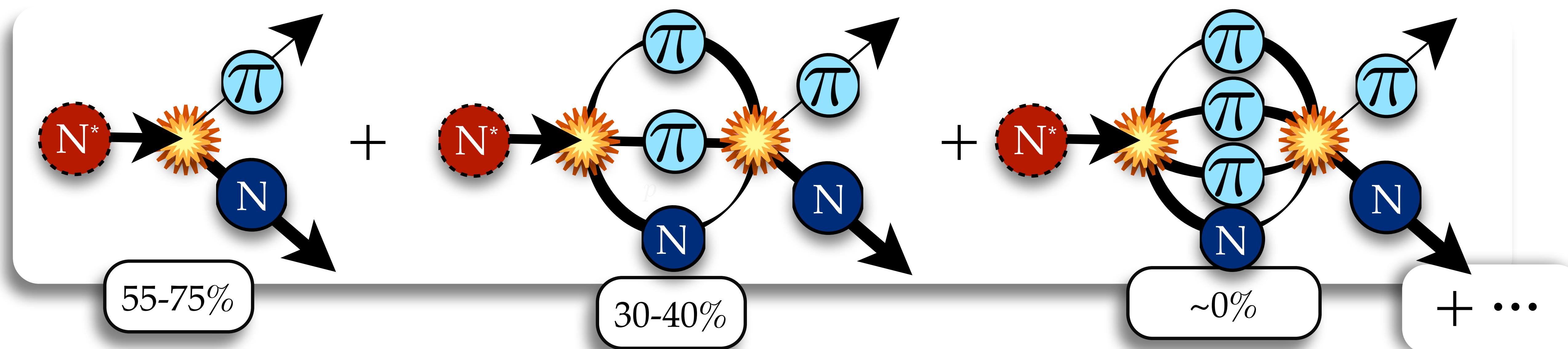
Why few-body physics?

"Few-body problems are present in many branches of physics..."

- particle physics
- nuclear physics

e.g., the *"Roper"*, $N^*(1440)$

Roper:
- $I\,(J^P) = \frac{1}{2}\,(\frac{1}{2}^+)$ resonance
- above the $N\pi$, $N\pi\pi$ and $N\pi\pi\pi$ thresholds



dominantly decays to two particles with significant overlap with three-particle states!

Why few-body physics?

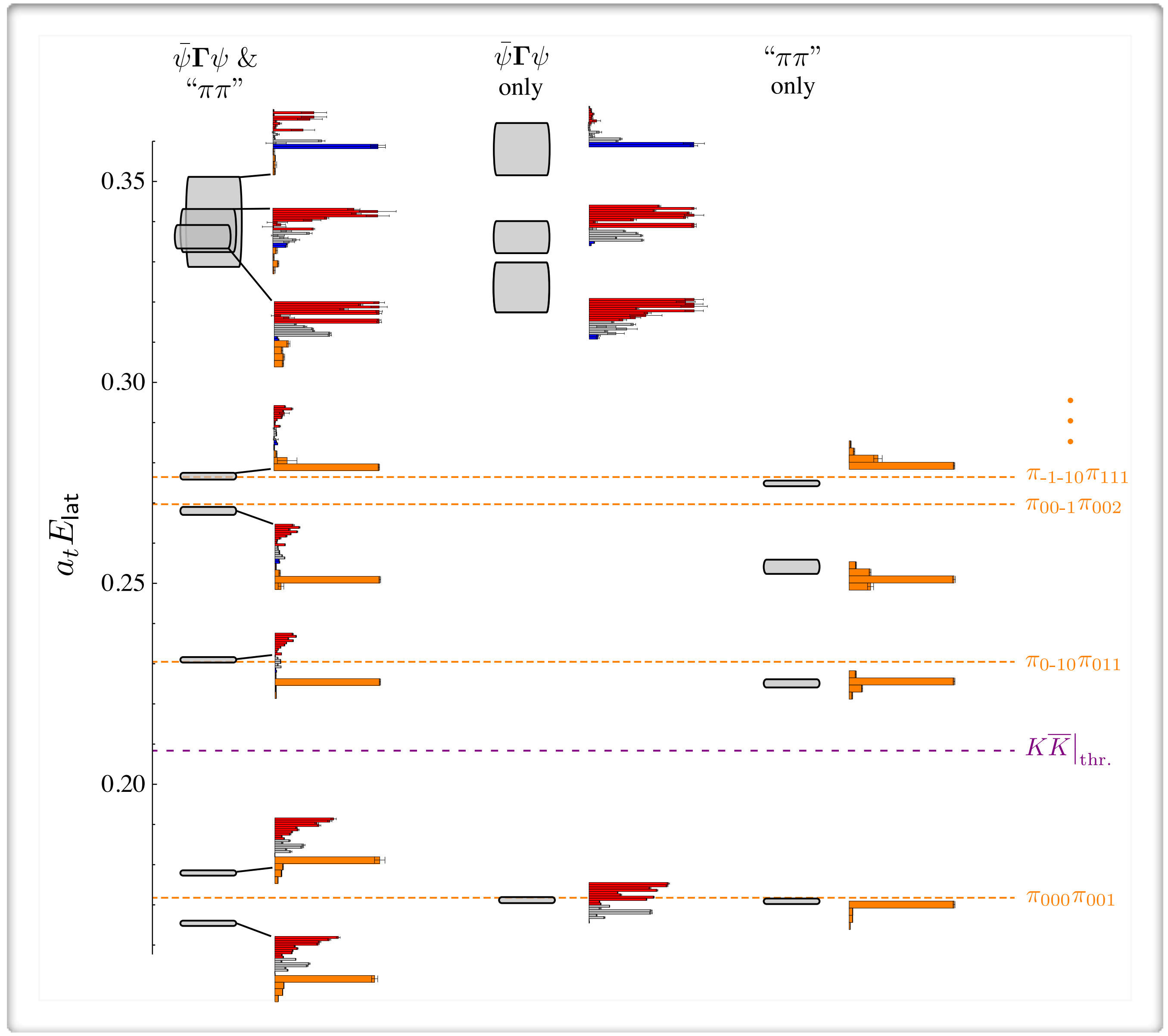
"Few-body problems are present in many branches of physics..."

- particle physics
- nuclear physics
- atomic physics
- condensed matter physics
- ...

Four main challenges with few-body systems on the lattice

1. Optimal operators
2. Poor signal / noise
3. Large number of contractions
4. Interpretation of observables

see talk by W. Kamleh on the implication of the five-quark operators on the nucleon spectrum, Wed. @ 09:00



[Hadron Spectrum Coll.] Dudek, Edwards, Thomas (2012)

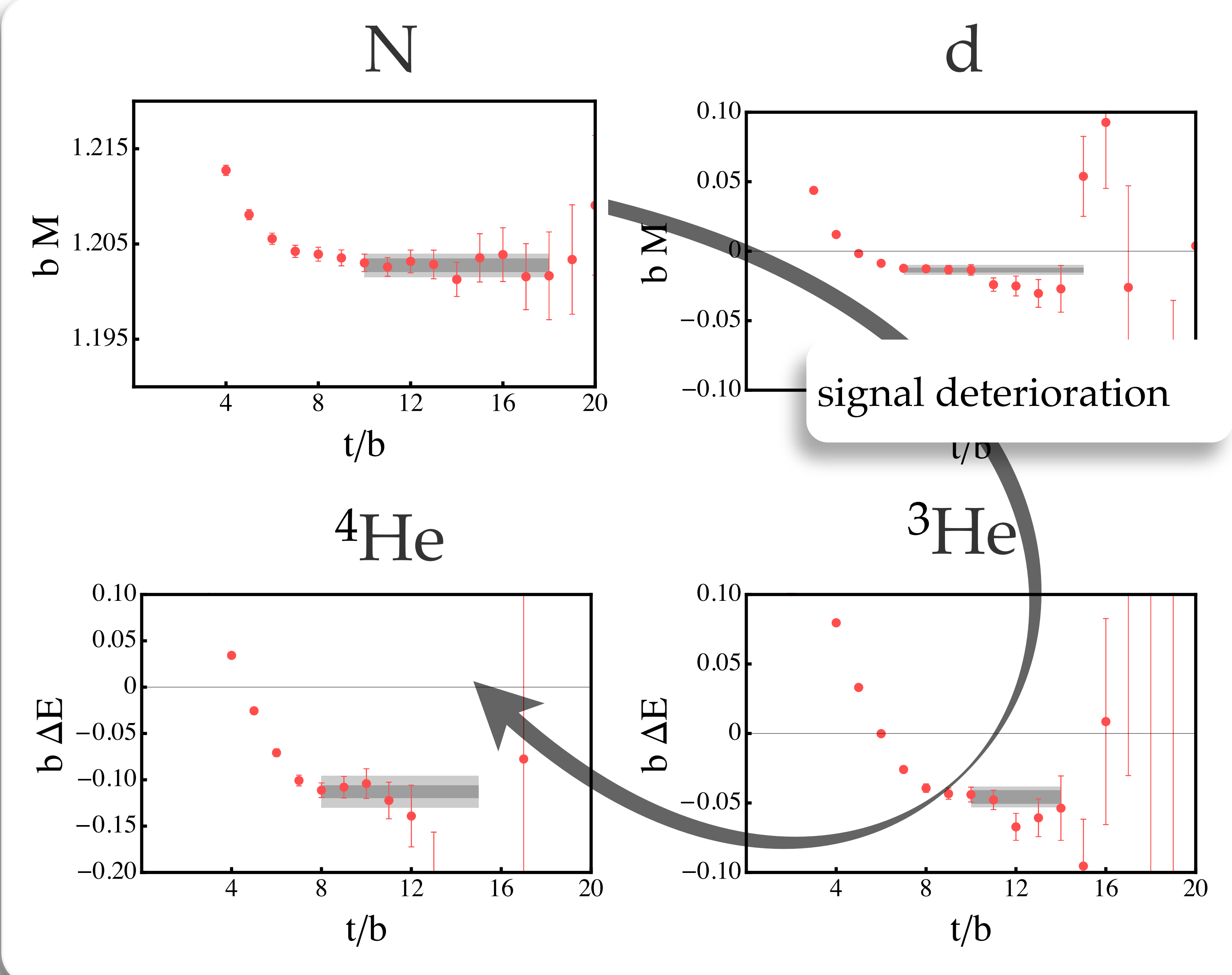
"without the right basis of operators, you simply get the wrong spectrum"

also see Lang & Verduci (2012)

Four main challenges with few-body systems on the lattice

1. Optimal operators
2. Poor signal / noise
3. Large number of contractions
4. ables

Lepage (1989)
M. J. Savage (2010)
Grabowska, Kaplan & Nicholson (2012)
...
Detmold and Endres (2014)



[NPLQCD Coll.] Beane, Chang, Cohen, Detmold, Lin, Luu, Orginos, Parreno, Savage, Walker-Loud (2012)

see M. Endres's talk Fri. @ 18:10, for signal/noise enhancement techniques

Four main challenges with few-body systems on the lattice

1. Optimal operators
2. Poor signal/noise
3. Large number of contractions

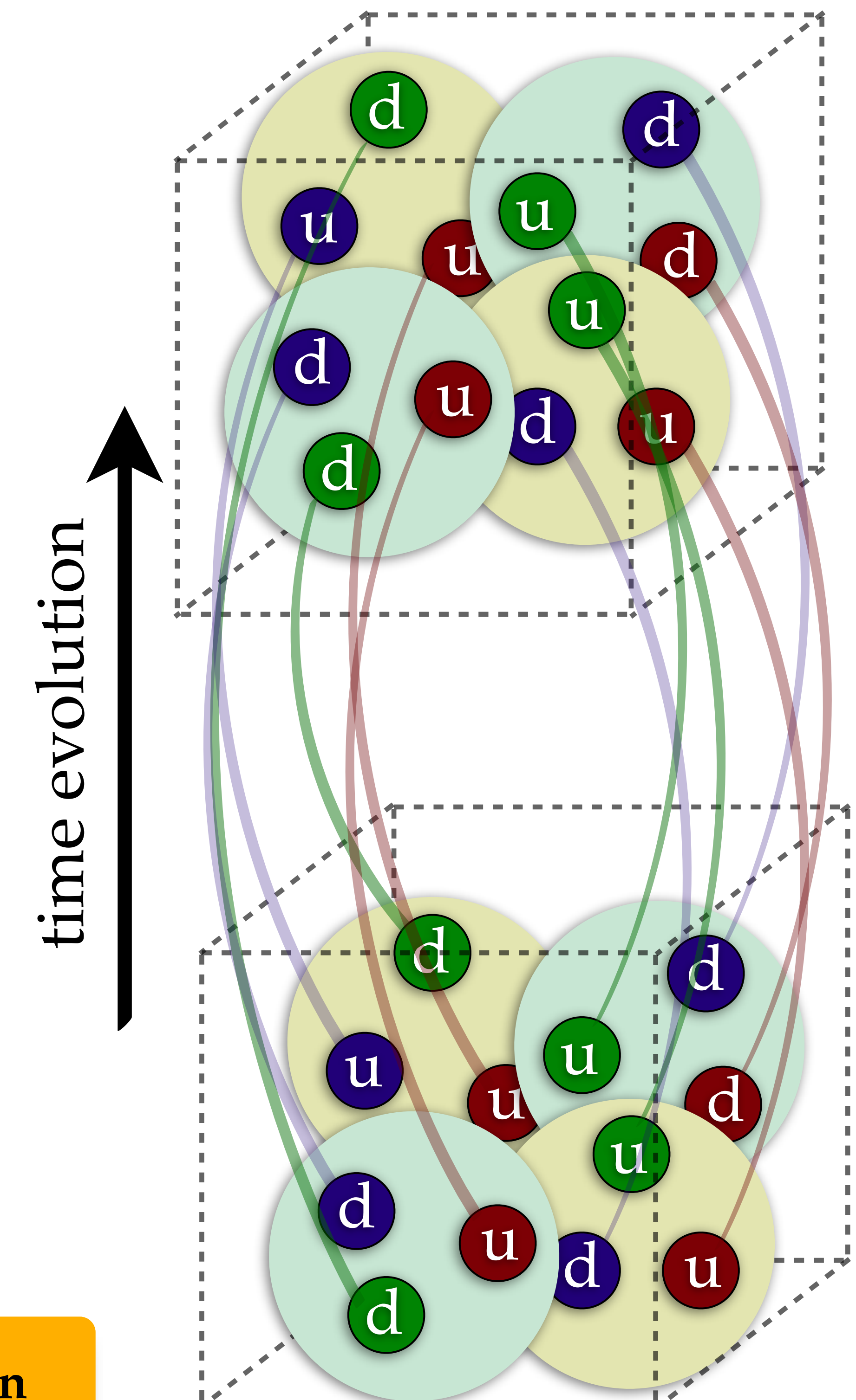
   e.g., naïvely ^{4}He has 6! x 6! = 518,400 contractions!

4. Interpretation of observables

Some clever tricks:
Detmold & Savage (2010)
Detmold, Orginos & Shi (2013)
Doi & Endres (2013)
Detmold & Orginos (2013)
Günther, Toth and Varnhorst (2013)

...

See poster by P. Vachaspati this afternoon



Four main challenges with few-body systems on the lattice

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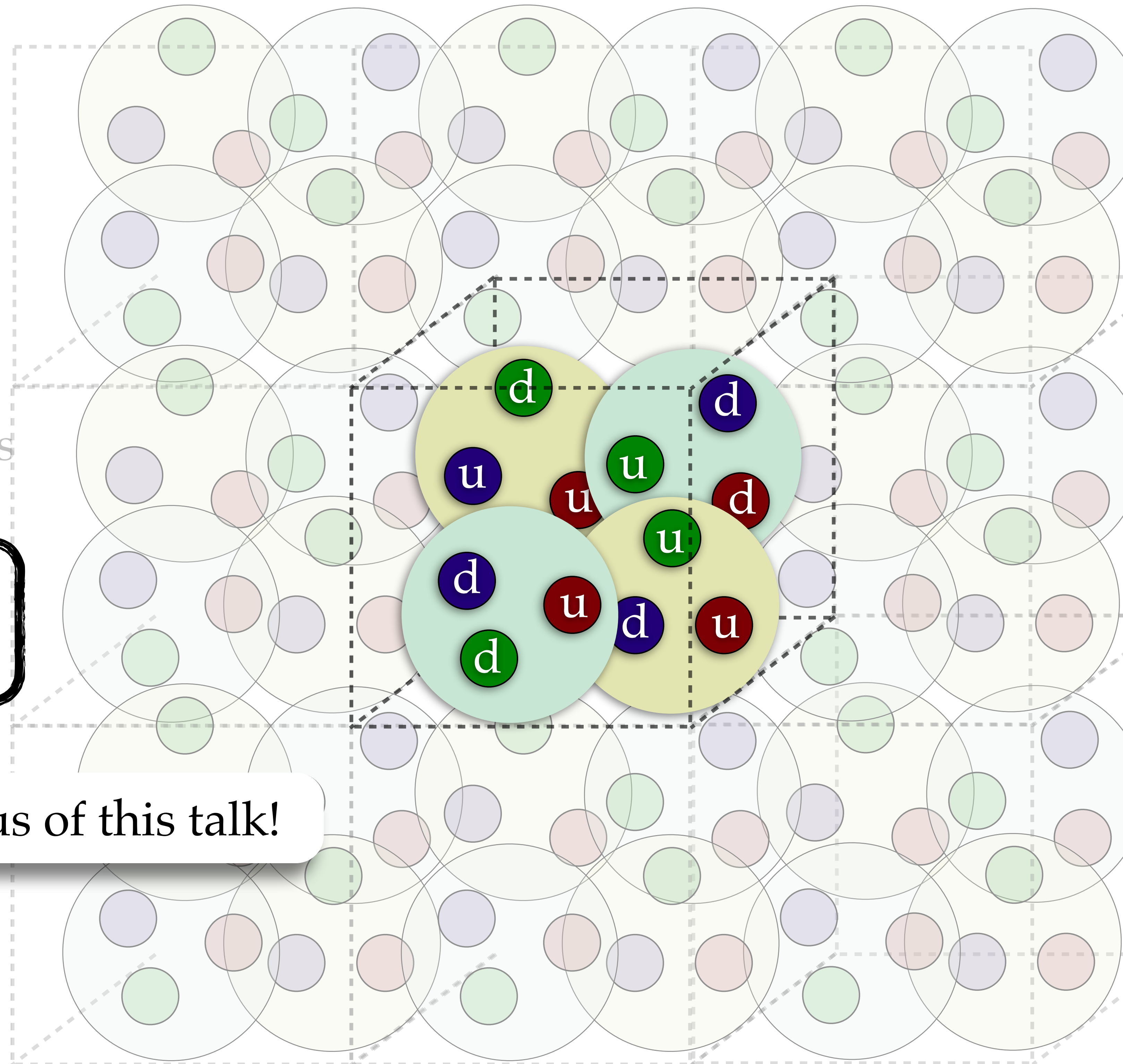
4. **Interpretation of observables**

focus of this talk!



Four main challenges with few-body systems on the lattice

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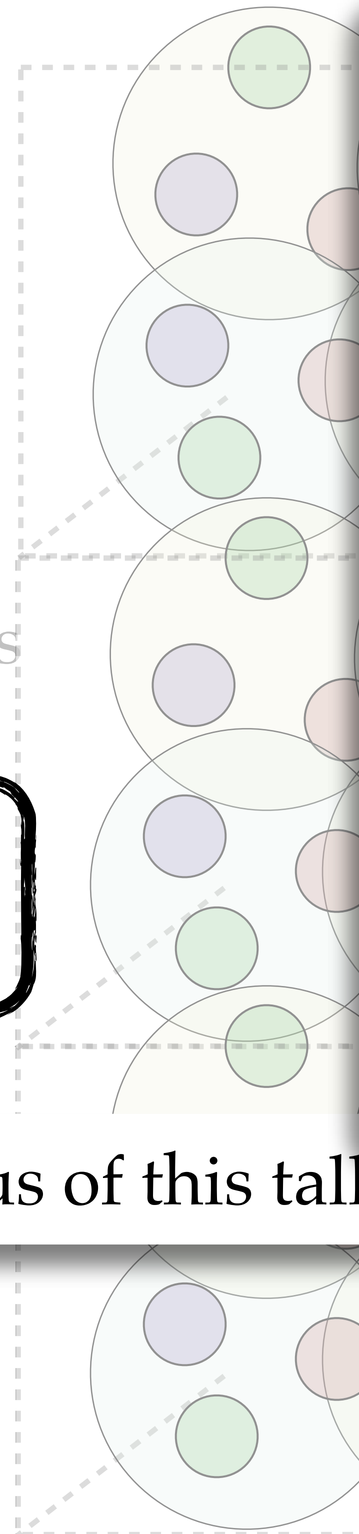

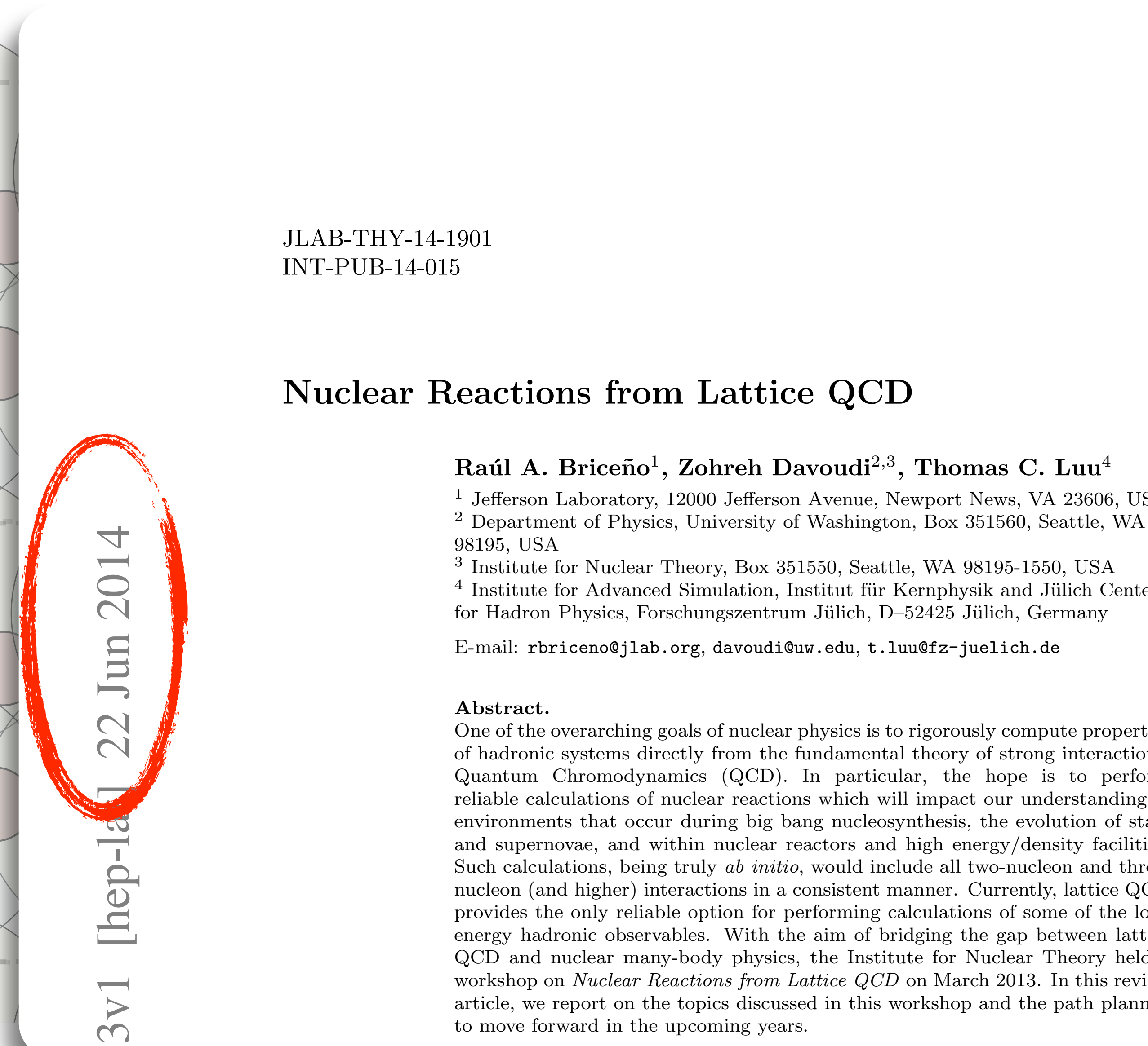

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Nuclear Reactions from Lattice QCD

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[3] Institute for Nuclear Theory, Box 351550, Seattle, WA 98195-1550, USA
[4] Institute for Advanced Simulation, Institut für Kernphysik and Jülich Cente for Hadron Physics, Forschungszentrum Jülich, D–52425 Jülich, Germany

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Abstract.
One of the overarching goals of nuclear physics is to rigorously compute propert of hadronic systems directly from the fundamental theory of strong interactio Quantum Chromodynamics (QCD). In particular, the hope is to perfo reliable calculations of nuclear reactions which will impact our understanding environments that occur during big bang nucleosynthesis, the evolution of st and supernovae, and within nuclear reactors and high energy/density faciliti Such calculations, being truly *ab initio*, would include all two-nucleon and thr nucleon (and higher) interactions in a consistent manner. Currently, lattice QC provides the only reliable option for performing calculations of some of the lo energy hadronic observables. With the aim of bridging the gap between latt QCD and nuclear many-body physics, the Institute for Nuclear Theory held workshop on *Nuclear Reactions from Lattice QCD* on March 2013. In this revi article, we report on the topics discussed in this workshop and the path plann to move forward in the upcoming years.

See RB, Davoudi & Luu (2014) for a very recent review on the status of few-body physics from the lattice!

How?

- Correlation functions: three basic representations

1

$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^\dagger_\lambda(y_0, -\mathbf{P})|0\rangle$$
$$= \delta_{\lambda,\lambda'} \sum_n e^{-E_{\lambda,n}(x_0-y_0)} \langle 0|\mathcal{O}'_\lambda(0, \mathbf{P})|E_{\lambda,n}\rangle\langle E_{\lambda,n}|\mathcal{O}^\dagger_\lambda(0, -\mathbf{P})|0\rangle$$

Operators could be different, but must have same quantum numbers (λ)

...explains how to extract observables

How?

- Correlation functions: three basic representations

1
$$C(x_0 - y_0, \mathbf{P}) = \langle 0|\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^\dagger_\lambda(y_0, -\mathbf{P})|0\rangle$$
$$= \delta_{\lambda,\lambda'} \sum_n e^{-E_{\lambda,n}(x_0-y_0)} \langle 0|\mathcal{O}'_\lambda(0, \mathbf{P})|E_{\lambda,n}\rangle\langle E_{\lambda,n}|\mathcal{O}^\dagger_\lambda(0, -\mathbf{P})|0\rangle$$

2
$$C(x_0 - y_0, \mathbf{P}) = \frac{1}{Z_{Eucl.}} \int \mathcal{D}[U, q, \bar{q}]\mathcal{O}'_{\lambda'}(x_0, \mathbf{P})\mathcal{O}^\dagger_\lambda(y_0, -\mathbf{P})\ e^{-S_{\mathrm{Eucl.}}}$$

... allows us to evaluate correlation functions numerically

How?

- Correlation functions: three basic representations

(1)
$$C(x_0 - y_0, \mathbf{P}) = \langle 0 | \mathcal{O}'_{\lambda'}(x_0, \mathbf{P}) \mathcal{O}^\dagger_\lambda(y_0, -\mathbf{P}) | 0 \rangle$$
$$= \delta_{\lambda,\lambda'} \sum_n e^{-E_{\lambda,n}(x_0 - y_0)} \langle 0 | \mathcal{O}'_\lambda(0, \mathbf{P}) | E_{\lambda,n} \rangle \langle E_{\lambda,n} | \mathcal{O}^\dagger_\lambda(0, -\mathbf{P}) | 0 \rangle$$

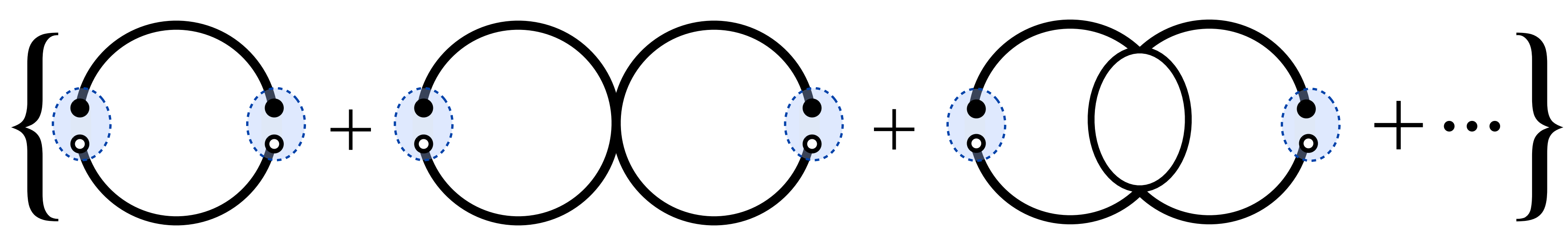
(2)
$$C(x_0 - y_0, \mathbf{P}) = \frac{1}{Z_{Eucl.}} \int \mathcal{D}[U, q, \bar{q}] \mathcal{O}'_{\lambda'}(x_0, \mathbf{P}) \mathcal{O}^\dagger_\lambda(y_0, -\mathbf{P}) \; e^{-S_{\mathrm{Eucl.}}}$$

(3) Sum over all Feynman diagrams:

e.g., $\pi\pi \to \pi\pi$

hadrons: *the low-energy degrees of freedom*

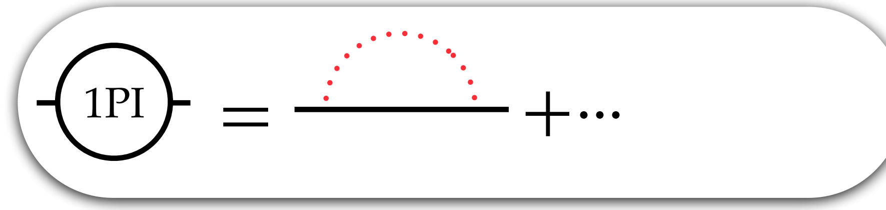
$$C(x_0 - y_0, \mathbf{P}) = \mathrm{F.T.} \Big\{ \cdots \Big\}$$

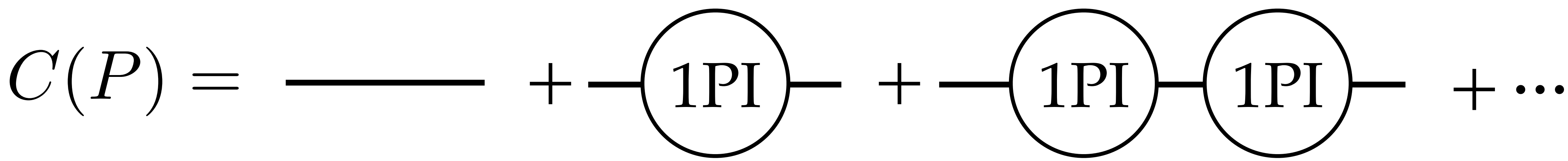


... gives ***meaning*** to the observables!

One particle in a finite volume

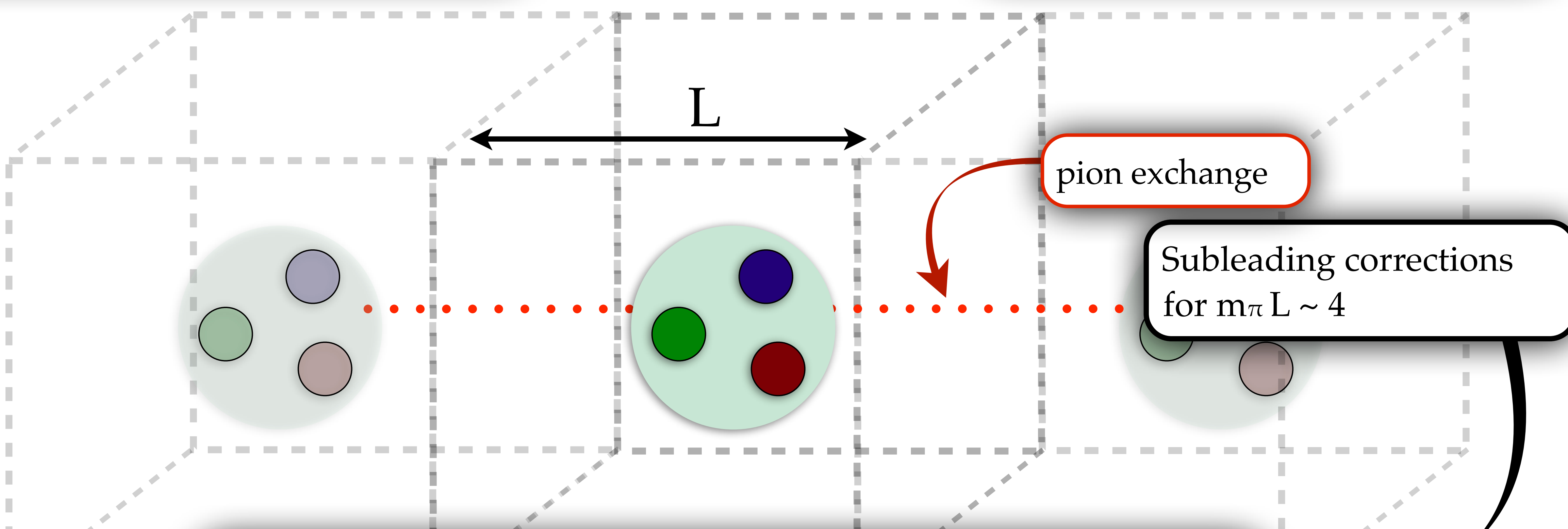
- One particle in a periodic finite volume:





Below multi-particle threshold intermediate particles cannot go on-shell

interaction with mirror images are suppressed (i.e., finite volume loops are can be replaced by infinite volume ones)



$$C(x_0 - y_0, \mathbf{0}) \longrightarrow Z_0 e^{-m_L(x_0 - y_0)} \approx Z_0 e^{-m_\infty(x_0 - y_0)}$$

Lüscher (1986)

One particle in a finite volume

- One particle in a periodic finite volume:

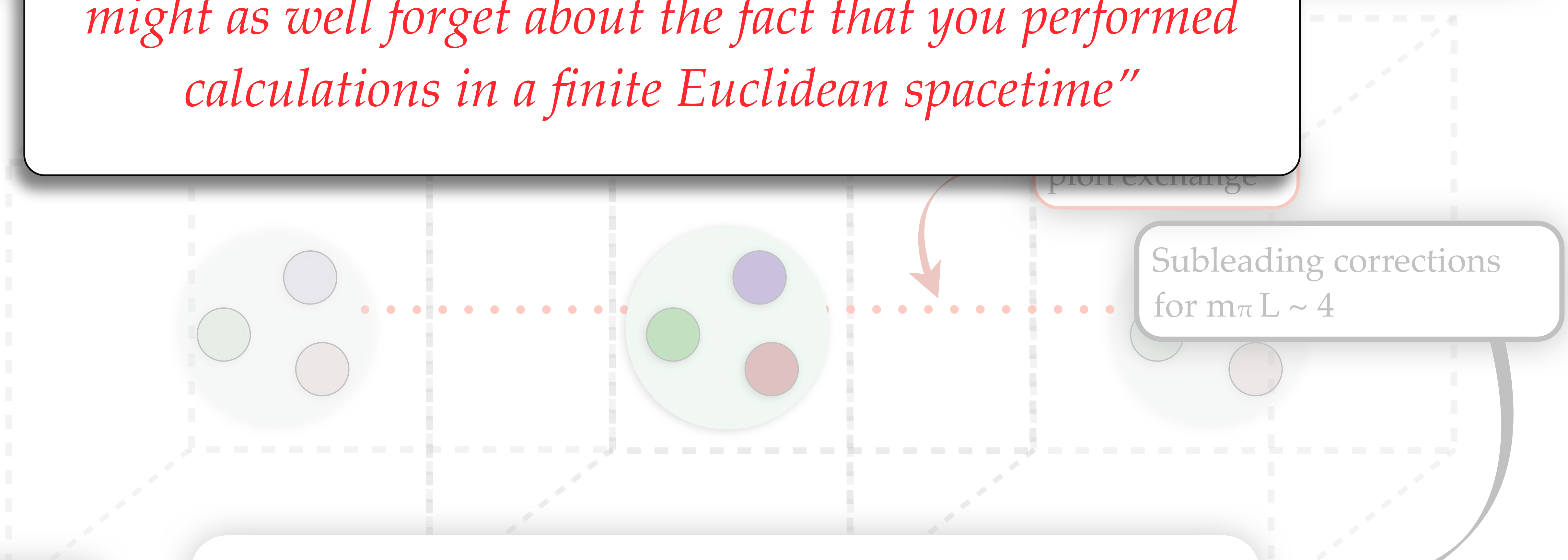
$$-\text{1PI}- = \text{(tadpole)} + \text{(sunset)} + \cdots$$

$$C(P) = \text{———} + -\text{1PI}- + -\text{1PI}-\text{1PI}- + \cdots$$

Below th… intermed…

…images are …ume loops are can …olume ones)

Take home message: *"get a big enough box and you might as well forget about the fact that you performed calculations in a finite Euclidean spacetime"*



Lüscher (1986)

$$C(x_0 - y_0, \mathbf{0}) \longrightarrow Z_0 e^{-m_L(x_0-y_0)} \approx Z_0 e^{-m_\infty(x_0-y_0)}$$

Bound states in a finite volume

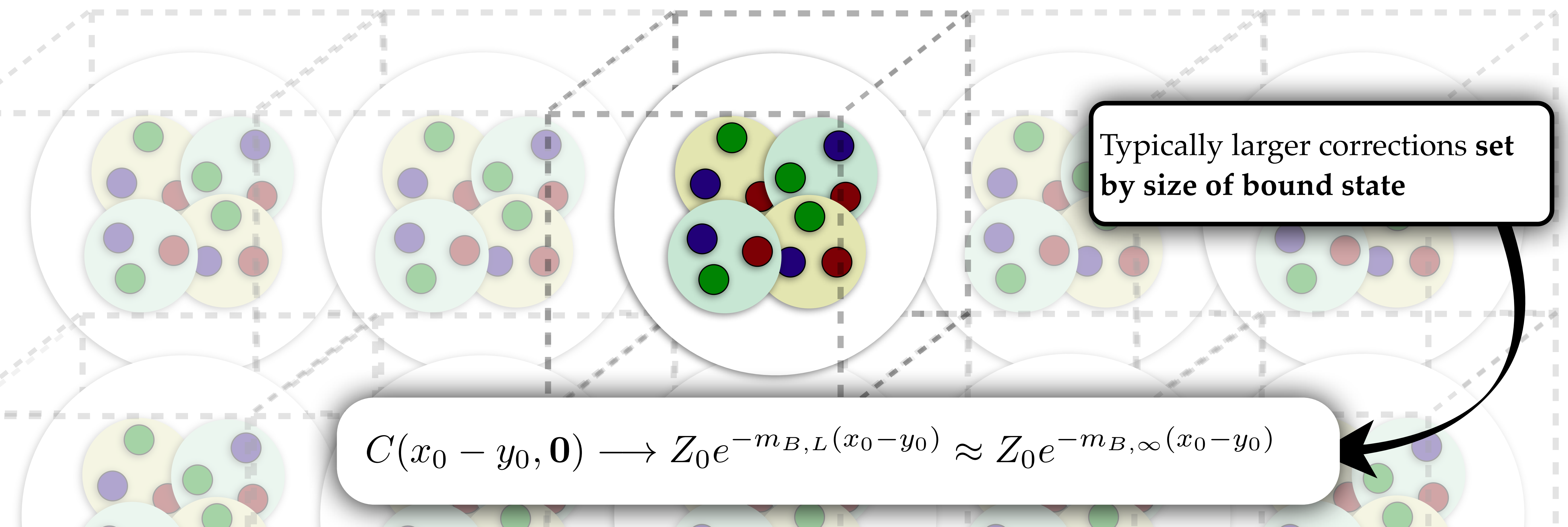
Get sufficiently large boxes and extrapolate to infinite volume

Formal studies supporting claim:

- Lüscher (1986)
- Beane, Bedaque, Parreno, and Savage (2004), (2005)
- Bour, Koenig, Lee, Hammer, and Meissner (2011)
- Kreuzer & Hammer (2008, 20009, 2010)
- Davoudi and Savage (2011) (2014)
- Kreuzer & Grießhammer (2013)
- RB, Davoudi, Luu and Savage (2013) ...

Some lattice QCD calculations involving bound

- Yamazaki, Ishikawa, Kuramashi, and Ukawa (2012)
- Beane *et al.* [NPLQCD] (2012)
- Hadron Spectrum Coll. (2014)
- HAL QCD



Typically larger corrections **set by size of bound state**

$$C(x_0 - y_0, \mathbf{0}) \longrightarrow Z_0 e^{-m_{B,L}(x_0-y_0)} \approx Z_0 e^{-m_{B,\infty}(x_0-y_0)}$$

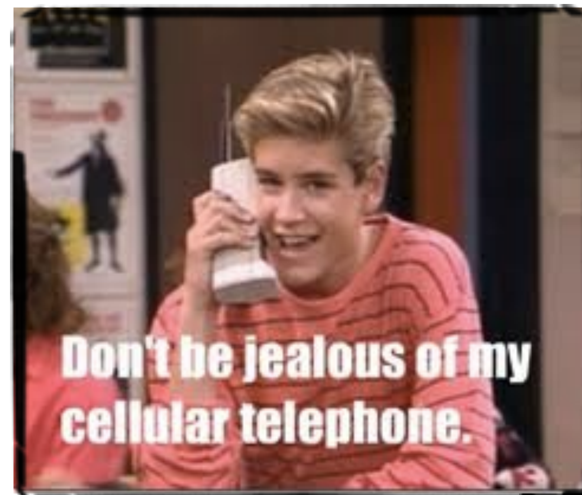
No-go theorem revisited

Calculation involving two particles or more, require additional formalism to relate lattice QCD quantities to infinite volume Minkowski observables:

- Maiani & Testa (1990)

⋮

- RB, Hansen & Walker-Loud (2014)



same thing with some modern *"bells & whistles"*

see A. Walker-Loud's talk, today @ 17:10

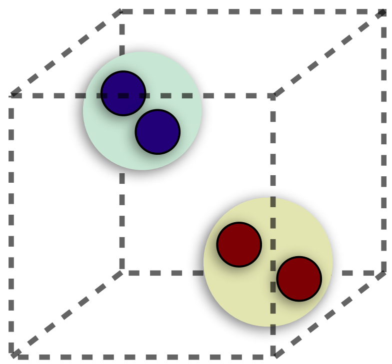
In a nutshell:

Minkowski: $\langle 0|\mathcal{O}_{\pi\pi}(t)\ \mathcal{O}^{\dagger}_{\pi\pi}(-t)|0\rangle \longrightarrow$ asymptotic, on-shell states

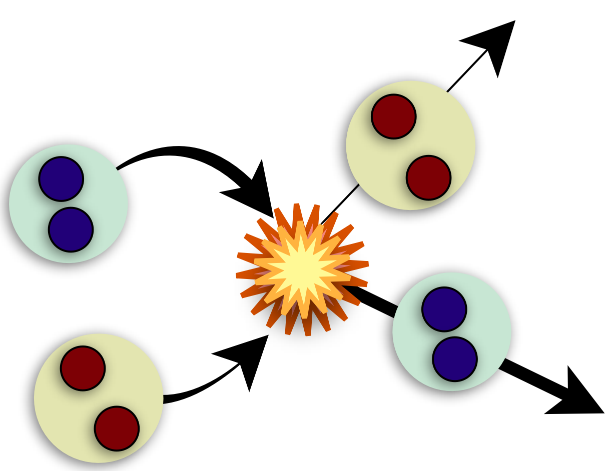
Euclidean: $\langle 0|\mathcal{O}_{\pi\pi}(t)\ \mathcal{O}^{\dagger}_{\pi\pi}(-t)|0\rangle \longrightarrow$ on-shell & off-shell states

A roadmap towards physics

1. Calculate finite volume spectrum

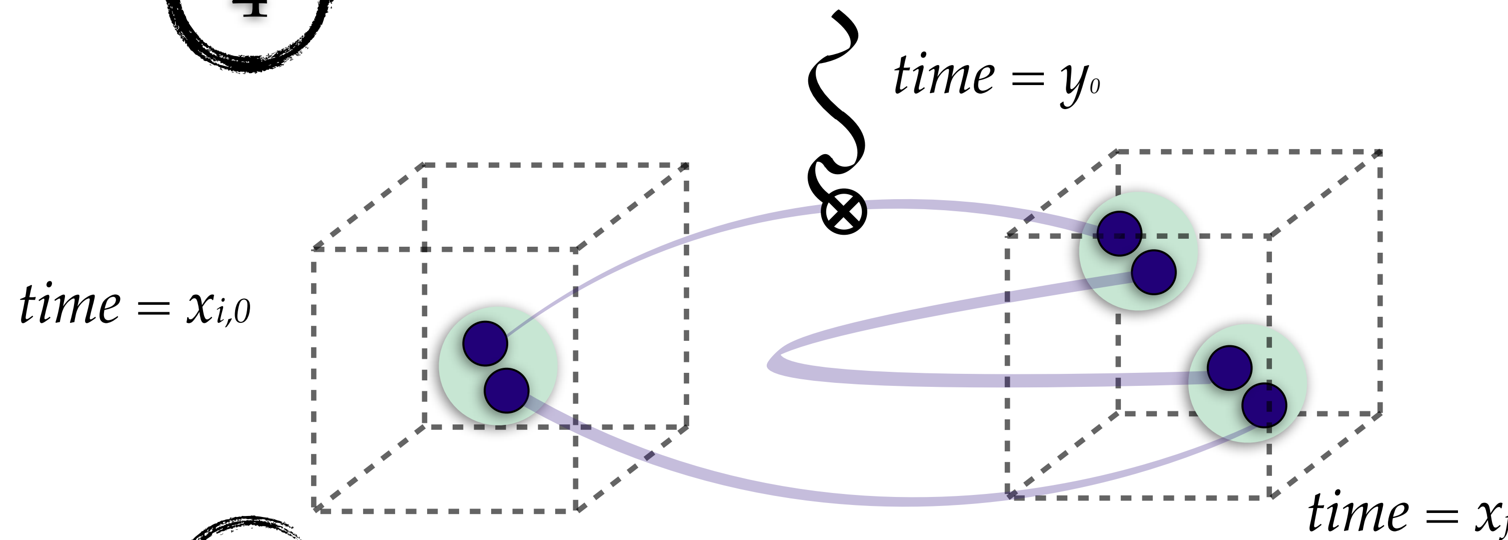


2. Plug into formalism
3. Out goes elastic & inelastic QCD scattering amplitudes

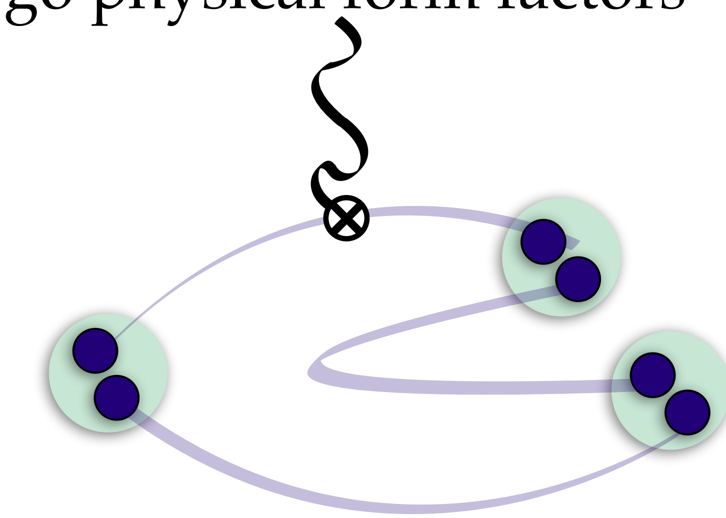


***à la mode de* Lüscher (1986)**

4. Calculate finite volume form factor



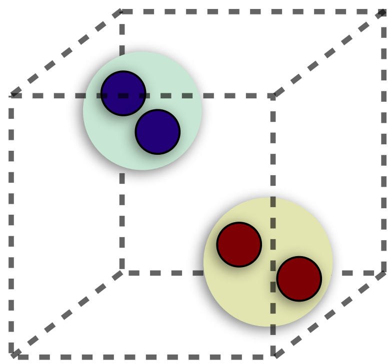
5. Plug spectrum, scattering parameters and finite volume form factor into formalism
6. Out go physical form factors



***à la mode de* Lellouch & Lüscher (2000)**

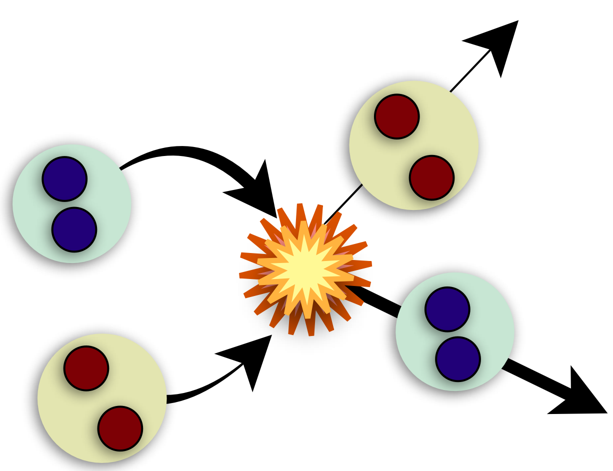
A roadmap towards physics

1 Calculate finite volume spectrum



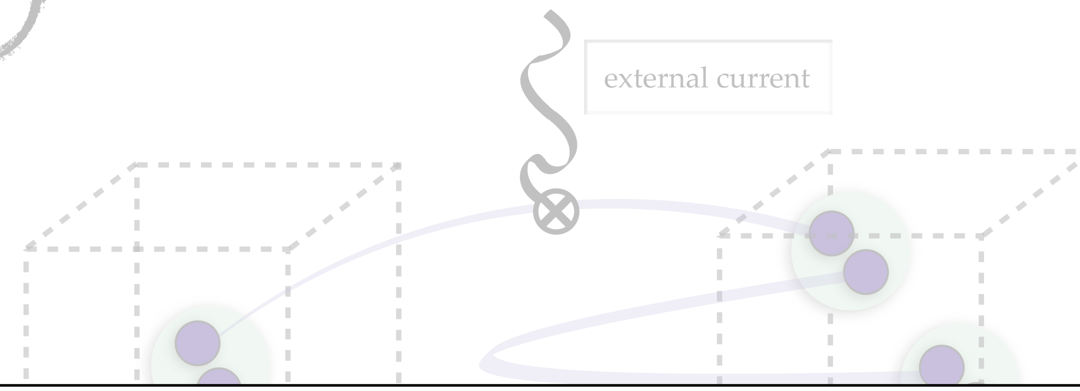
2 Plug into formalism

3 Out goes elastic & inelastic QCD scattering amplitudes



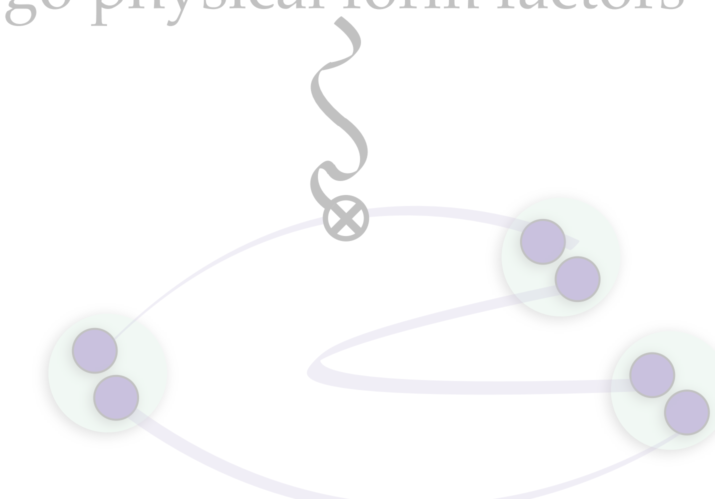
à la mode de **Lüscher (1986)**

4 Calculate finite volume form factor



let's review what is known regarding the spectrum first

6 Out go physical form factors



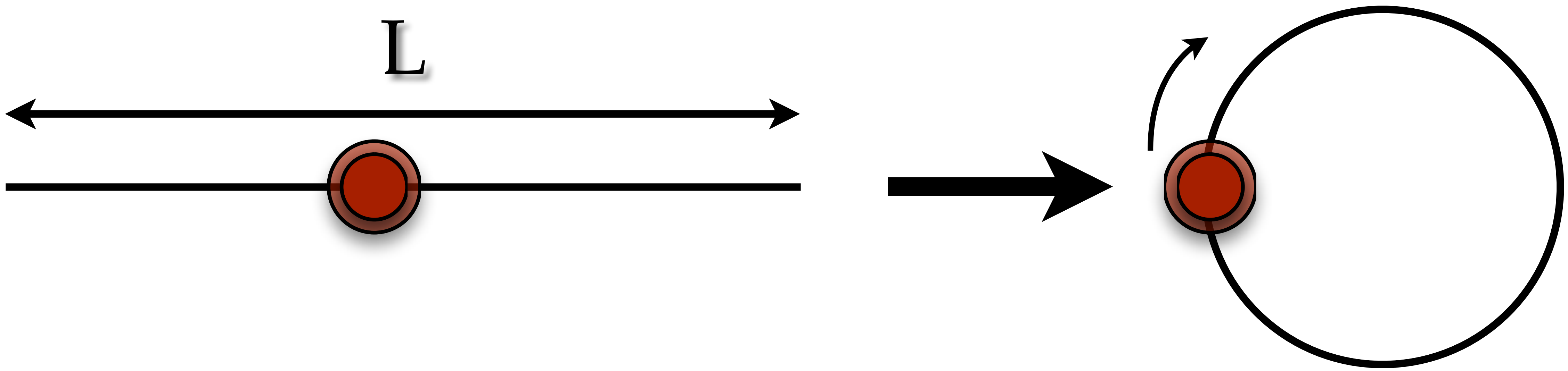
à la mode de **Lellouch & Lüscher (2000)**

A long list of extensions of the Lüscher formalism

- **Lüscher (1986), (1991)** *("Lüscher Formalism")*
- Maiani and Testa (1990)
- Rummukainen and Gottlieb (1995)
- Beane, Bedaque, Parreno, and Savage (2004), (2005)
- Bedaque (2004)
- Li and Liu (2004)
- Detmold and Savage (2004)
- Feng, Li, and Liu (2004)
- Christ, Kim, and Yamazaki (2005)
- Kim, Sachrajda, and. Sharpe (2005)
- Bernard, Lage, Meissner, and Rusetsky (2008)
- Ishizuka (2009)
- Bour, Koenig, Lee, Hammer, and Meissner (2011)
- Davoudi and Savage (2011) (2014)
- Leskovec and Prelovsek (2012)
- Gockeler, Horsley, Lage, Meissner, Rakow (2012)
- Polejaeva and Rusetsky (2012)
- Hansen and Sharpe (2012), (2013)
- RB and Davoudi (2012), (2013)
- Li and Liu (2013)
- Guo, Dudek, Edwards, and Szczepaniak (2013)
- RB, Davoudi, and Luu (2013)
- RB, Davoudi, Luu and Savage (2013)
- Bernard, Lage,Meissner, and Rusetsky (2011)
- RB (2014)
- Li, Li, Liu (2014)
- …

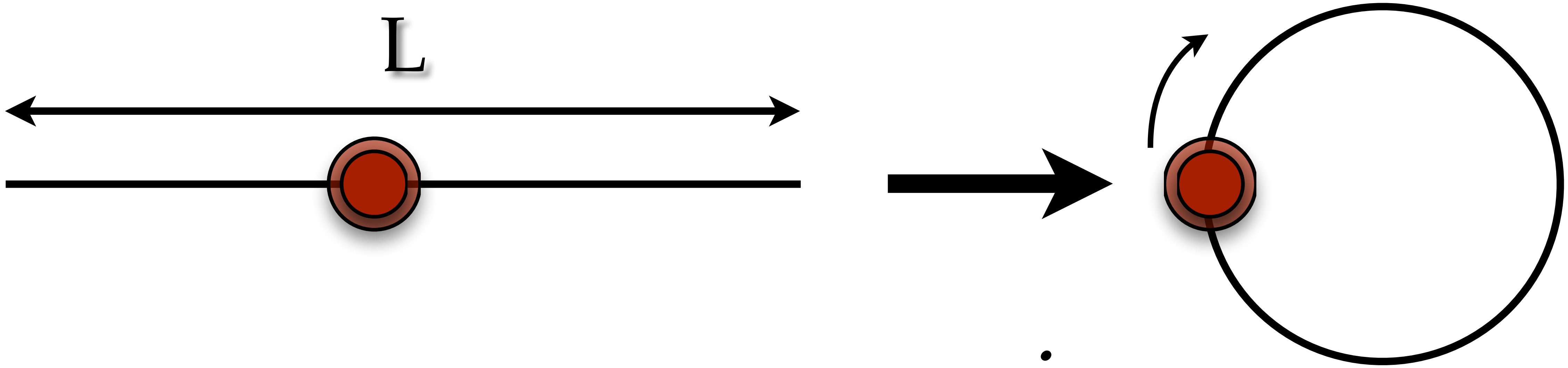
Reinventing the *quantum-mechanical* wheel

(in 1+1 dimensions)



Reinventing the *quantum-mechanical* wheel

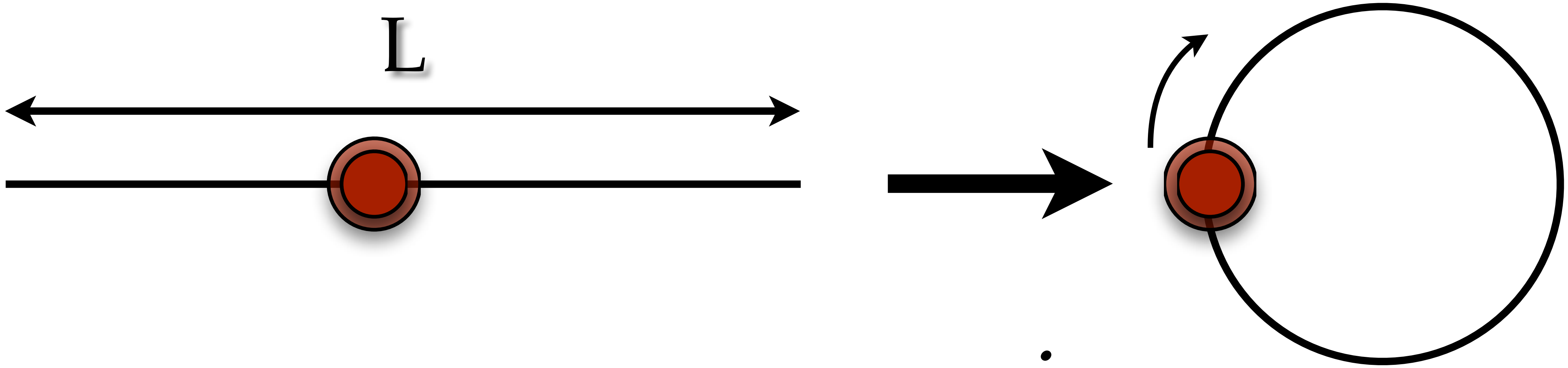
(in 1+1 dimensions)



$$\phi(x) \sim e^{ipx}$$

Reinventing the *quantum-mechanical* wheel

(in 1+1 dimensions)



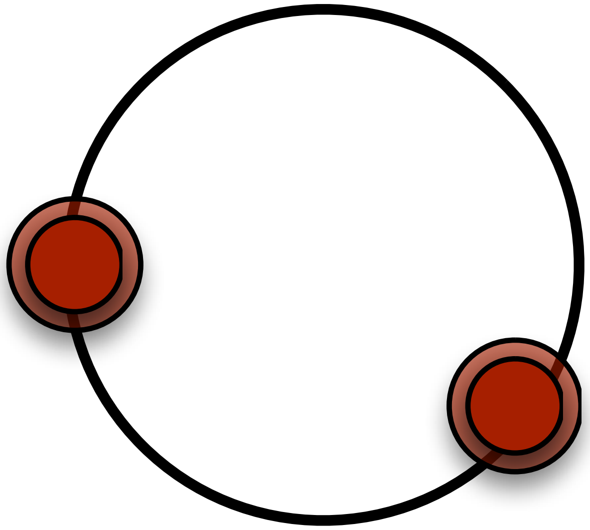
$$\phi(x) \sim e^{ipx}$$

Periodicity: $\phi(L) = \phi(0)$

Quantization condition: $\boxed{L\ p_n = 2\pi n}$

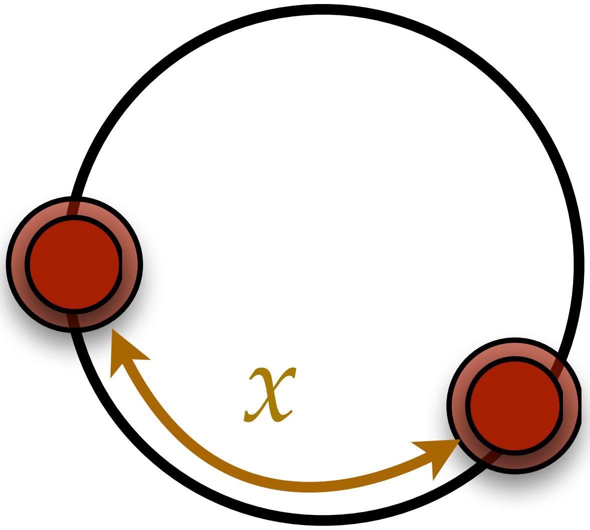
Reinventing the *quantum-mechanical* wheel

Two particles:



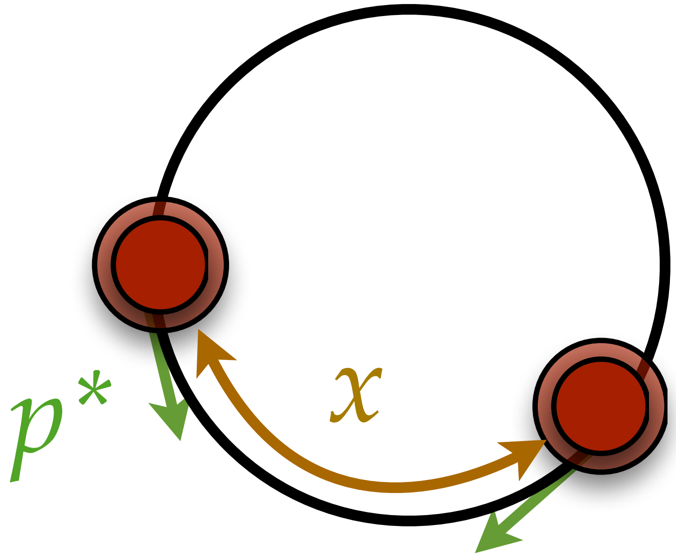
Reinventing the *quantum-mechanical* wheel

Two particles:



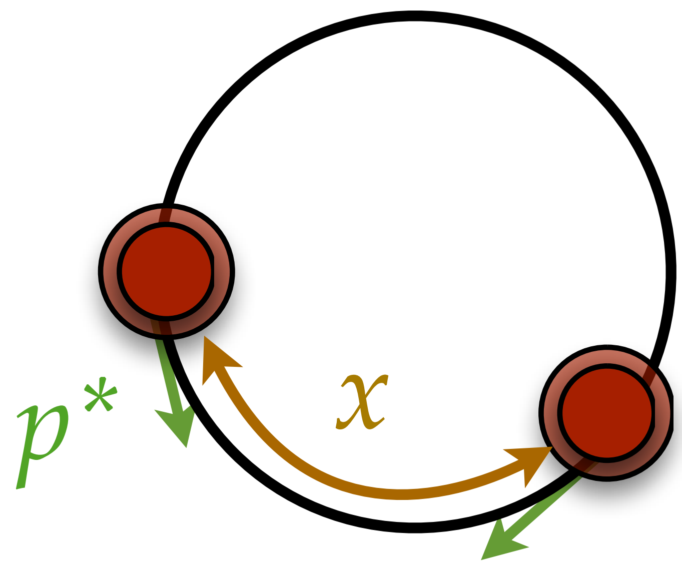
Reinventing the *quantum-mechanical* wheel

Two particles:



Reinventing the *quantum-mechanical* wheel

Two particles:



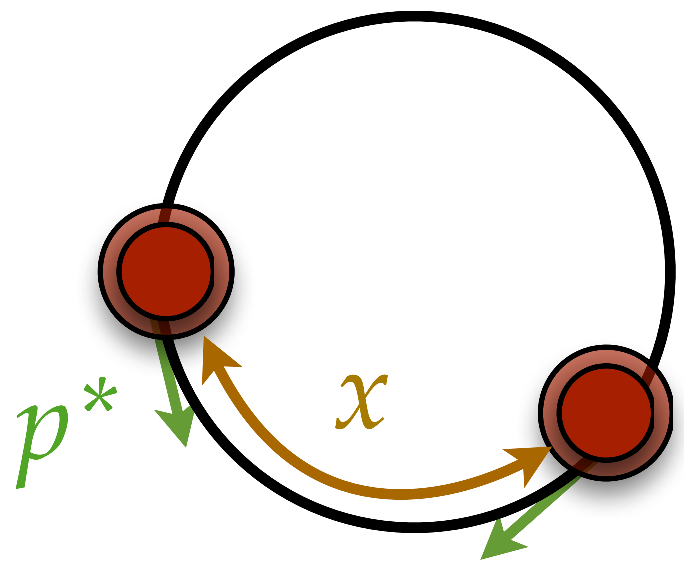
infinite volume scattering phase shift

$$\psi(x) \sim e^{ip^*x + i2\delta(p^*)}$$

Asymptotic wavefunction

Reinventing the *quantum-mechanical* wheel

Two particles:



$$\psi(x) \sim e^{ip^* x + i2\delta(p^*)}$$

infinite volume scattering phase shift

Asymptotic wavefunction

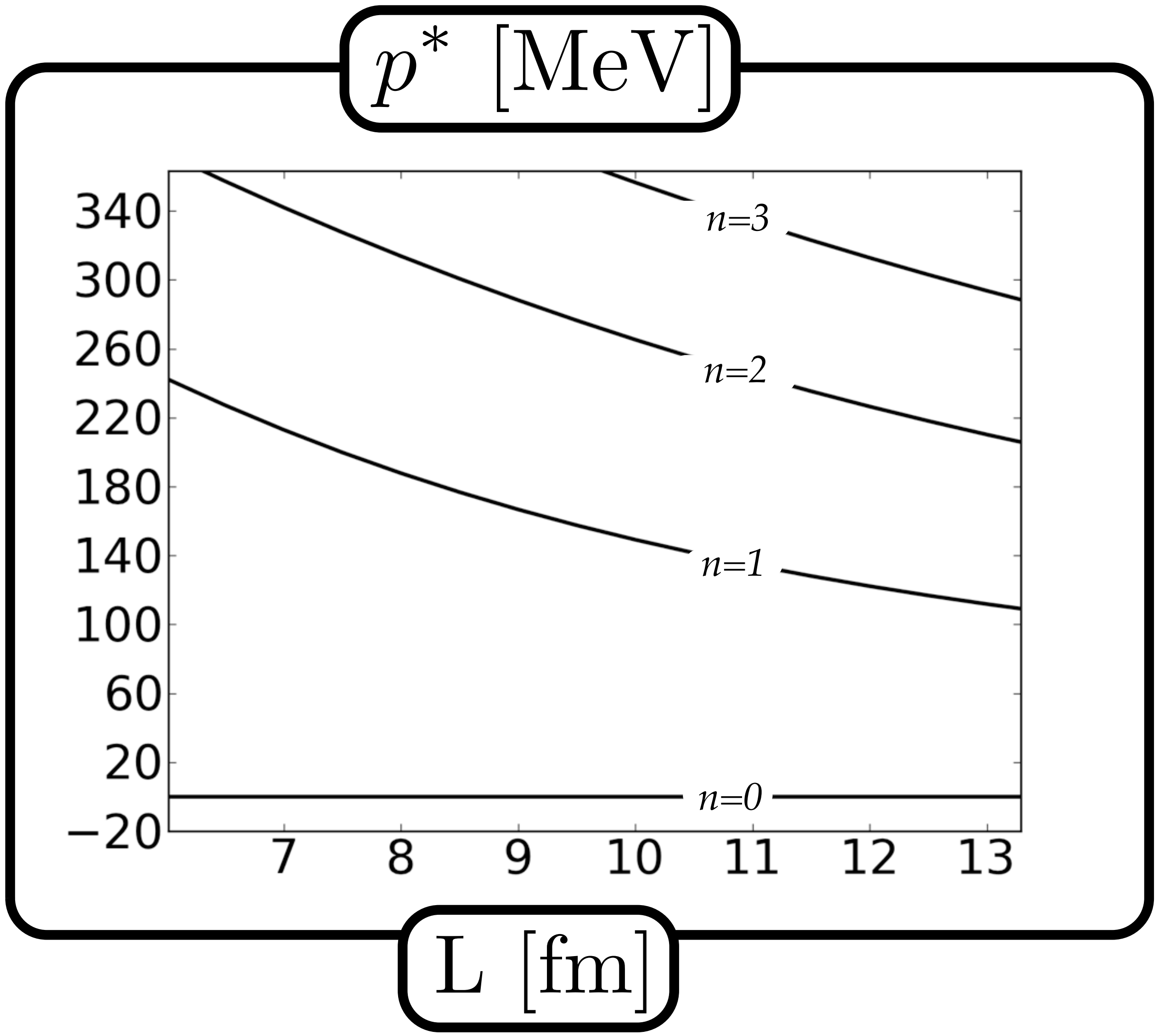
Periodicity: $\psi(L) = \psi(0)$

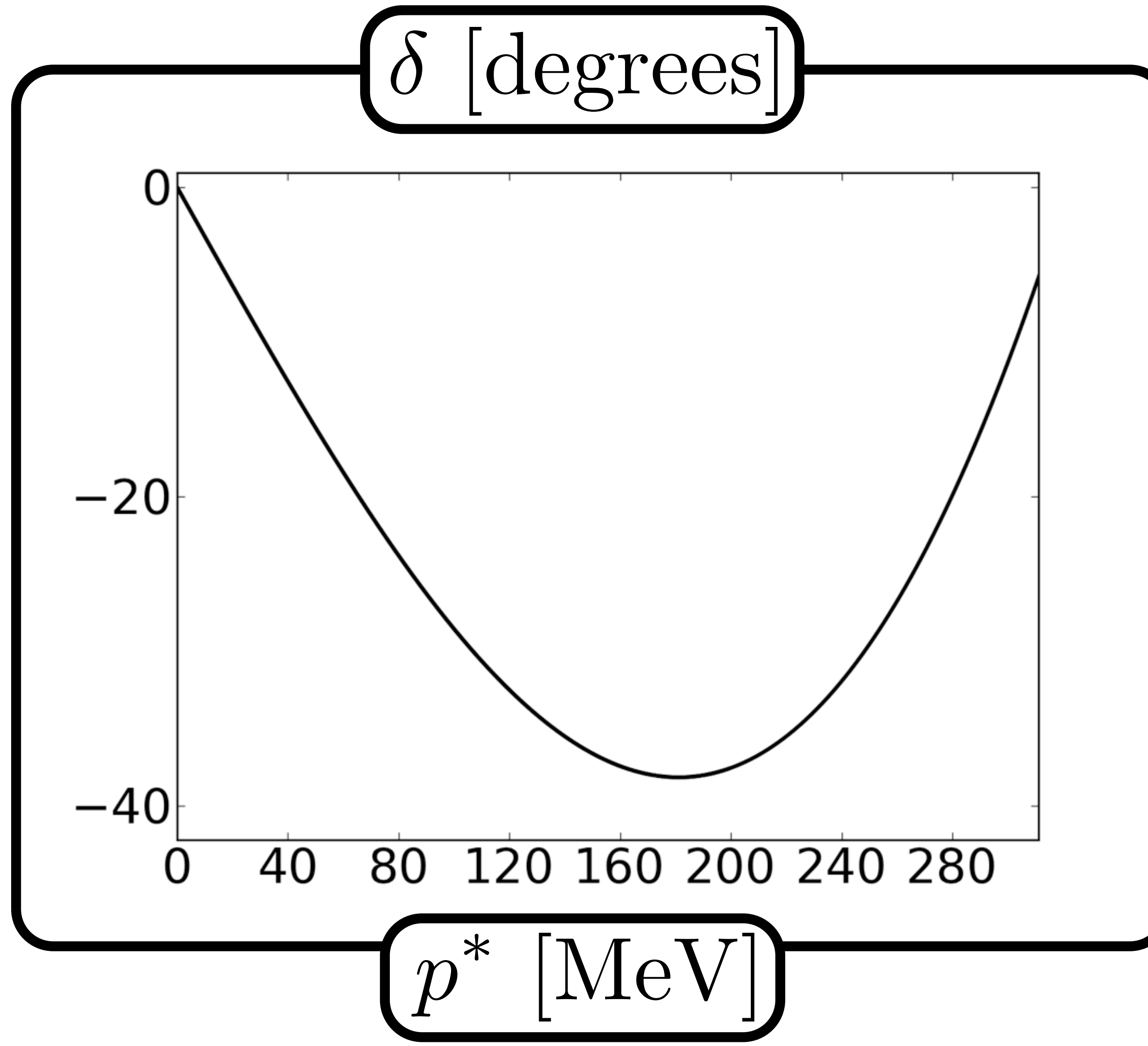
Quantization condition:

$$L\ p_n^* + 2\delta(p_n^*) = 2\pi n$$

Reinventing the *quantum-mechanical* wheel

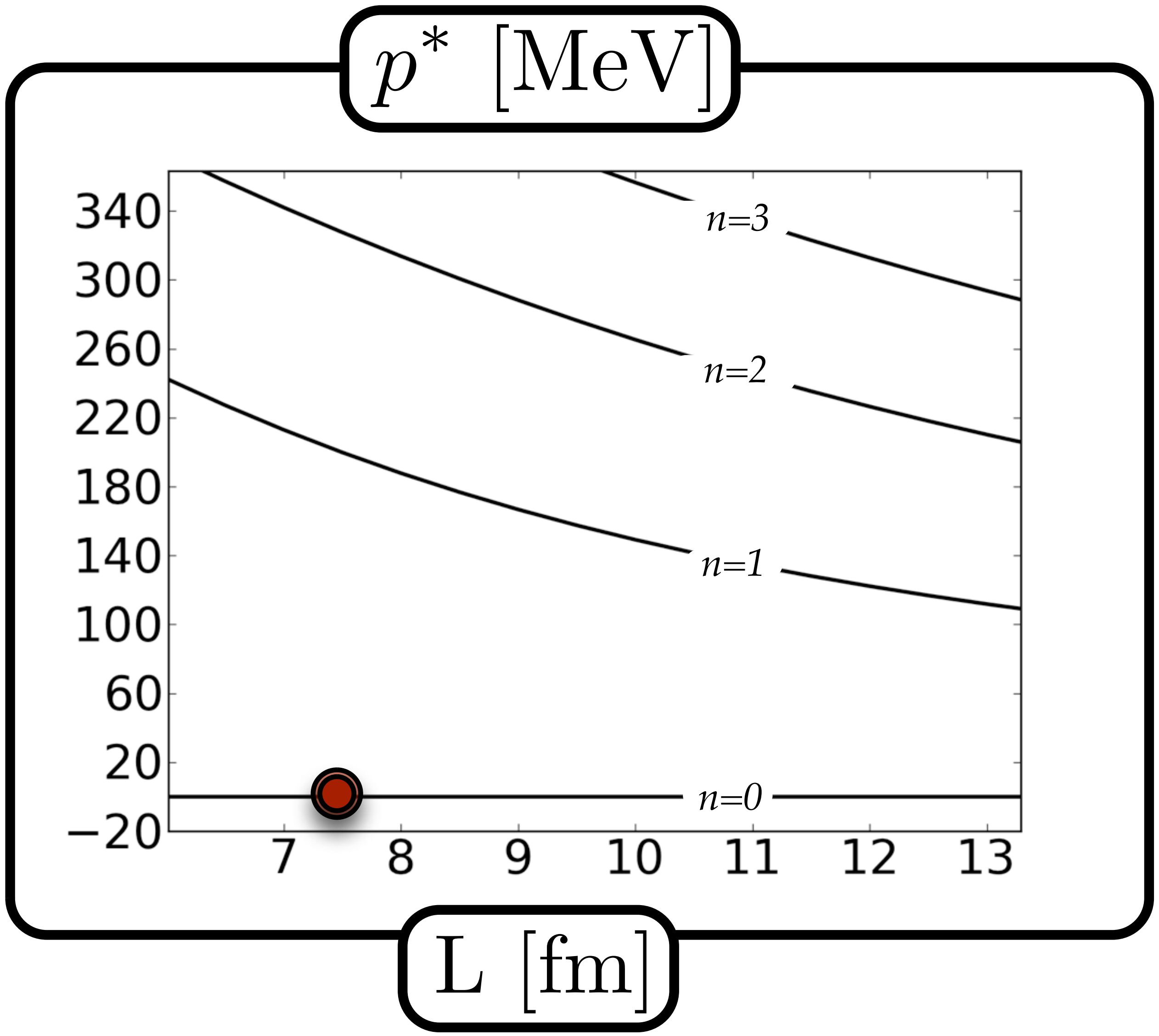
$$L\ p_n^* + 2\delta(p_n^*) = 2\pi n$$

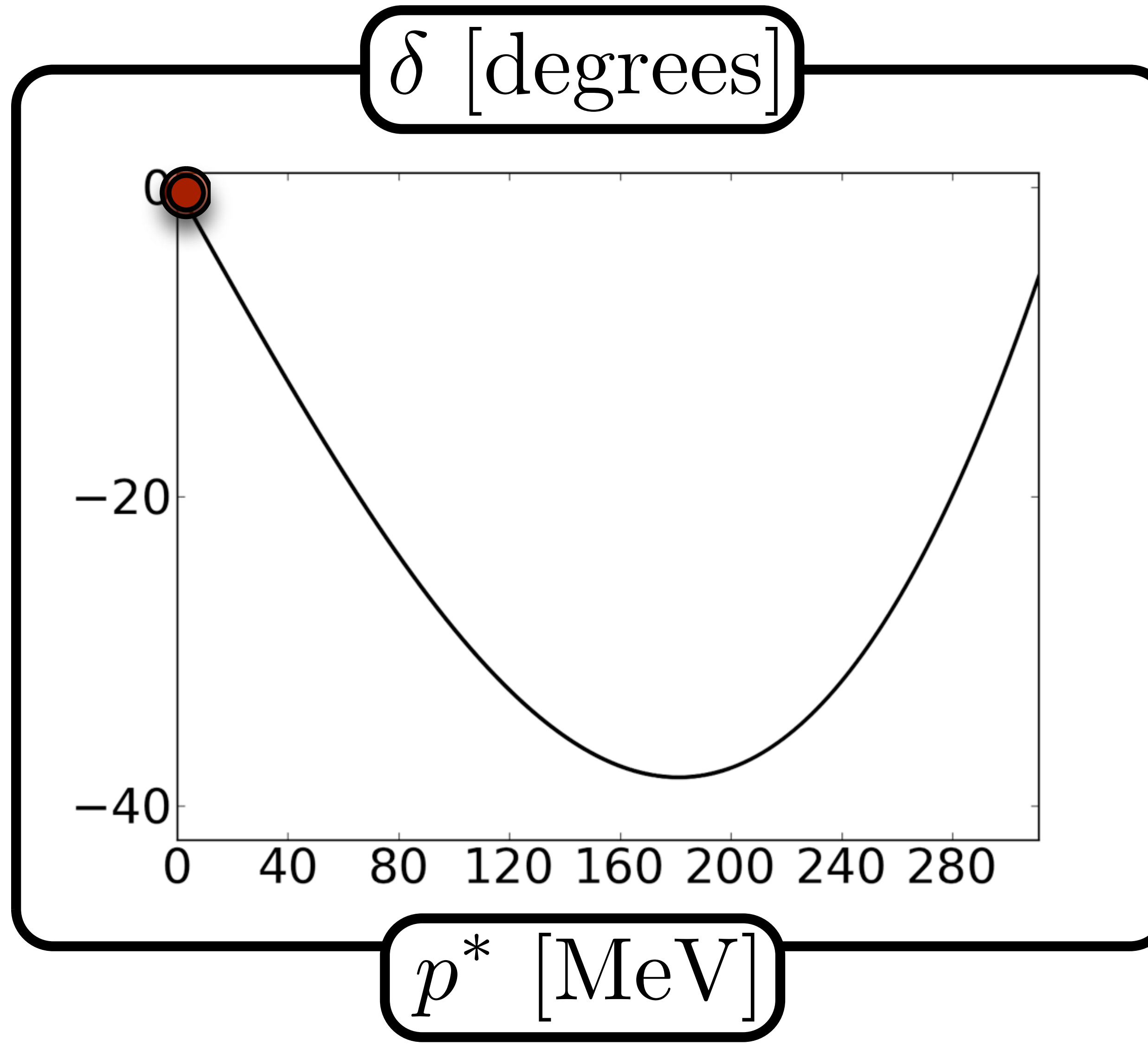




Reinventing the *quantum-mechanical* wheel

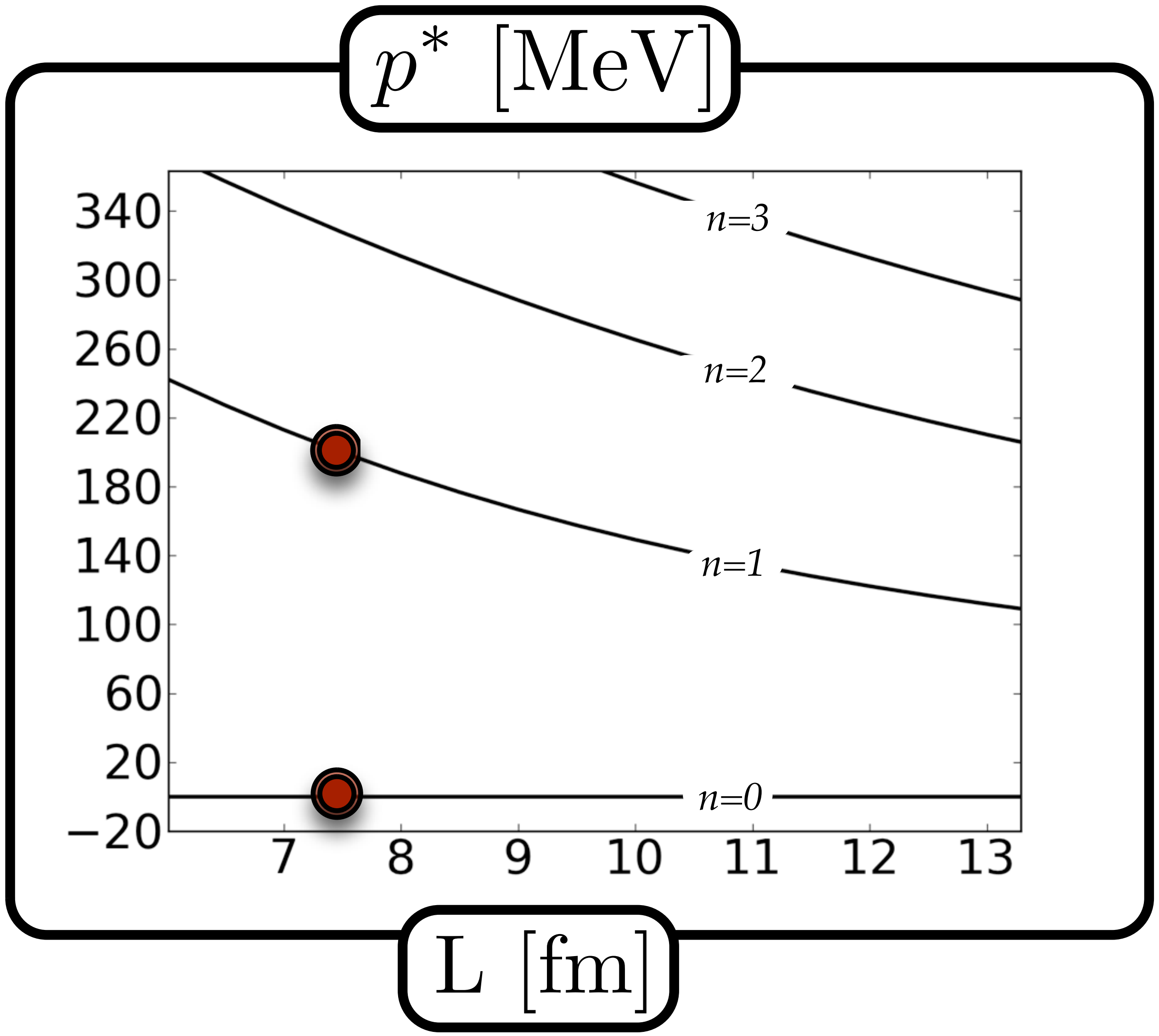
$$L\ p_n^* + 2\delta(p_n^*) = 2\pi n$$

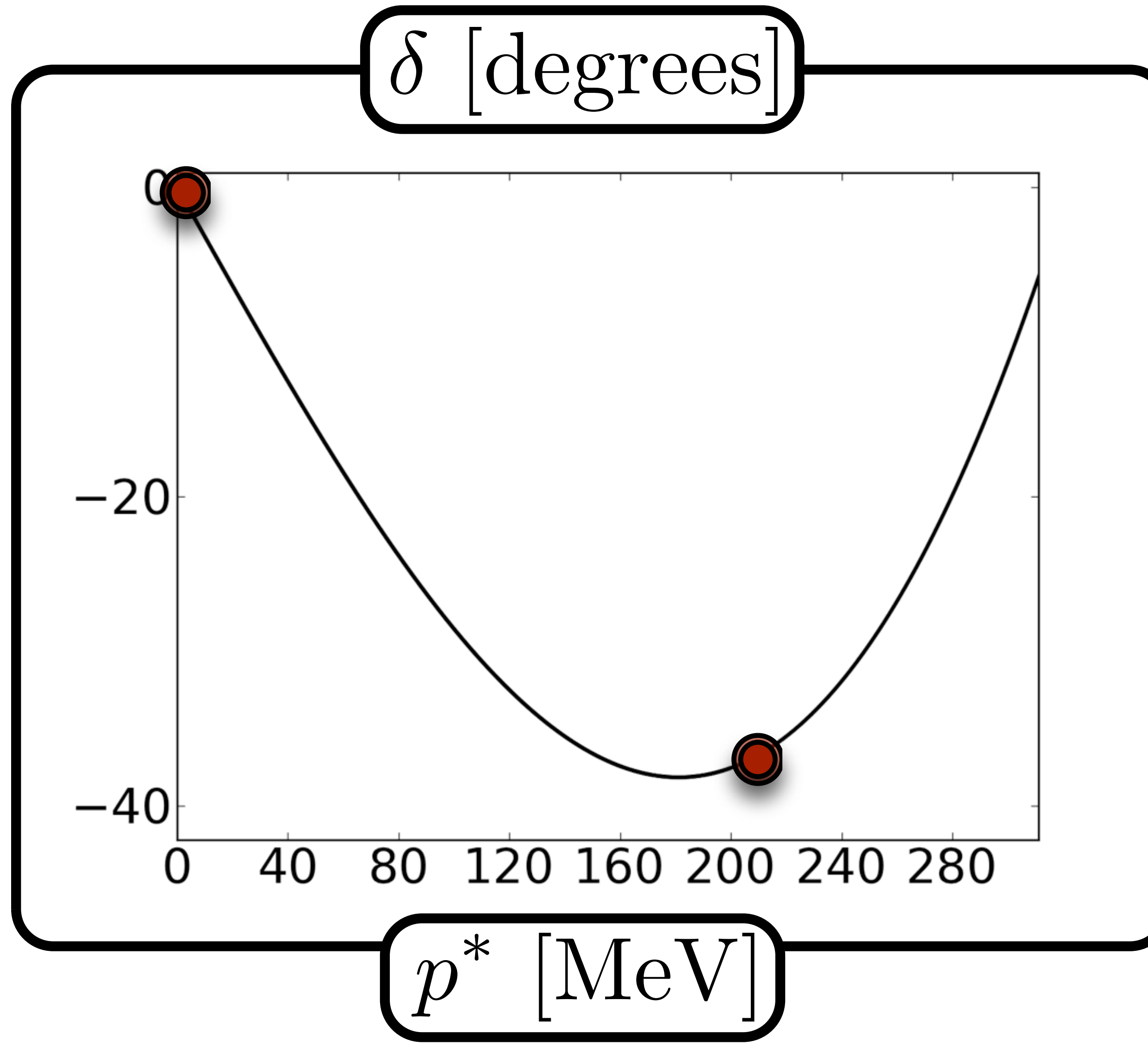




Reinventing the *quantum-mechanical* wheel

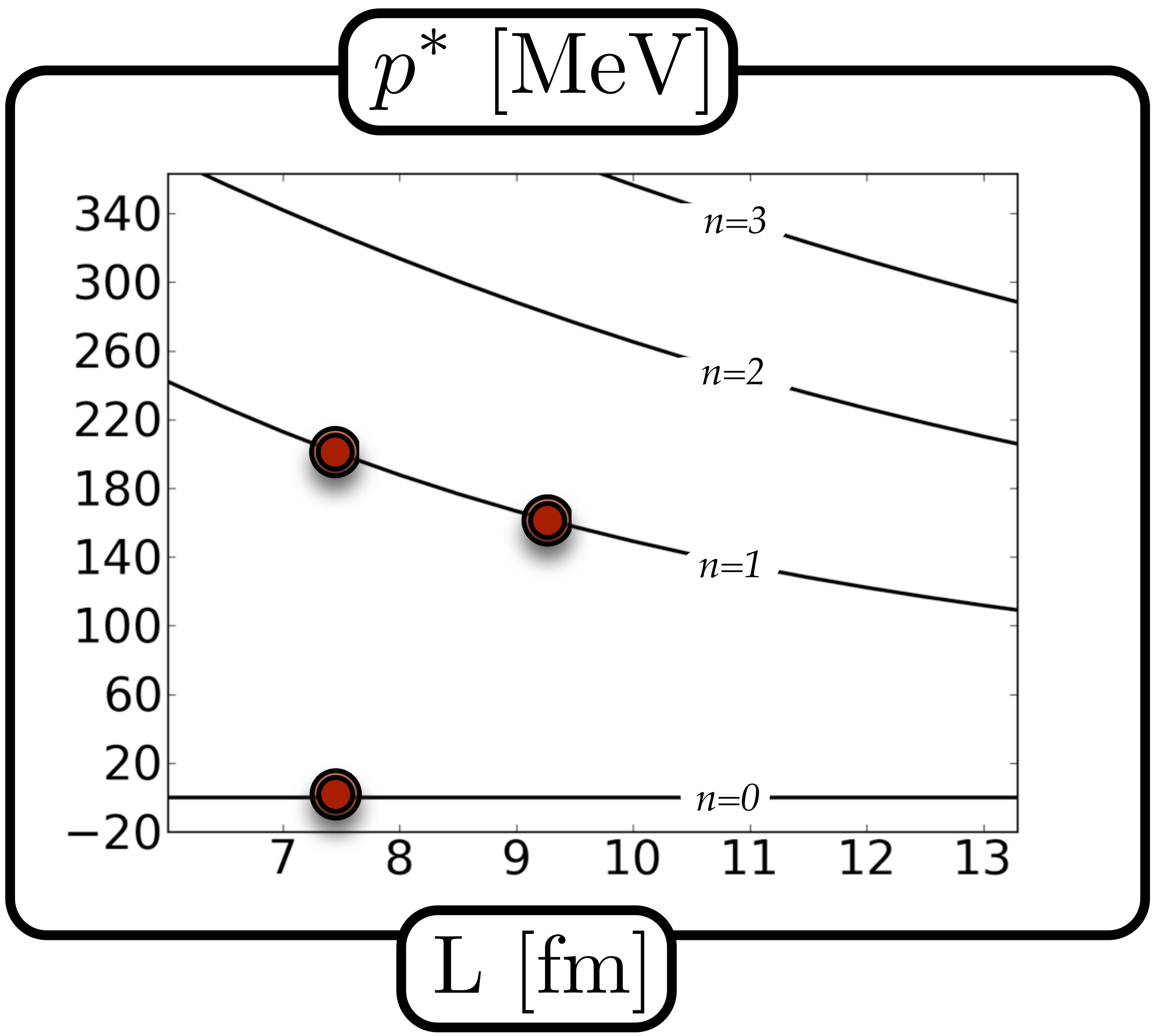
$$L\ p_n^* + 2\delta(p_n^*) = 2\pi n$$

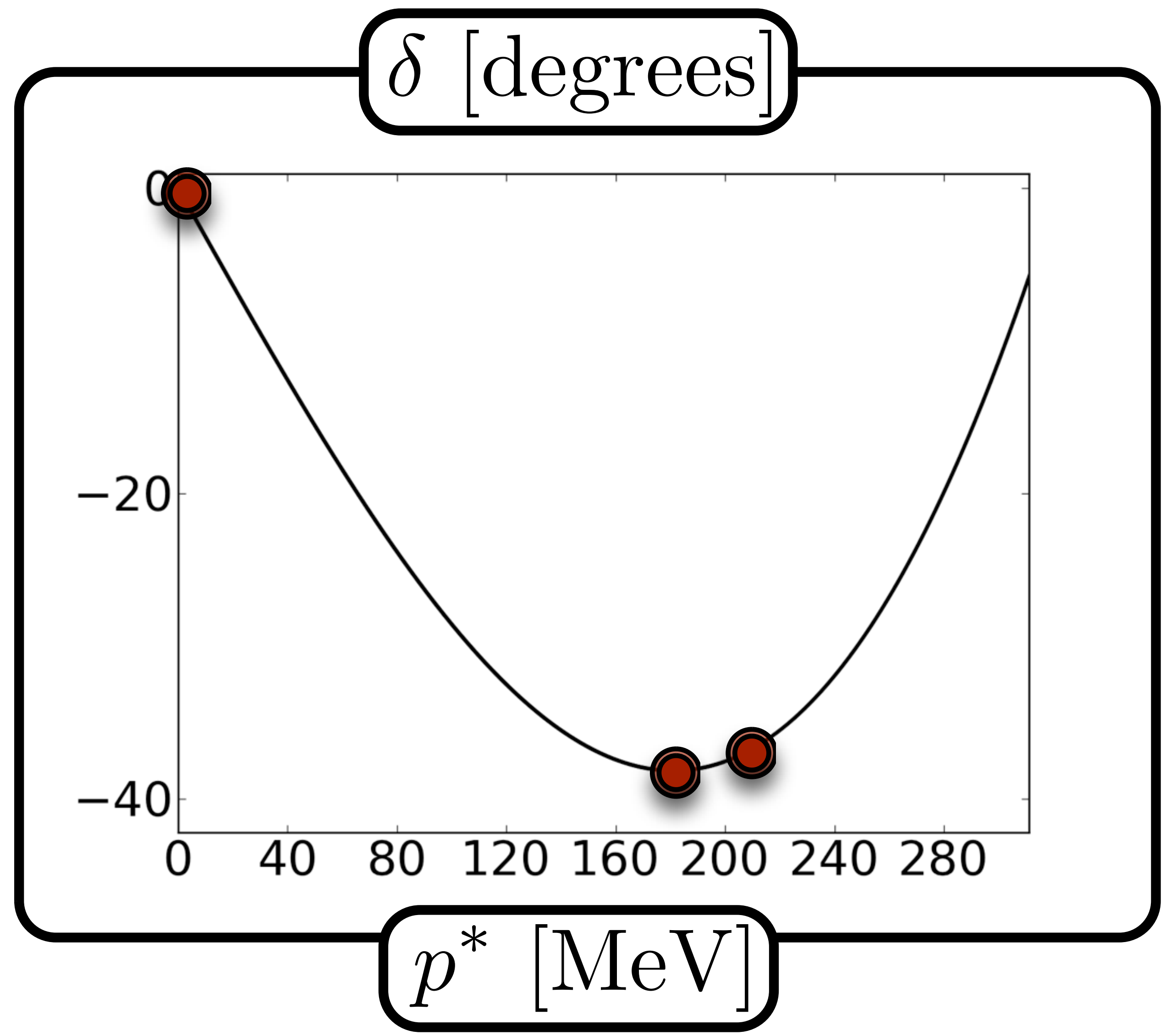




Reinventing the *quantum-mechanical* wheel

$$L\ p_n^* + 2\delta(p_n^*) = 2\pi n$$





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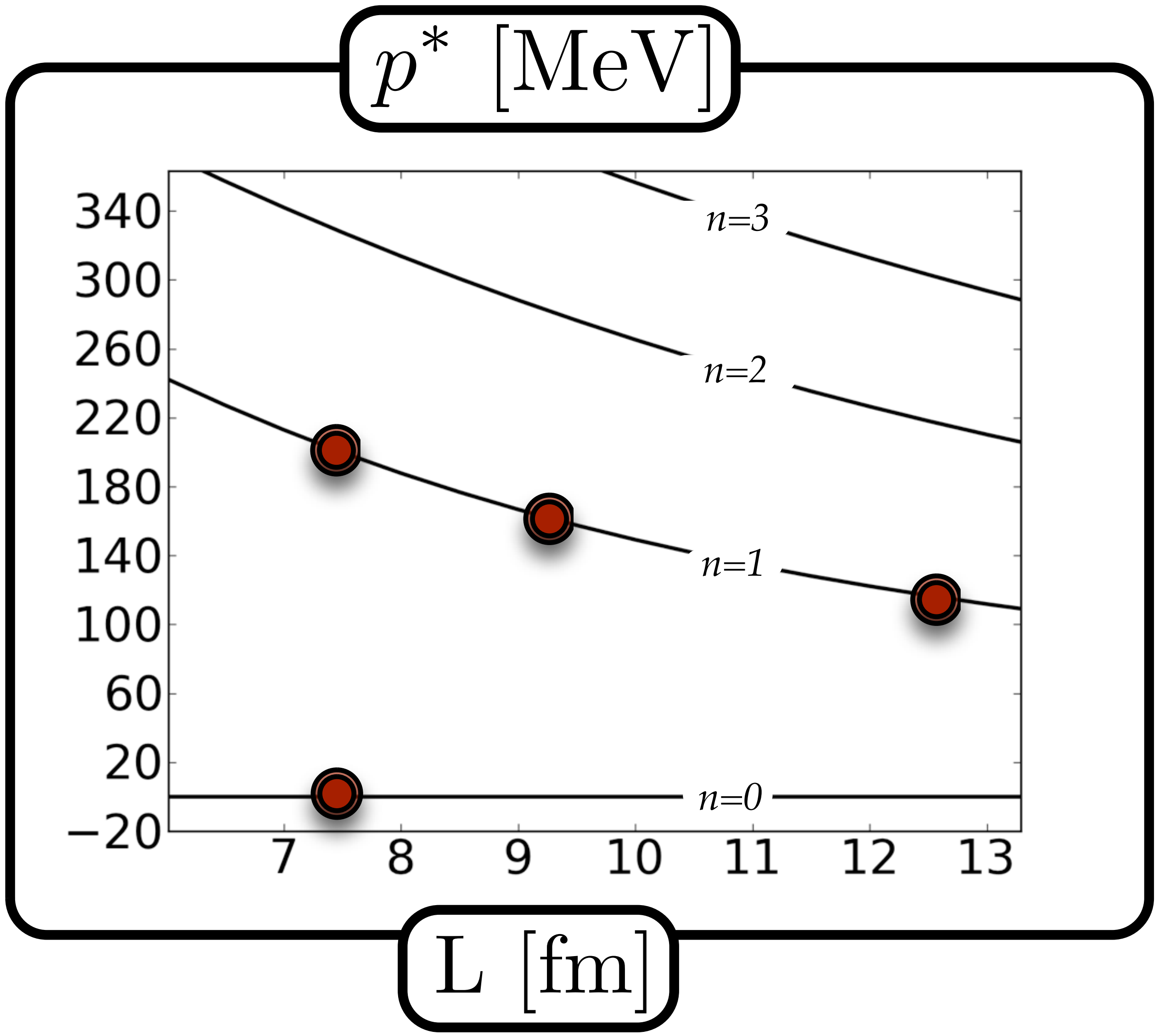
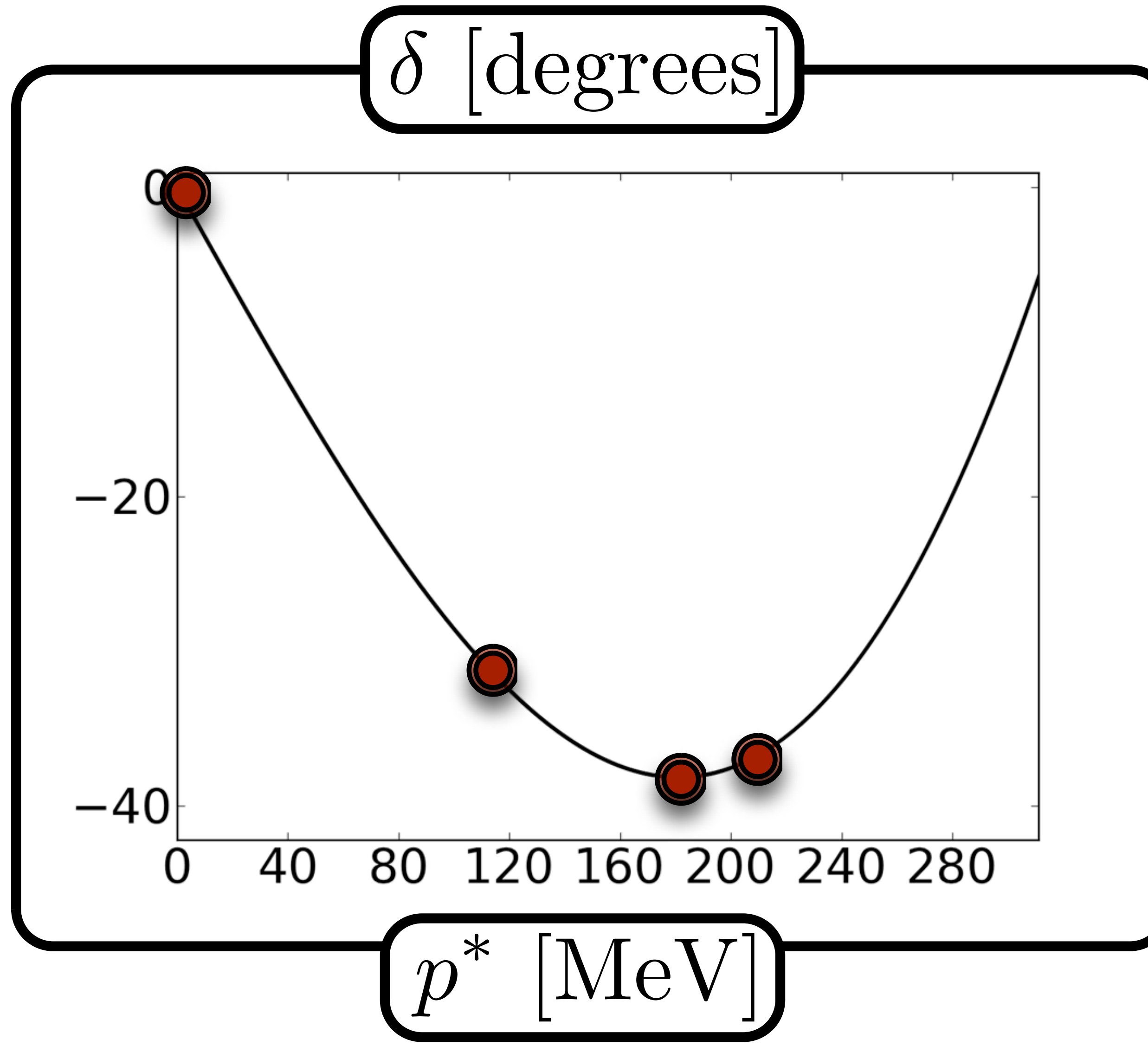


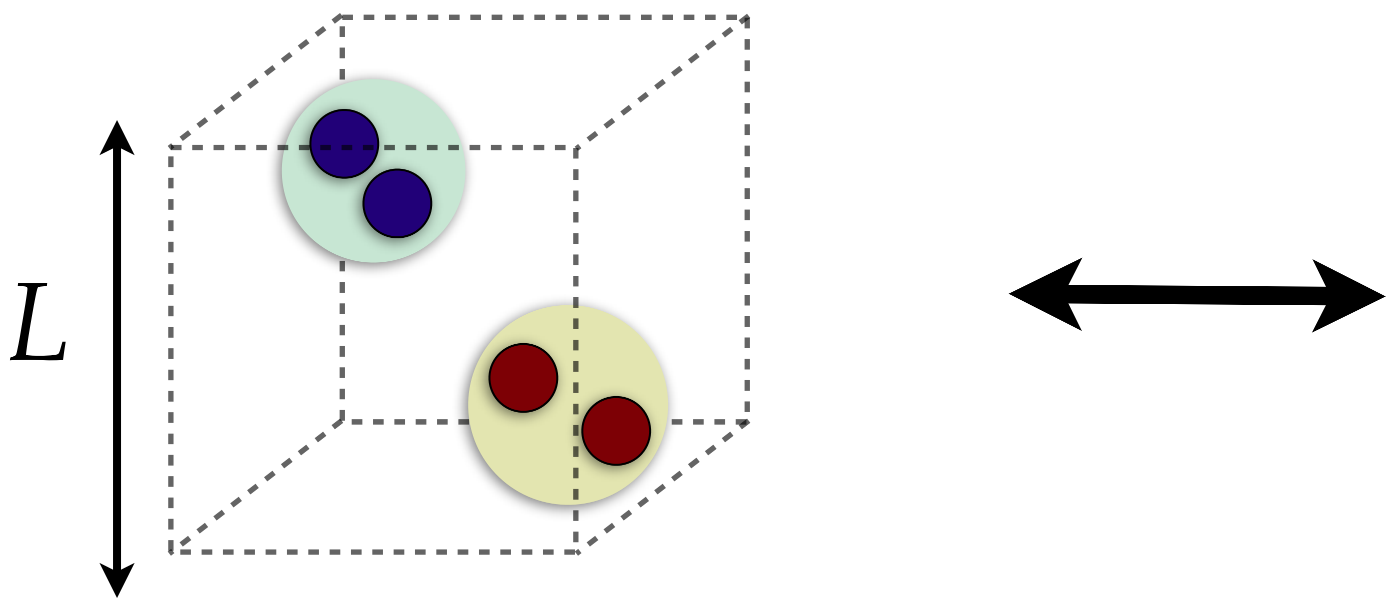


3+1D result

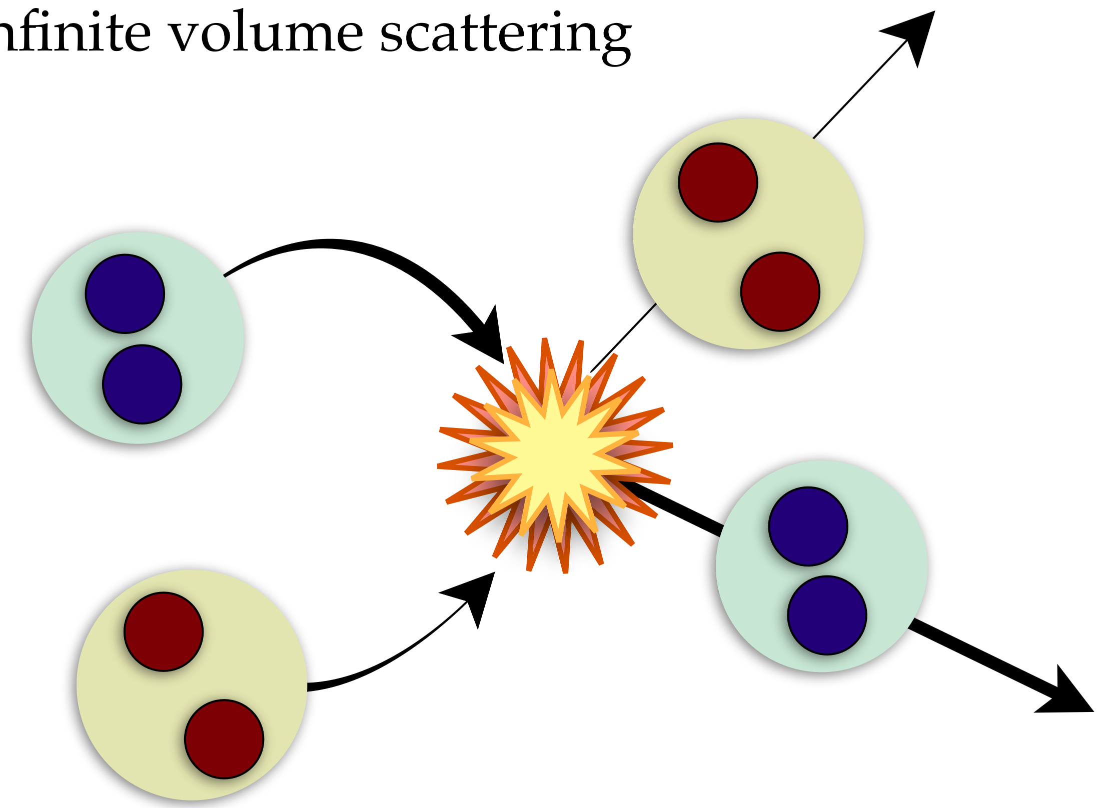
RB [PRD] (2014)

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^V\right] = 0$$

Finite volume spectrum



Infinite volume scattering



- **Model independent & non-perturbative**
- **Universal:** nuclear physics, atomic physics, etc
- **Arbitrary quantum numbers:** relativity, spin, masses, momenta, angular momentum, inelasticities, etc
- **General volumes with any boundary conditions:** periodic, anti-periodic, or any linear combination on any rectangular prism

3+1D result

RB [PRD] (2014)

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

Finite volume spectrum

Infinite volume scattering

Compactly summarizes & generalizes all that has been written on the two-body sector in the literature

A long list of extensions of the Lüscher formalism

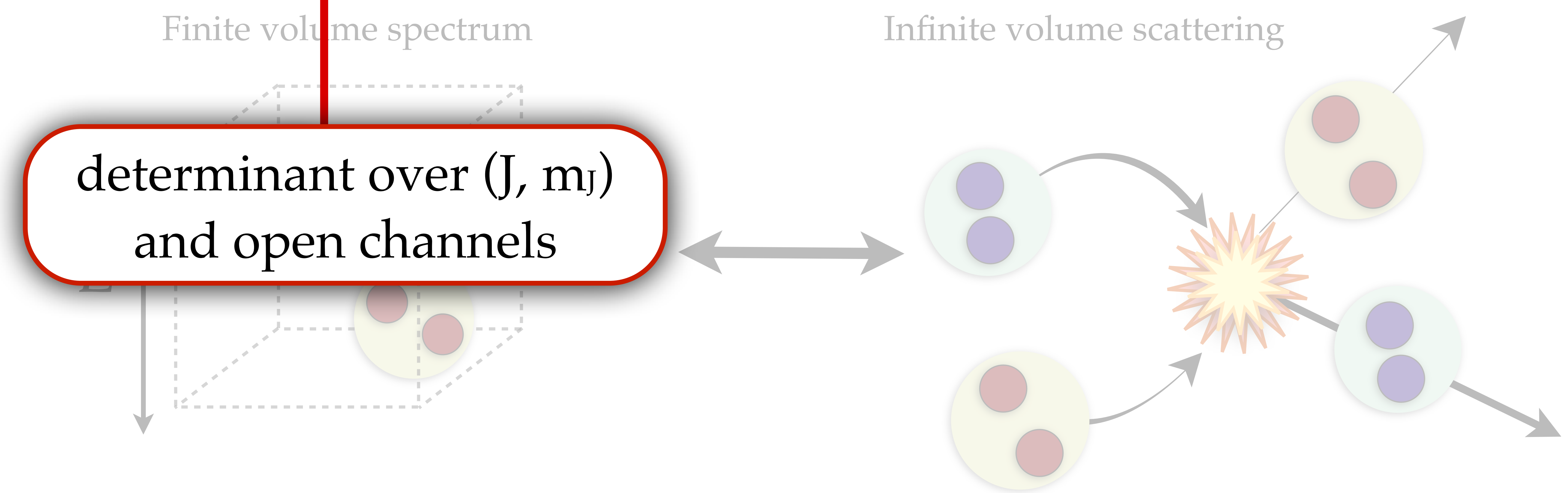
- **M. Lüscher (1086), (1991)** *("Lüscher Formalism")*
- L. Maiani and M. Testa (1990)
- K. Rummukainen and S. A. Gottlieb (1995)
- S. Beane, P. Bedaque, A. Parreno, and M. Savage (2004), (2005)
- P. Bedaque (2004)
- X. Li and C. Liu (2004)
- W. Detmold and M. J. Savage (2004)
- X. Feng, X. Li, and C. Liu (2004)
- N. H. Christ, C. Kim, and T. Yamazaki (2005)
- C. Kim, C. Sachrajda, and S. R. Sharpe (2005)
- V. Bernard, M. Lage, U.-G. Meissner, and A. Rusetsky (2008)
- N. Ishizuka (2009)
- S. Bour, S. Koenig, D. Lee, H.-W. Hammer, and U.-G. Meissne (2011)
- Z. Davoudi and M. J. Savage (2011) (2014)
- L. Leskovec and S. Prelovsek (2012)
- M. Gockeler, R. Horsley, M. Lage, U.-G. Meissner, P. Rakow (2012)
- K. Polejaeva and A. Rusetsky (2012)
- M. T. Hansen and S. R. Sharpe (2012), (2013)
- RB and Z. Davoudi (2012), (2013)
- N. Li and C. Liu (2013)
- P. Guo, J. Dudek, R. Edwards, and A. P. Szczepaniak (2013)
- RB, Z. Davoudi, and T. C. Luu (2013)
- RB, Z. Davoudi, T. C. Luu and M. J. Savage (2013) (2013)
- V. Bernard, M. Lage, U.-G. Meissner, and A. Rusetsky (2011)
- N. Li, S. Y. Li, C. Liu (2014)
- RB (2014)
- Ning Li, Song-Yuan Li, Chuan Liu (2014)
- ...

3+1D result

RB [PRD] (2014)

$$\det\left[\mathcal{M}^{-1}+\delta\mathcal{G}^V\right]=0$$

determinant over (J, m_J) and open channels



3+1D result

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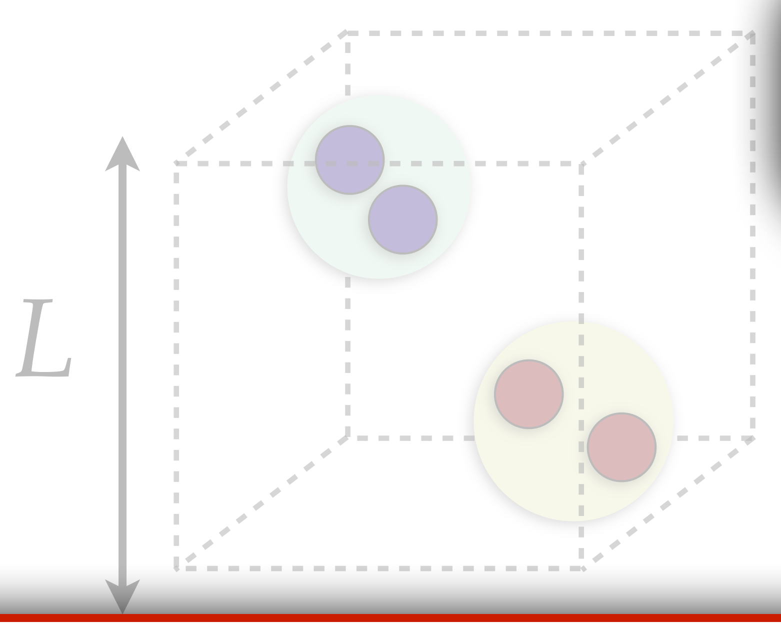
Finite volume spectrum

Infinite volume scattering



Physical scattering amplitude

(Can couple any number of channels)

When fitting the spectrum one may choose to parametrize the scattering amplitude using:

- low-energy effective field theory
- effective range expansion
- analyticity
- potential method
- ...

Warning: modeling may creep in here

e.g. positive parity, isosinglet, two-nucleon channel (deuteron...)

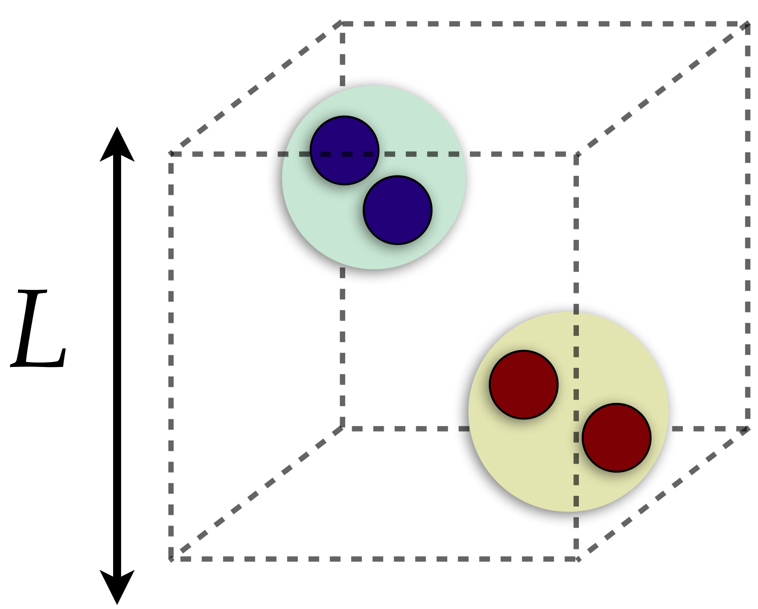
$$\begin{pmatrix} \mathcal{M}_1^S & \mathcal{M}_1^{SD} & 0 & \\ \mathcal{M}_1^{DS} & \mathcal{M}_1^D & 0 & \\ 0 & 0 & \mathcal{M}_3^D & \\ & & & \ddots \end{pmatrix}$$

3+1D result

RB [PRD] (2014)

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^V\right] = 0$$

Finite volume spectrum



Infinite volume scattering

$$\begin{pmatrix} \delta G^V_{00} & \delta G^V_{01} & \delta G^V_{02} & \\ \delta G^V_{10} & \delta G^V_{11} & \delta G^V_{12} & \\ \delta G^V_{20} & \delta G^V_{21} & \delta G^V_{22} & \\ & & & \ddots \end{pmatrix}$$

Typically a sparse matrix, but in general partial waves do mix (as they should!)

e.g. S-wave at rest

$$k^* \cot \delta_S = \frac{1}{\pi L} \sum_{\mathbf{n}} \frac{1}{\mathbf{n}^2 - (k^* L/2\pi)^2}$$

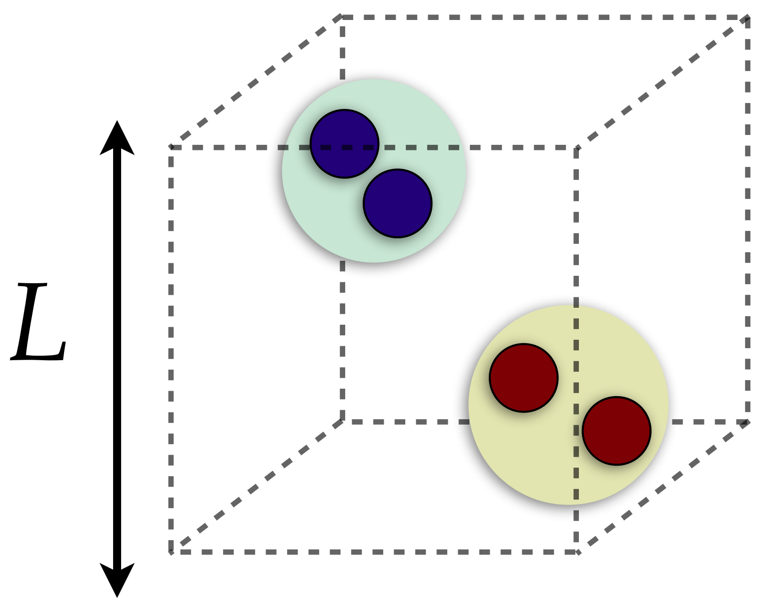
3+1D result

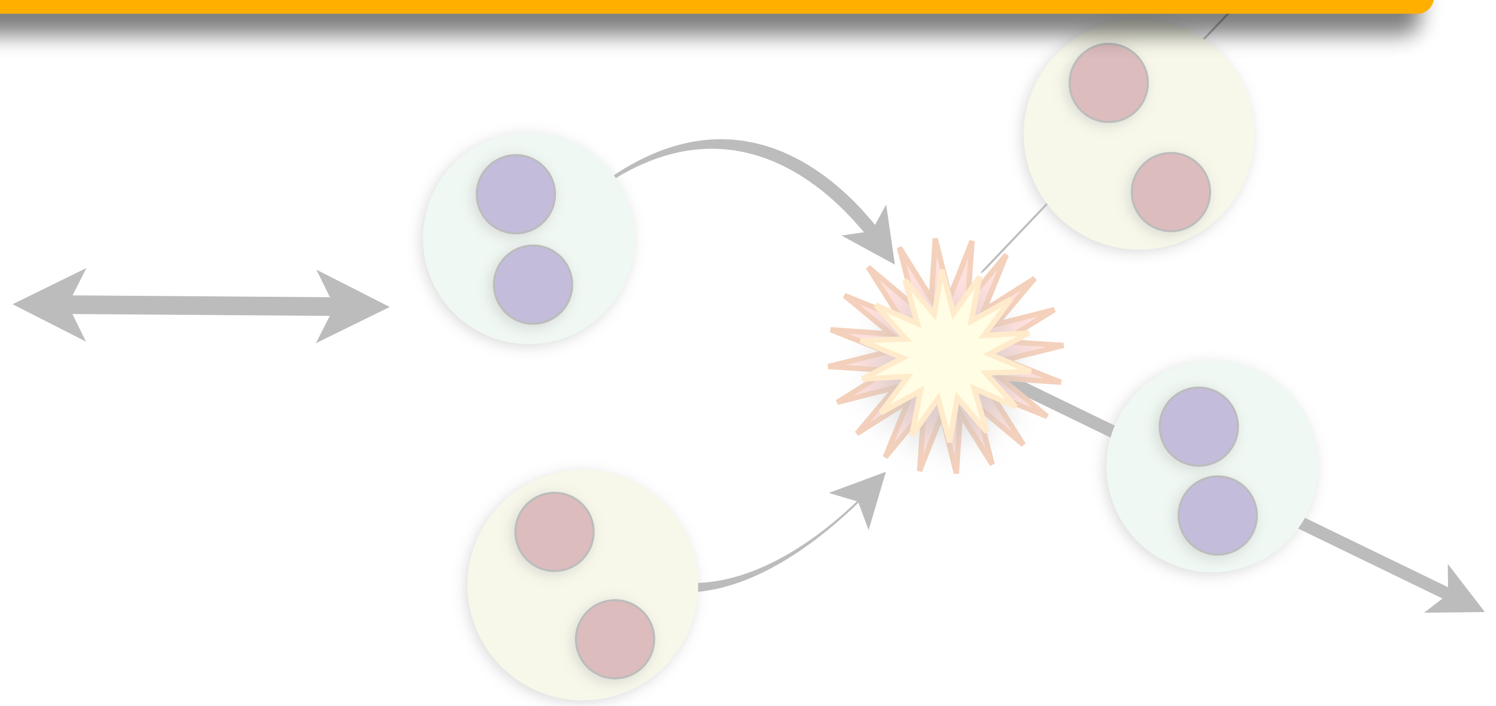
RB [PRD] (2014)

$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^{V}\right] = 0$$

See talk by **Z. Davoudi** on *"Two-baryon systems with twisted boundary"*, Wed. @ 12:50

Finite volume spectrum



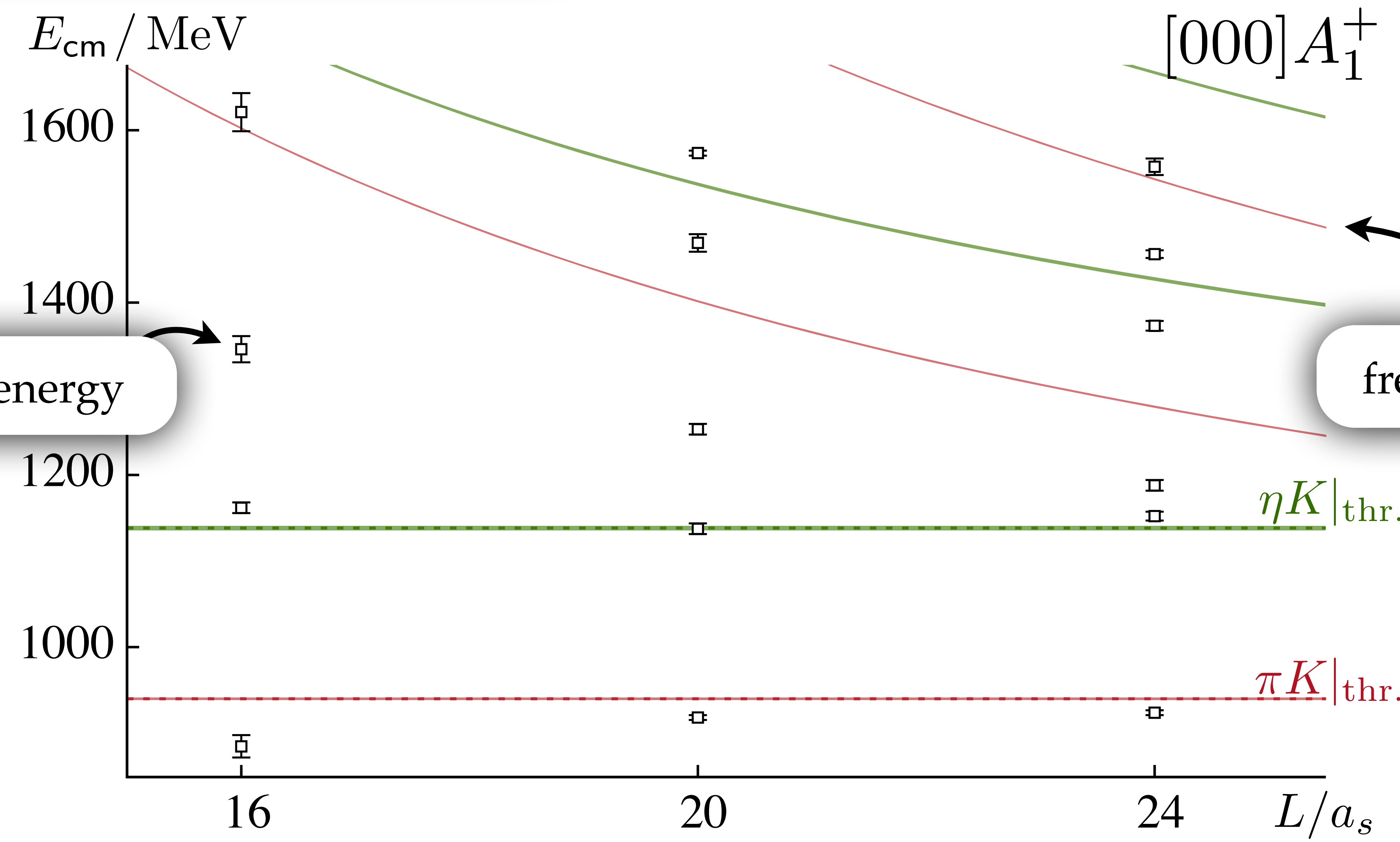


One example: Kπ-Kη

1

Determine finite volume spectra, e.g., Kπ-Kη spectrum using m_π~390MeV

by ***David Wilson***, Dudek, Edwards & Thomas (2014) [Hadron Spectrum Coll]



Lattice QCD energy

free energy level

unboosted
d=**P**L/2π=[000]

Over 100 energy levels determined using **3** different volumes and 5 different types of boosts, **d={[000],[001],[011],[111],[002]}** and allowed cubic rotations.

One example: Kπ-Kη

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Determine finite volume spectra, e.g., Kπ-Kη spectrum using $m_\pi \sim 390$MeV

by ***David Wilson***, Dudek, Edwards & Thomas (2014) [Hadron Spectrum Coll]

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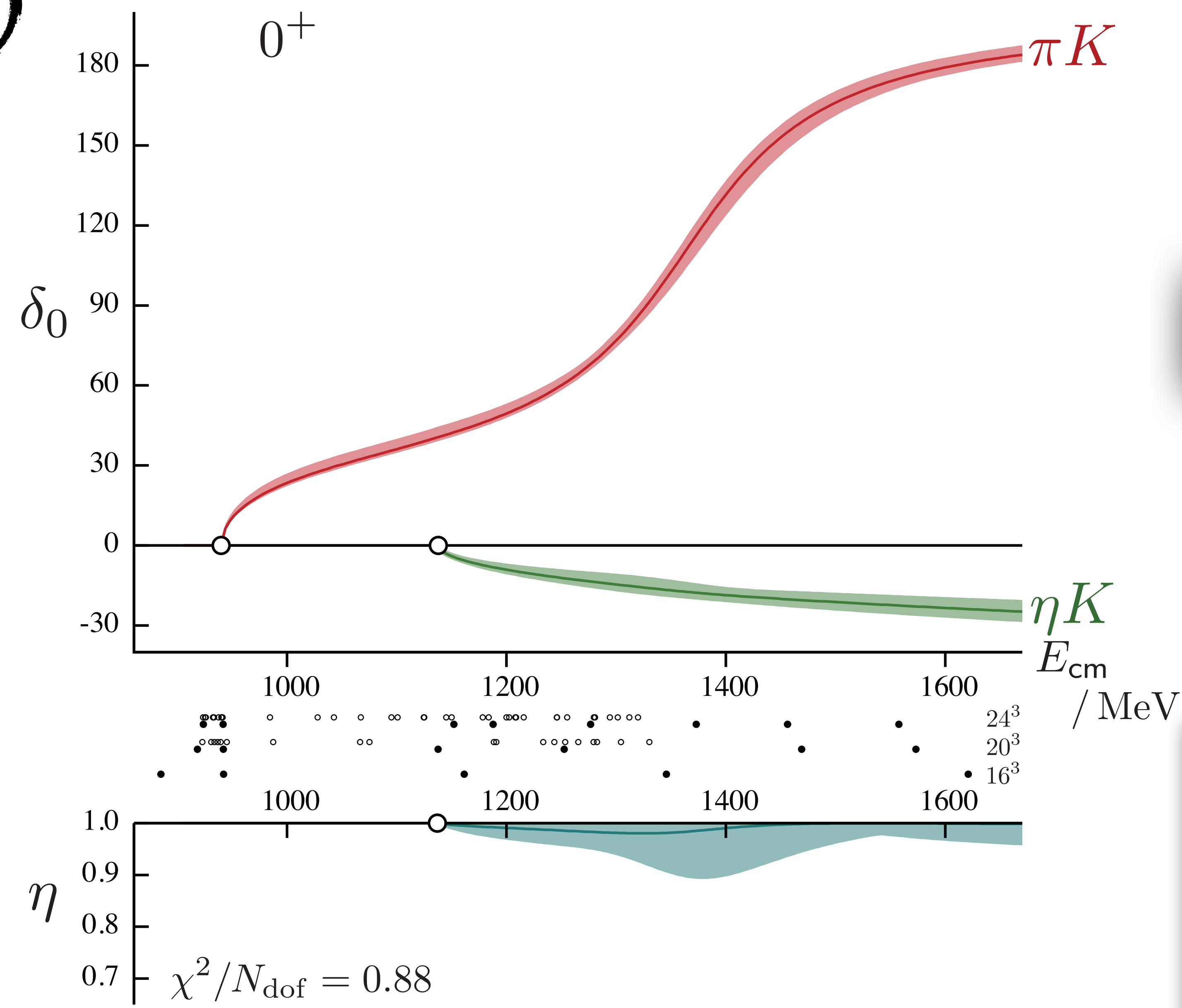
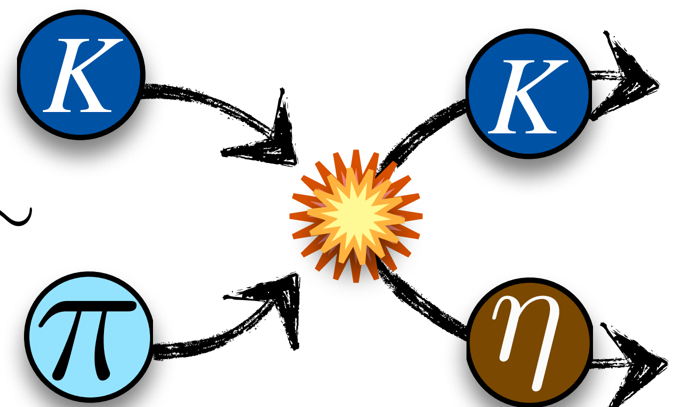
$$\det \left[\mathcal{M}^{-1} + \delta\mathcal{G}^V\right] = 0$$

3



S-wave phase shifts

$$\sqrt{1-\eta^2} \sim$$



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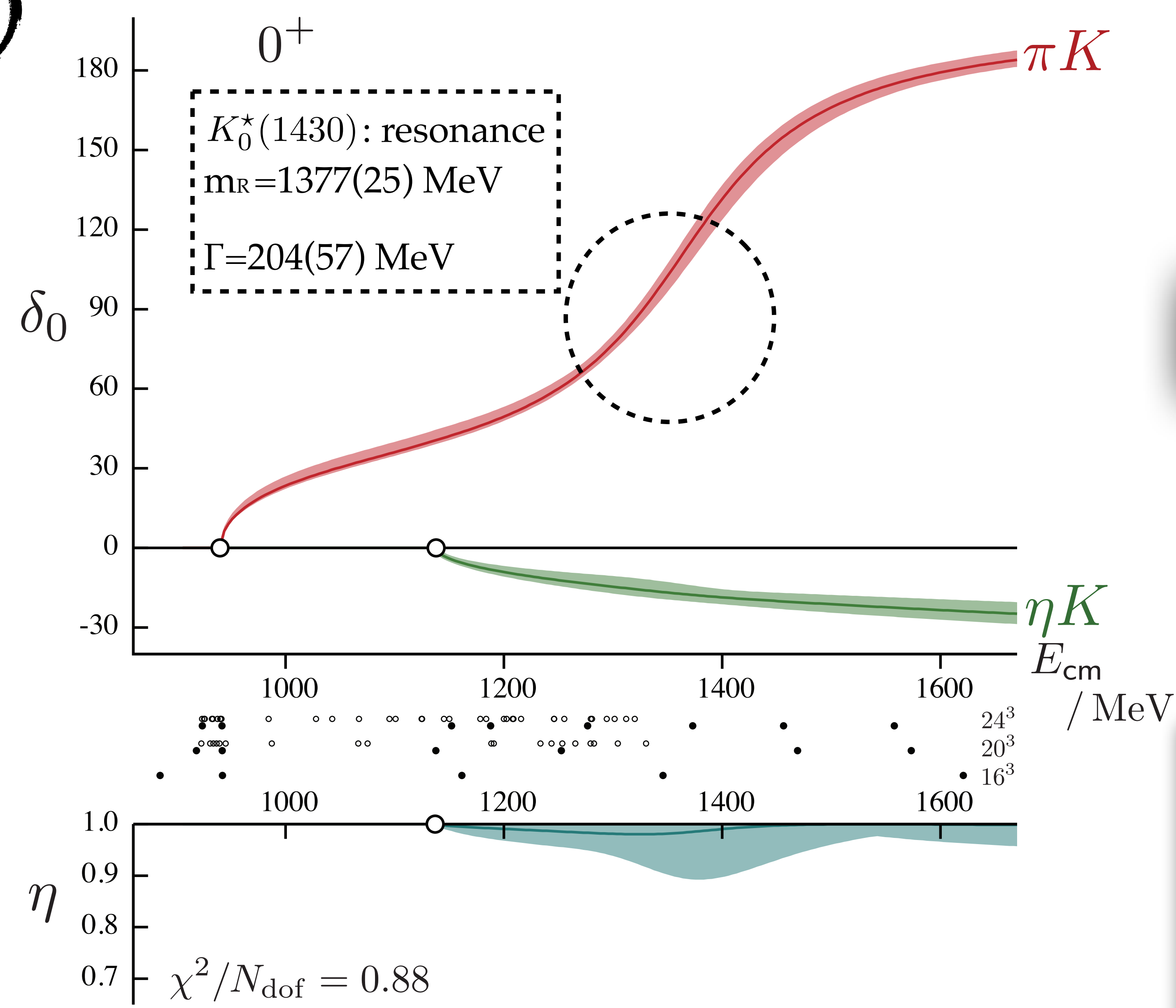

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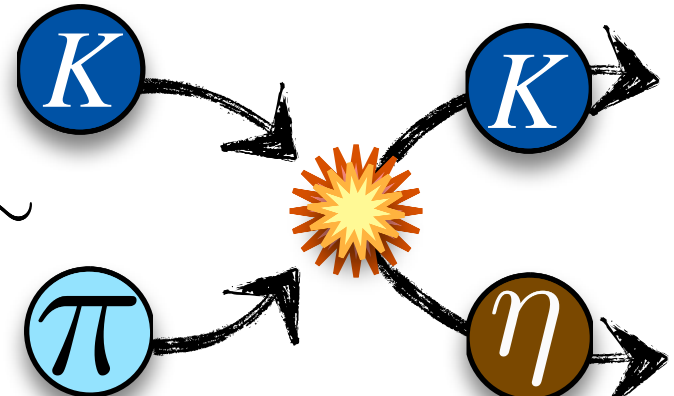

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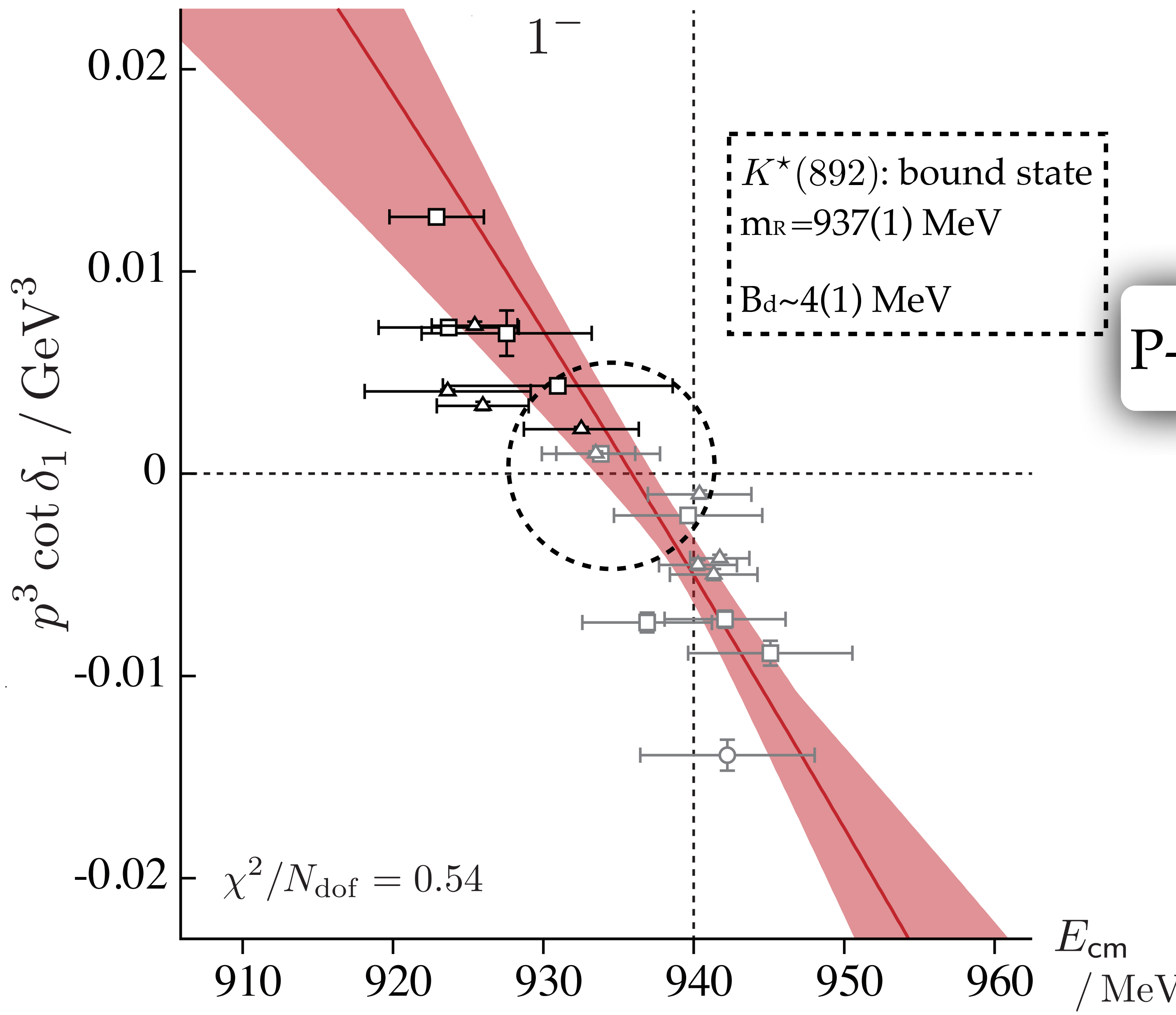
3. 

P-wave phase shifts

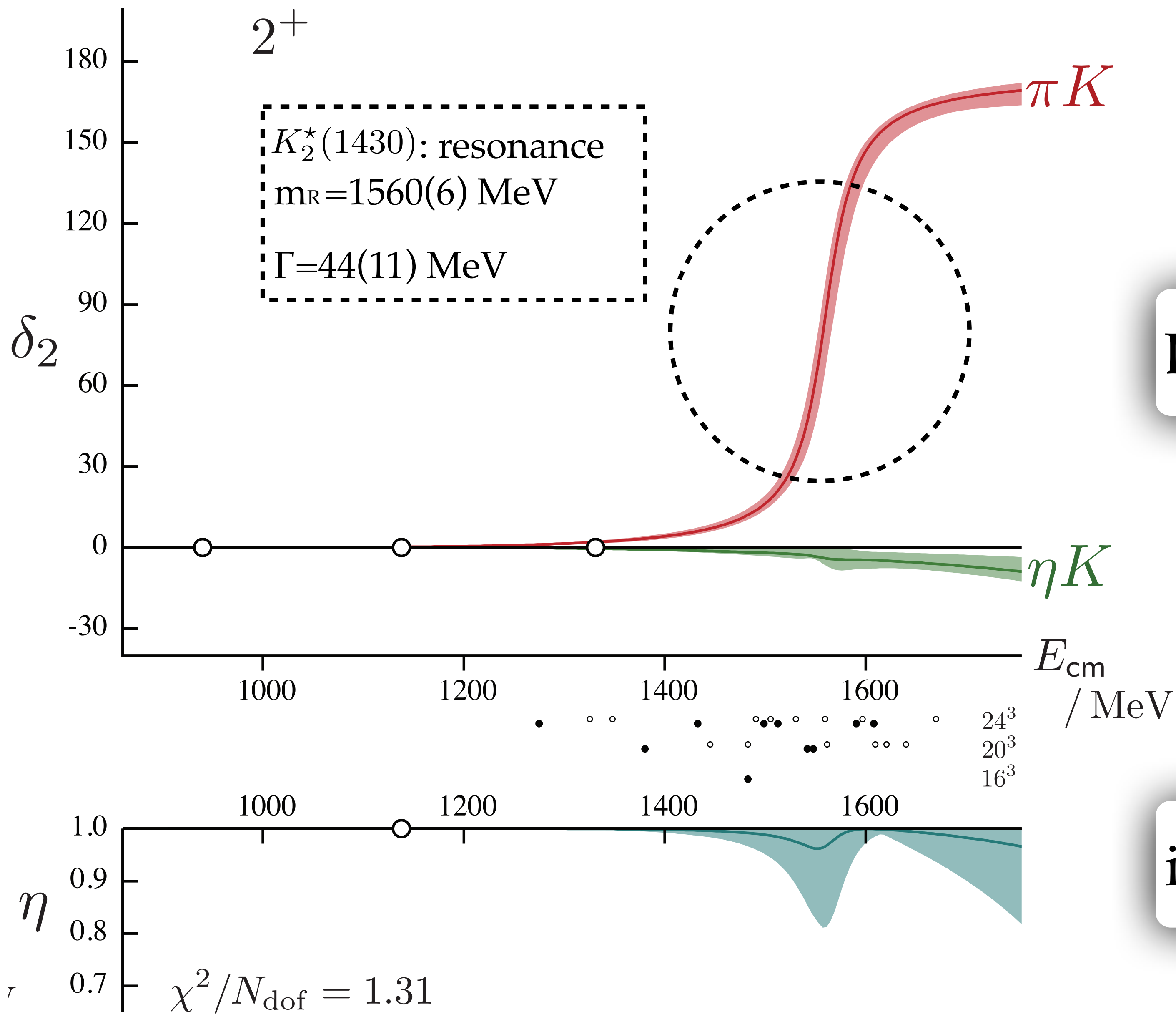
One example: Kπ-Kη

1 Determine finite volume spectra, e.g., Kπ-Kη spectrum using m_π~390MeV

by ***David Wilson***, Dudek, Edwards & Thomas (2014) [Hadron Spectrum Coll]

2 $$\det\ [\mathcal{M}^{-1} + \delta\mathcal{G}^V] = 0$$

3



D-wave phase shifts

inelasticity

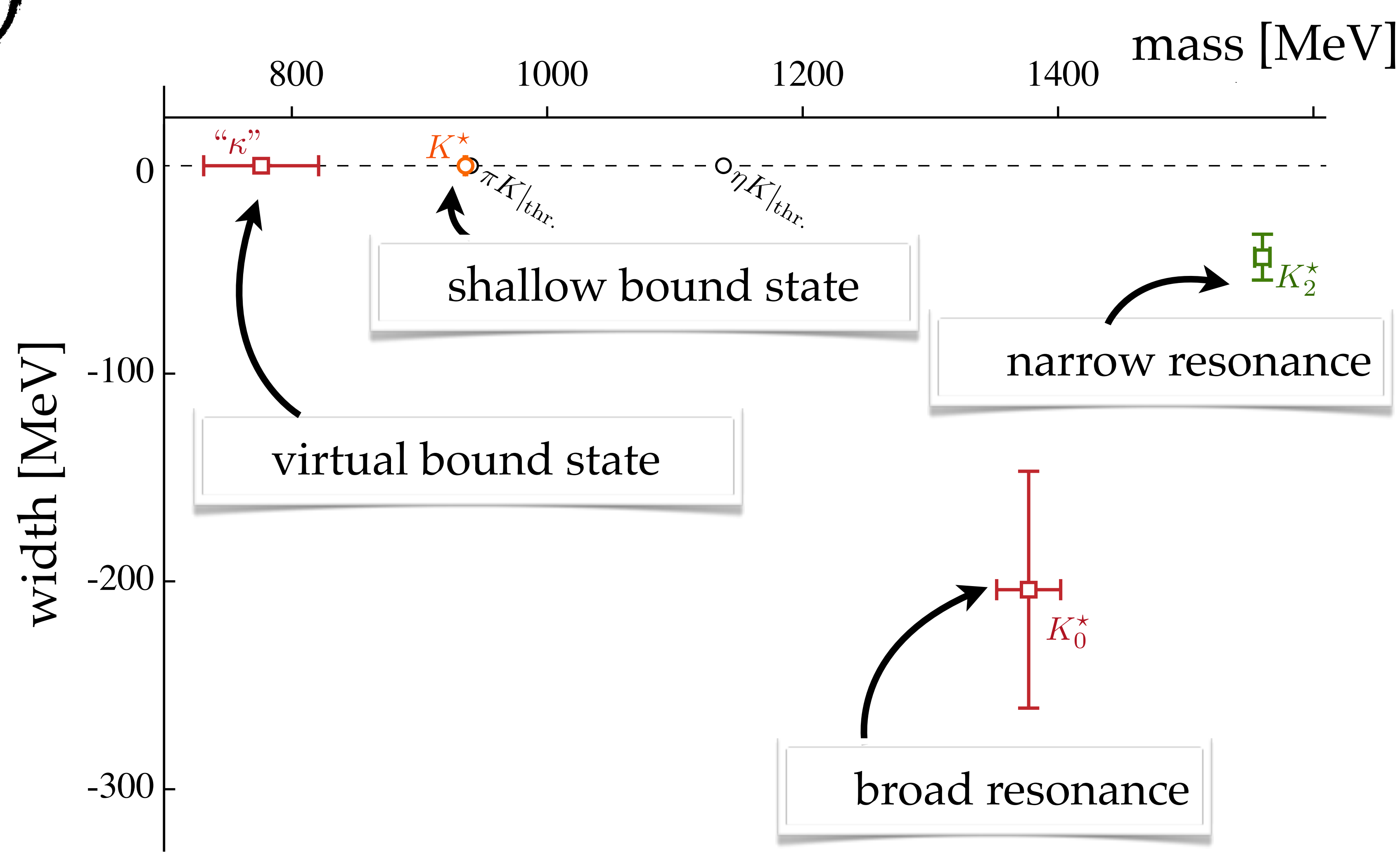
One example: Kπ-Kη

1 Determine finite volume spectra, e.g., Kπ-Kη spectrum using $m_\pi \sim 390$MeV

by ***David Wilson***, Dudek, Edwards & Thomas (2014) [Hadron Spectrum Coll]

2
$$\det\left[\mathcal{M}^{-1} + \delta\mathcal{G}^V\right] = 0$$

3



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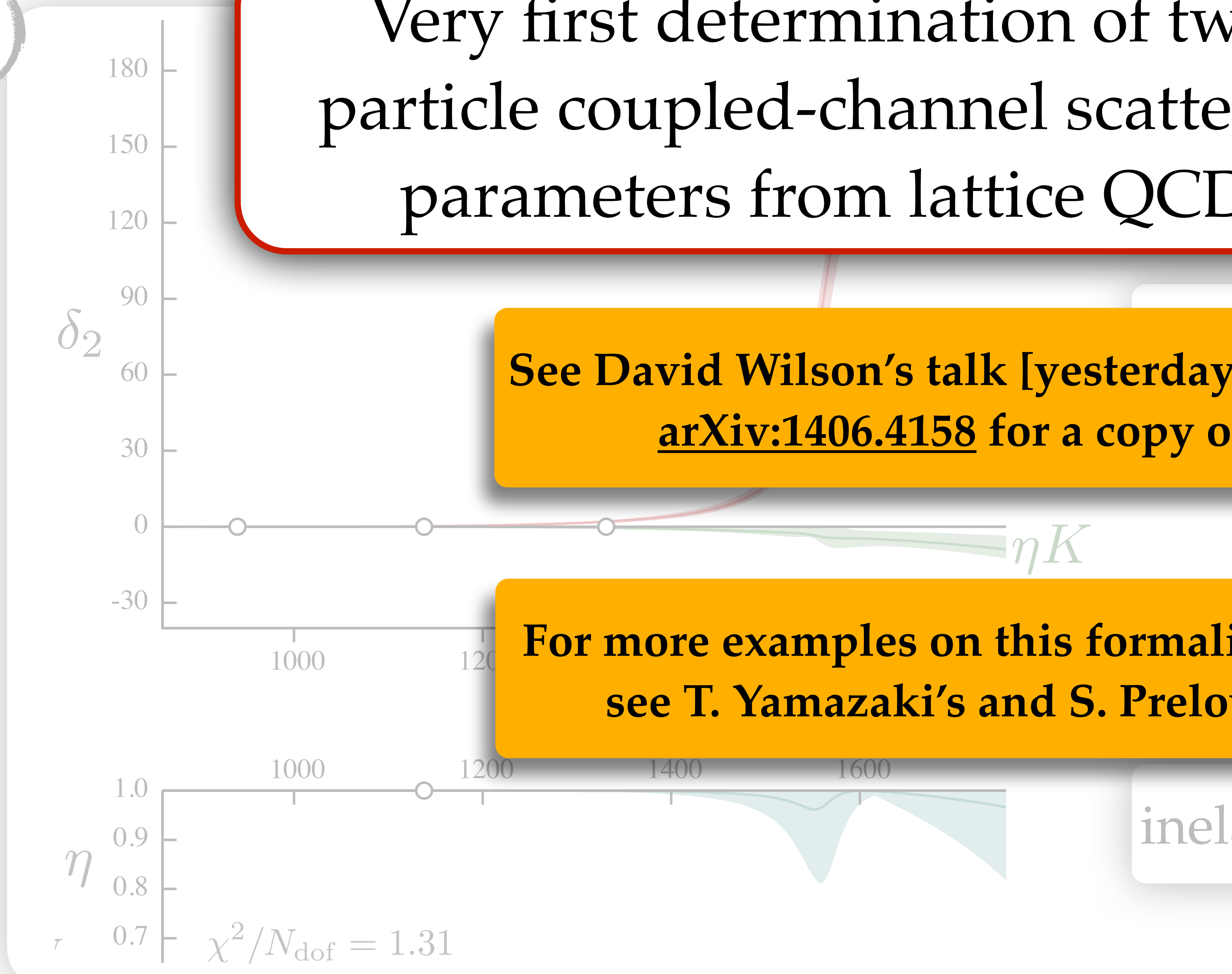


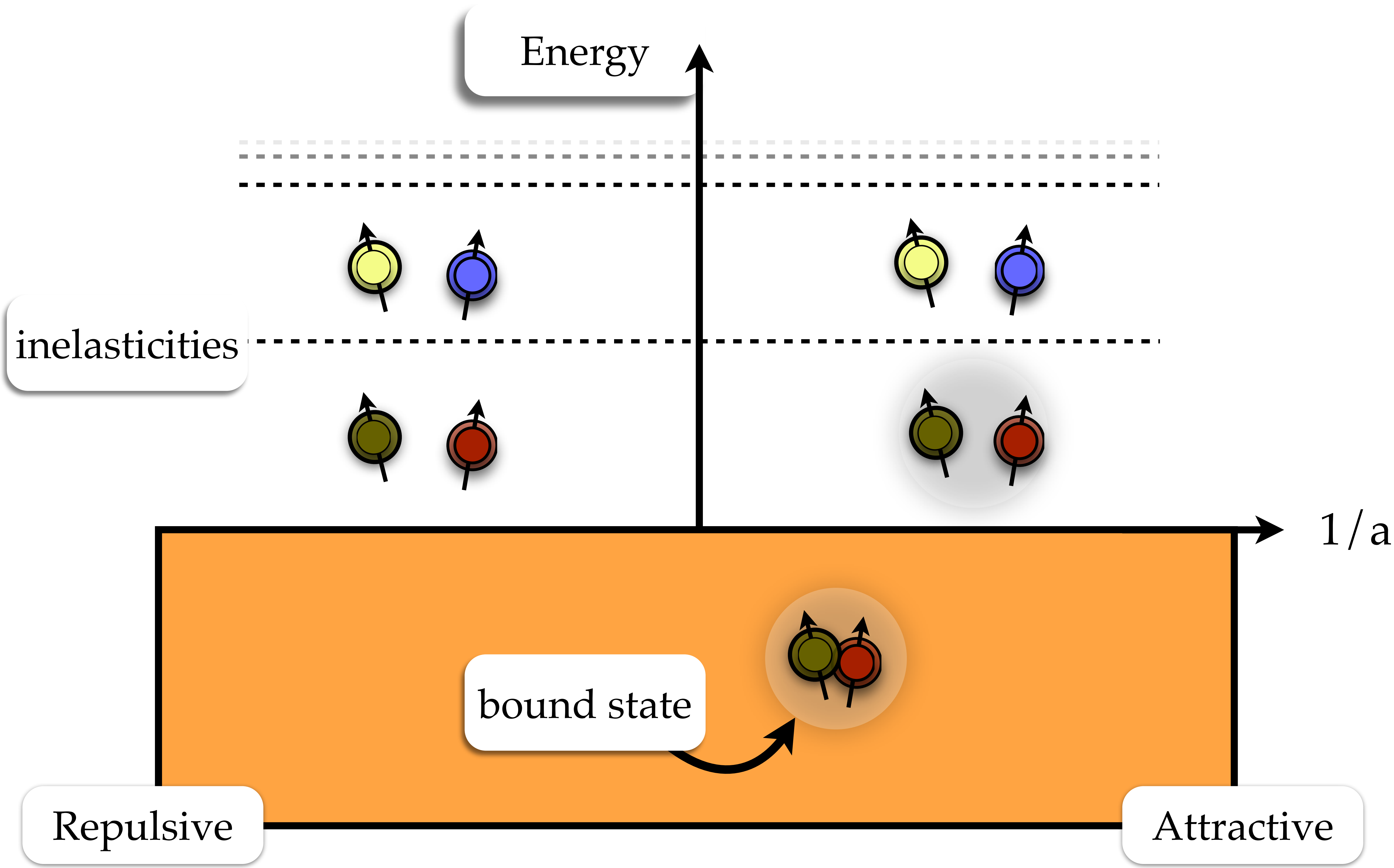
inelasticity

Very first determination of two-particle coupled-channel scattering parameters from lattice QCD!

See David Wilson's talk [yesterday!] for further details and arXiv:1406.4158 for a copy of the manuscript.

For more examples on this formalism being implemented see T. Yamazaki's and S. Prelovsek's plenary talks

Spectrum 2-body system in a box



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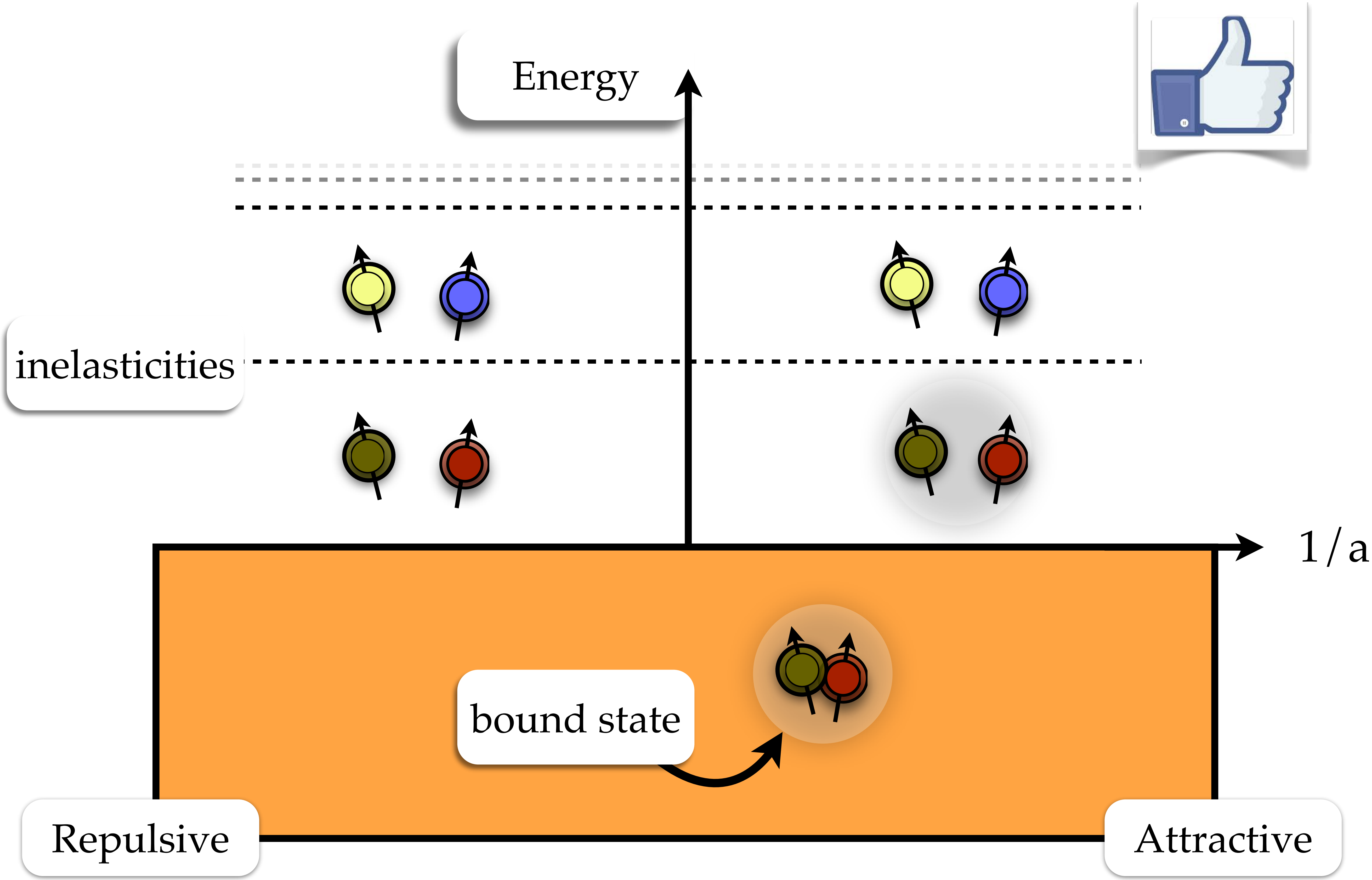
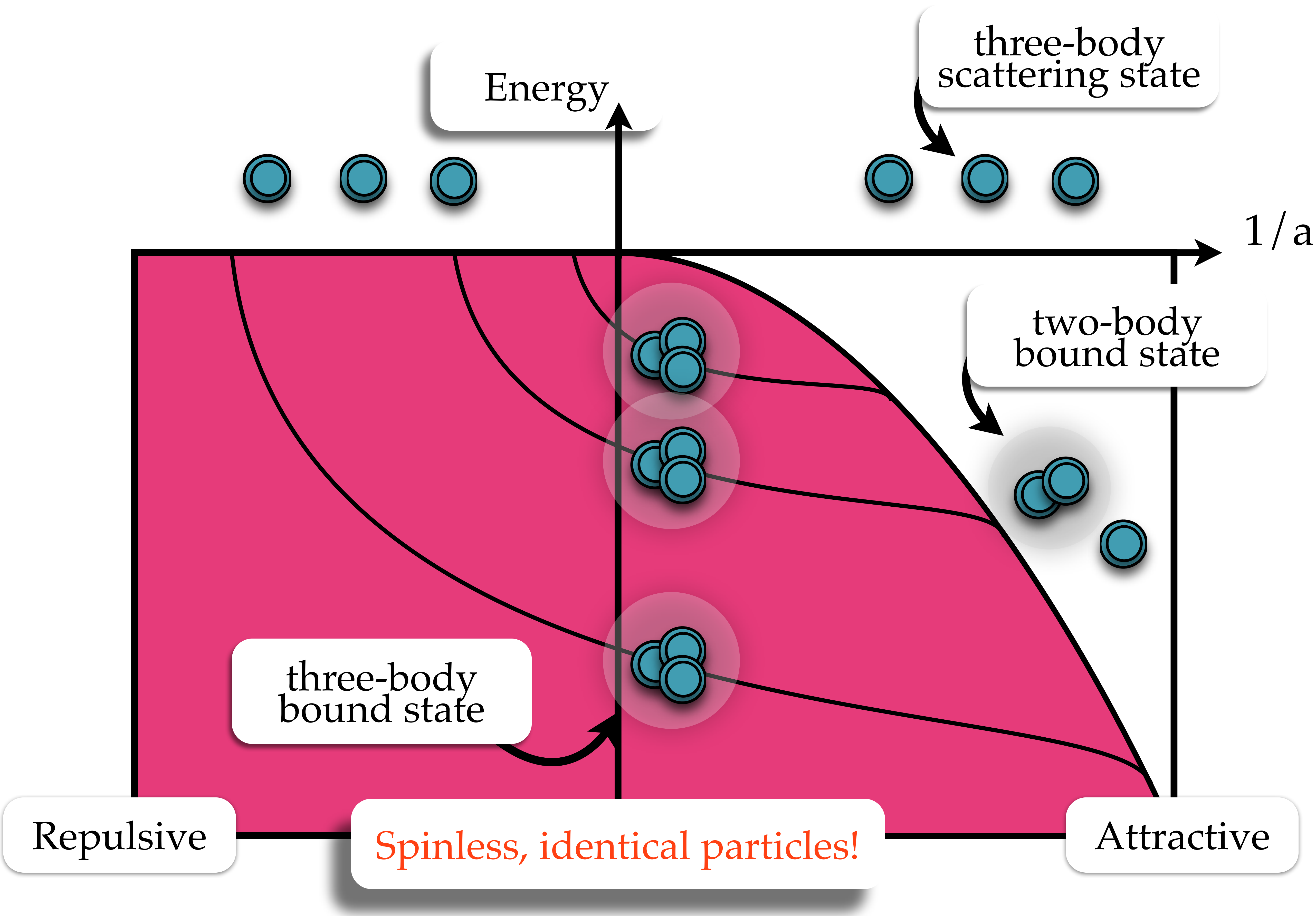
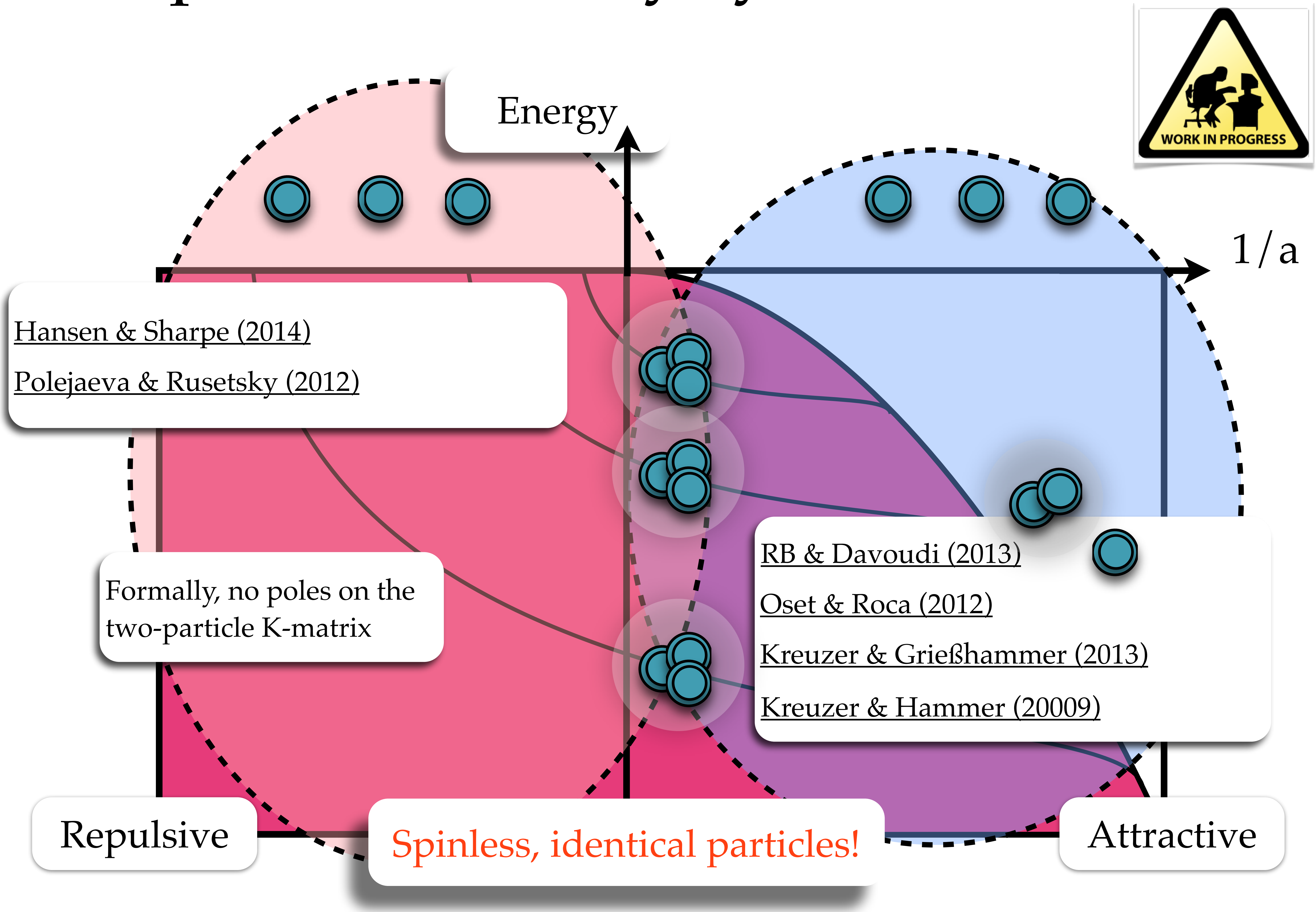


Spectrum 3-body system in a box



Spectrum 3-body system in a box



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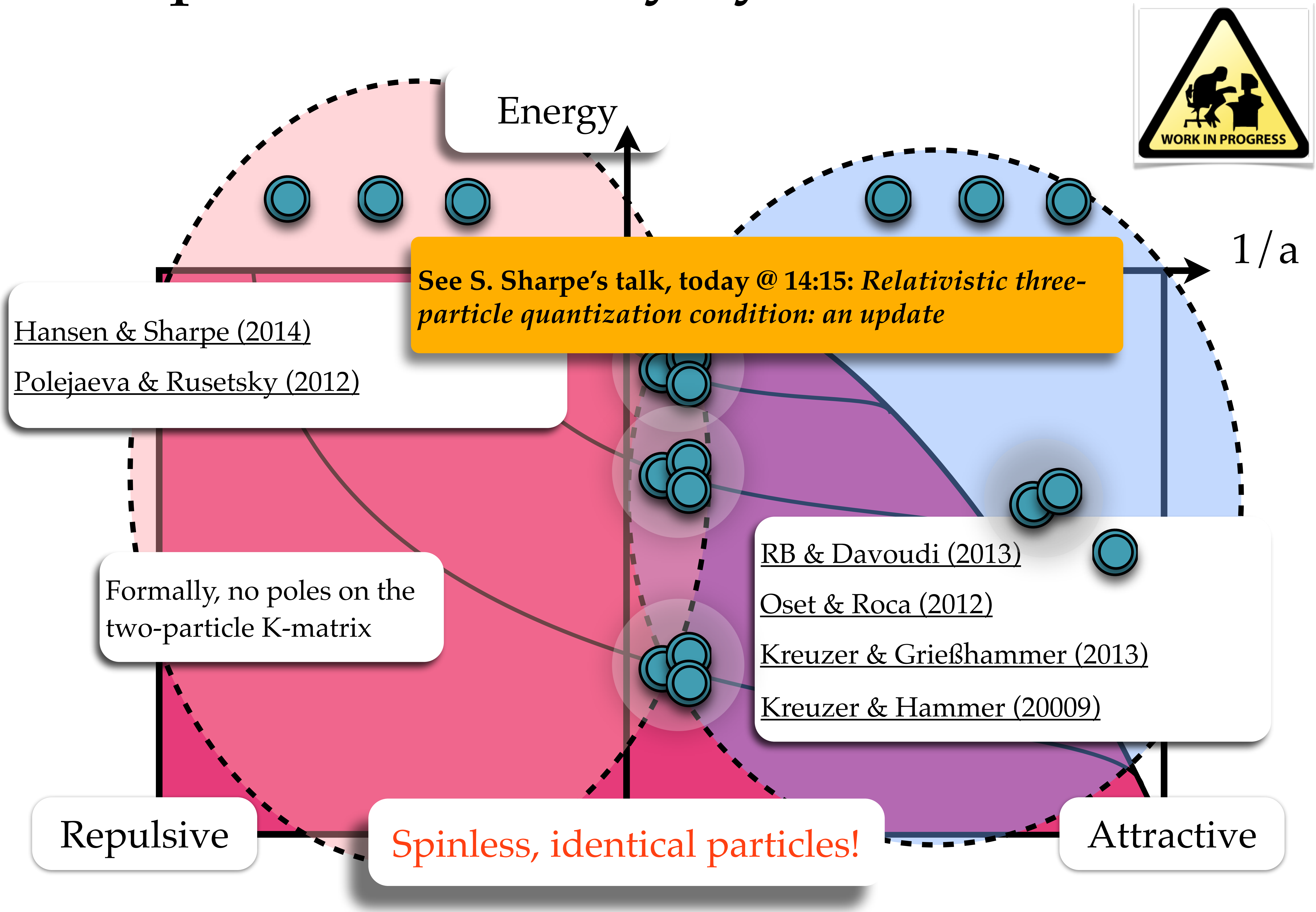
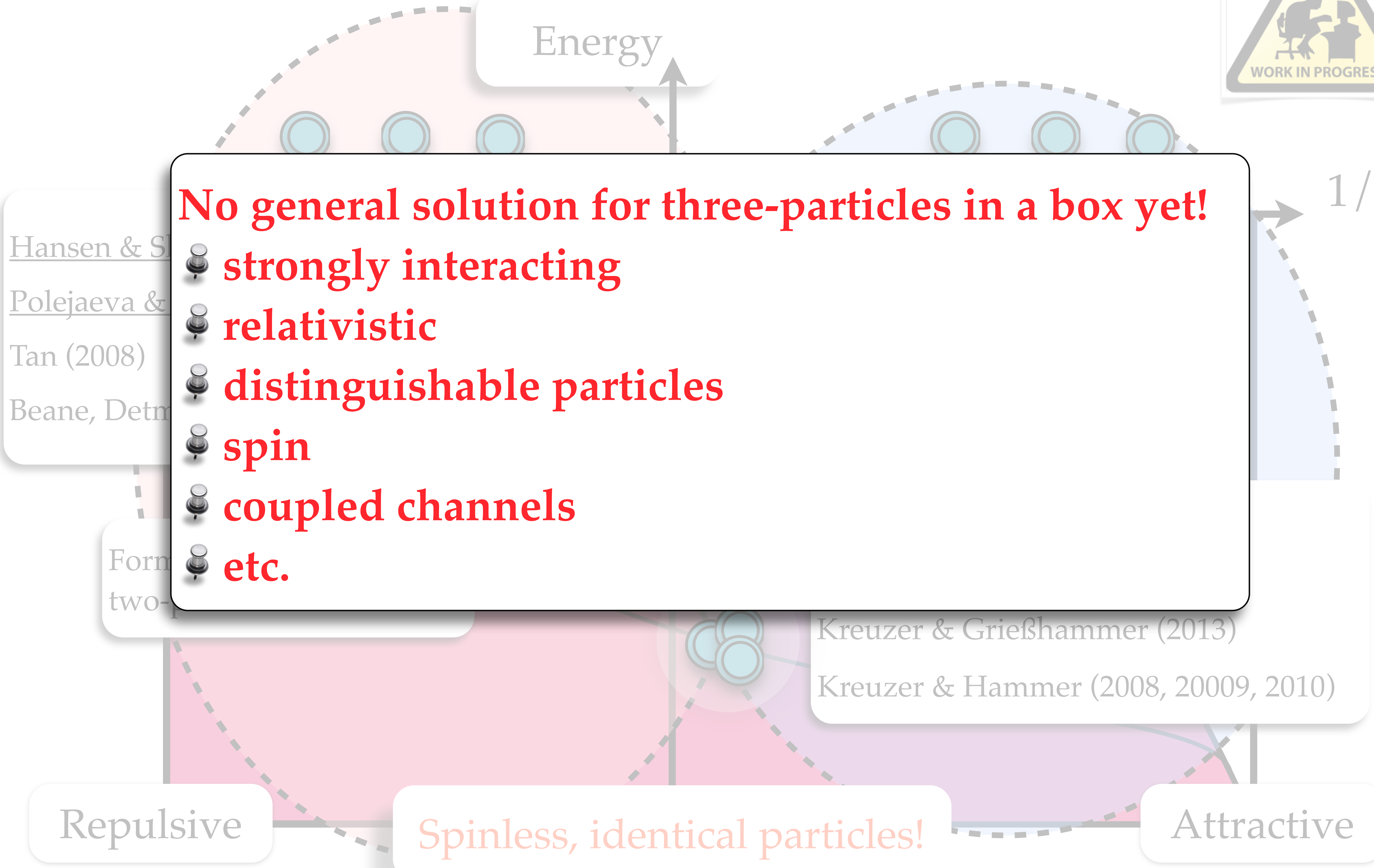


Spectrum 3-body system in a box

No general solution for three-particles in a box yet!

- **strongly interacting**
- **relativistic**
- **distinguishable particles**
- **spin**
- **coupled channels**
- **etc.**



N-Body system in a box

Weakly interacting N-bosons (two species):

- Smigielski & Wasem (2008)
- Tan (2008)
- Beane, Detmold, & Savage (2007)

Weakly interacting N-bosons + 1 baryon:

- Detmold & Nicholson (2013)

Deeply bound N-particles:

- Yamazaki, Ishikawa, Kuramashi, and Ukawa (2012)
- Beane *et al.* [NPLQCD] (2012)

See J. Green's talk, Wed @ 12:30: *H-dibaryon searches*

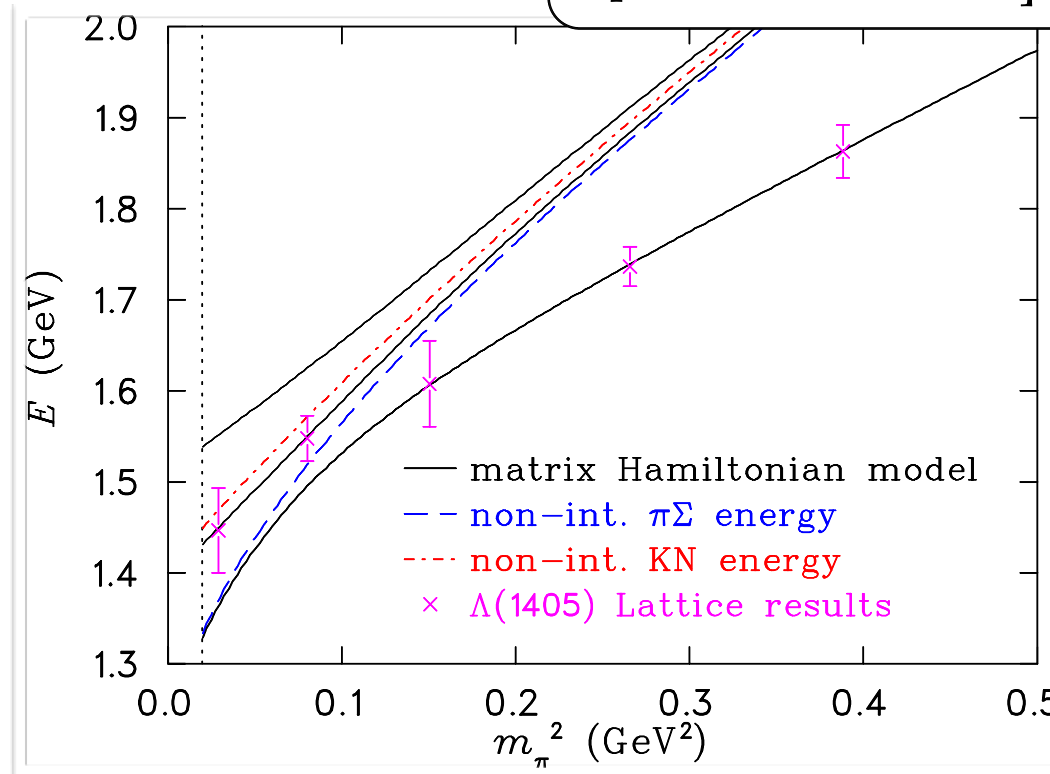
Alternative techniques

Finite-volume Hamiltonian method:

- Technique for parametrizing the interaction between particles in a finite volume
- Nπ in Δ channel [Hall, Hsu, Leinweber, Thomas & Young (2013)]
- ππ-KK coupled channel [Wu, Lee, Thomas & Young (2014)]

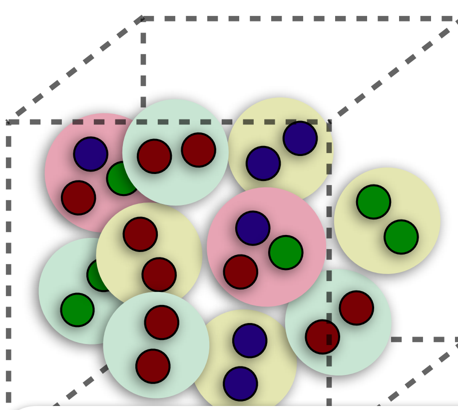
See D. Leinweber's talk today @ 16:50

Σπ-KN **Spectrum**
[Leinweber et al.]



Non-relativistic potential method for N-body:

- Arbitrary number of non-relativistic particles
- Relativistic limit holds for two particles
- **HAL QCD (2012)**



e.g., see Walker-Loud (2014)

See T. Doi's talk Thursday @ 15:55

Alternative techniques

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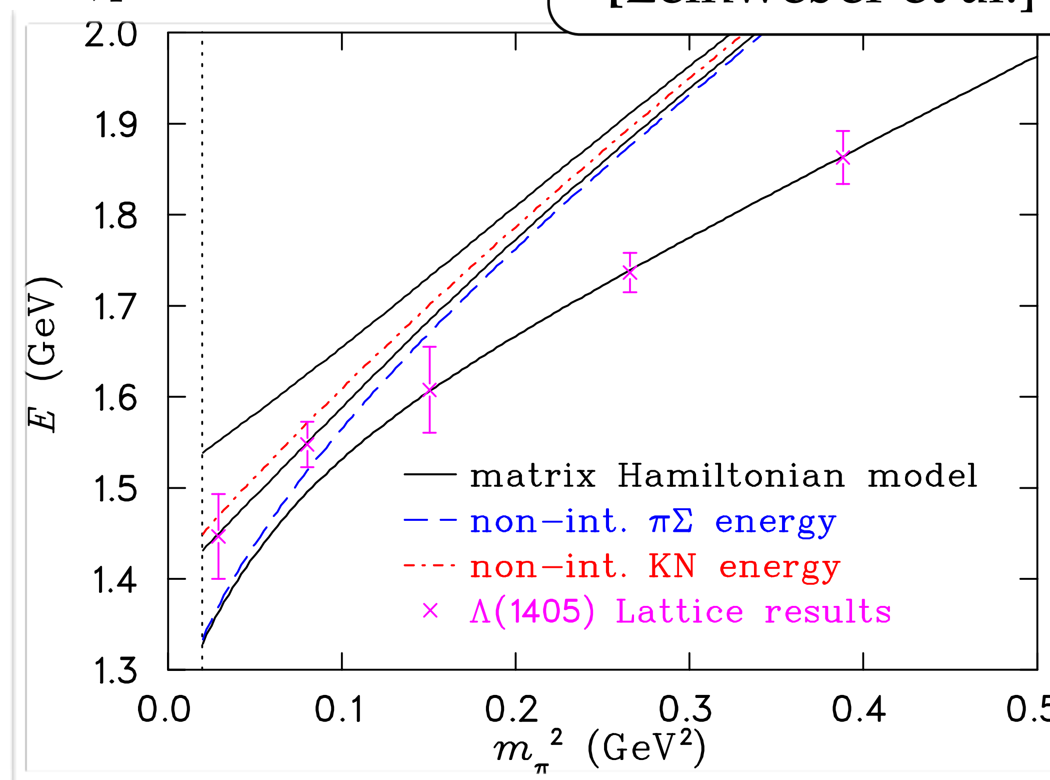
See D. Leinweber's talk today @ 16:50

Not distinct from Lüscher! Just another way to parametrize the scattering amplitude!

$\Sigma\pi$-KN **Spectrum**
[Leinweber et al.]



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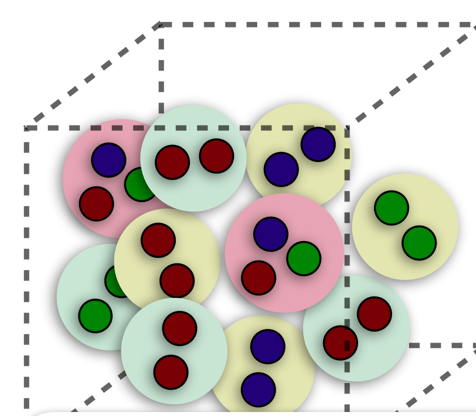

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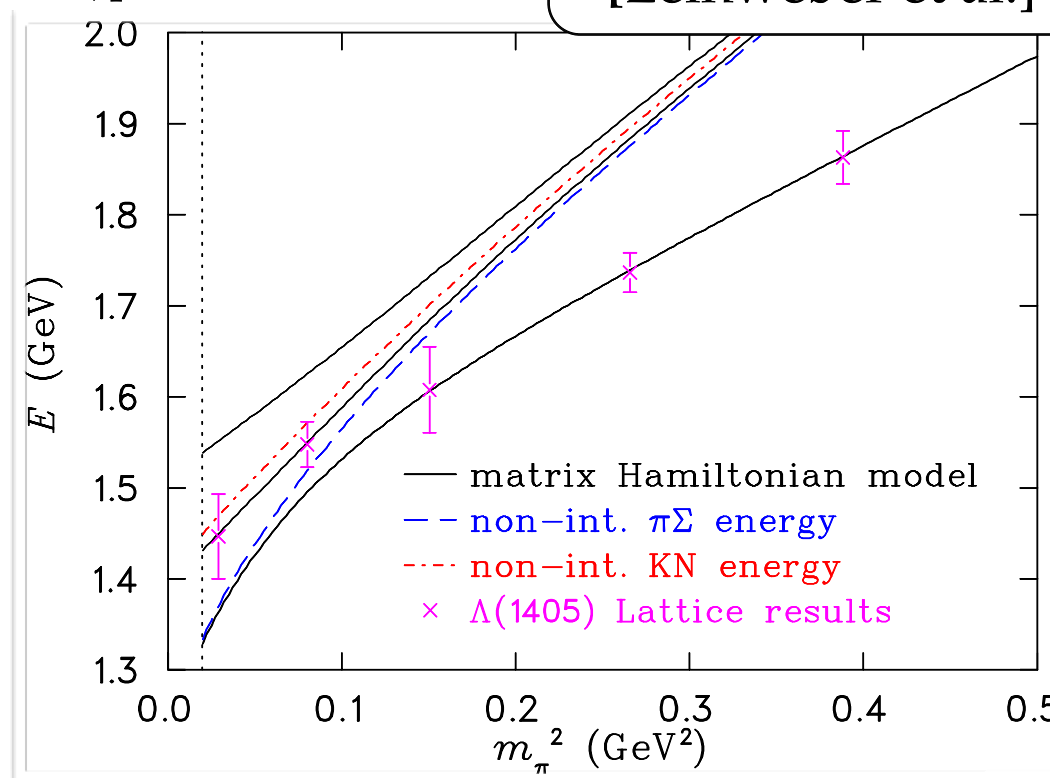

Non-relativistic potential method for N-body:

- Arbitrary number of non-relativistic particles
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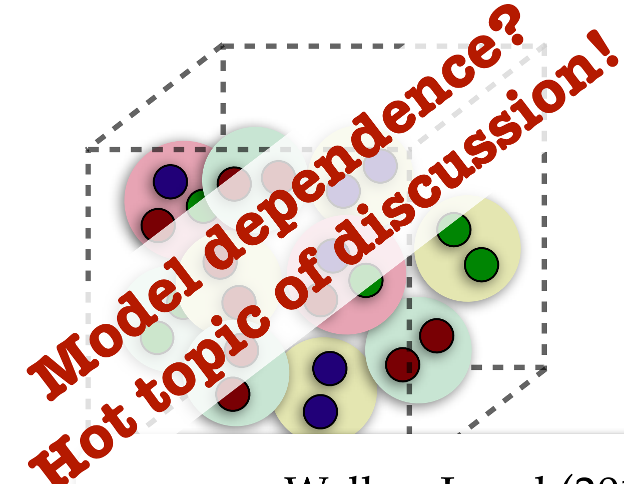
Paraphrase: *maybe a little competition would do us all some good*

Adam Smith
"father of capitalism"

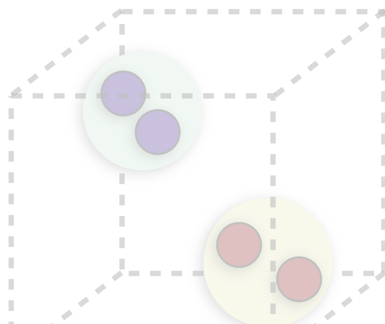
See T. Doi's talk Thursday @ 15:55



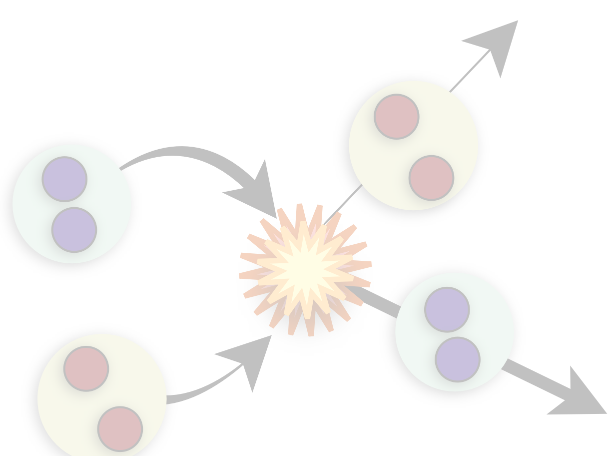
e.g., see Walker-Loud (2014)

A roadmap towards physics

1 Calculate finite volume spectrum

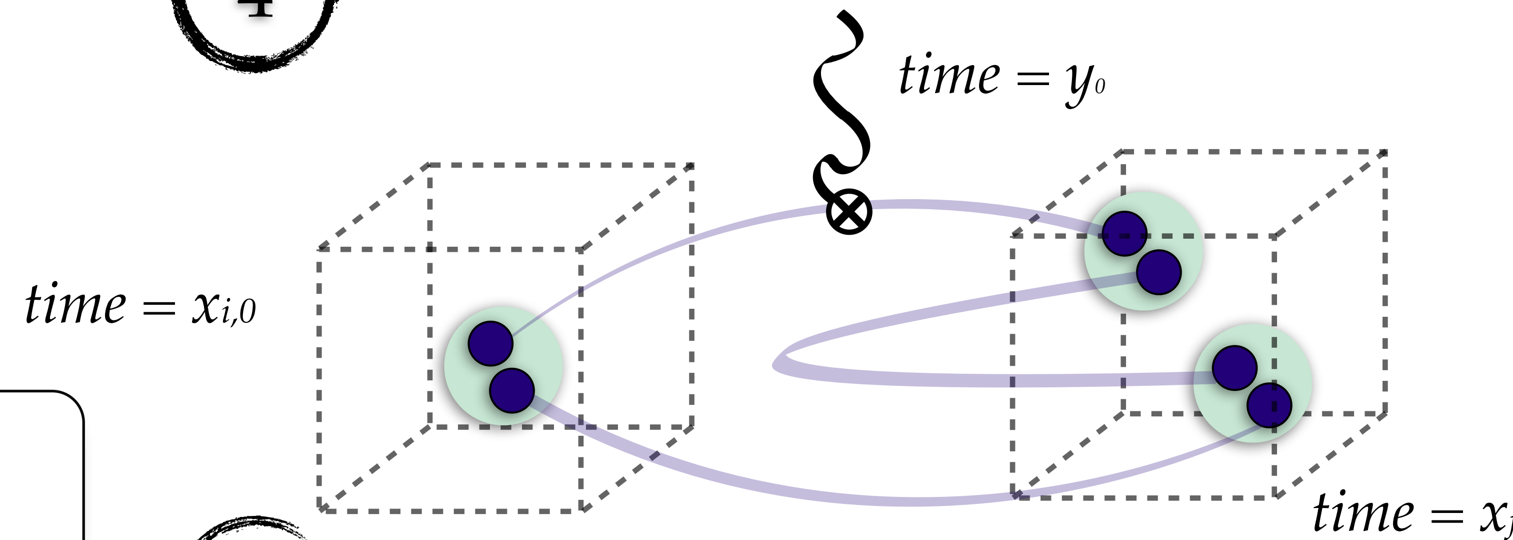


What about form factors of unstable particles, two particle or more?



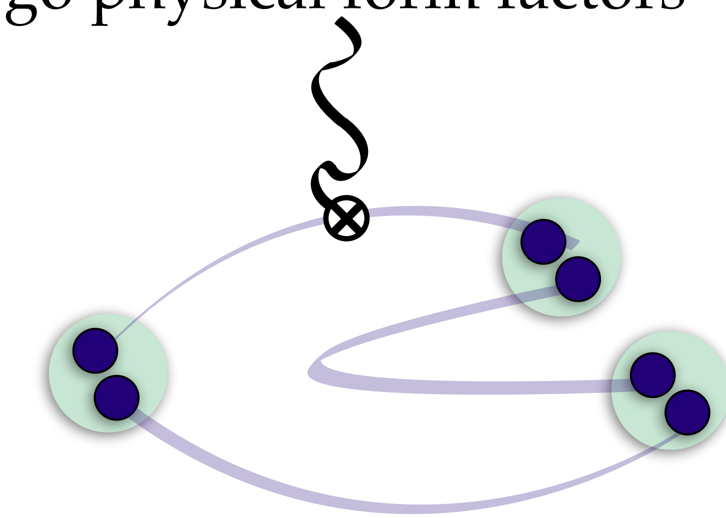
à la mode de **Lüscher (1986)**

4 Calculate finite volume form factor



5 Plug spectrum, scattering parameters and finite volume form factor into formalism

6 Out go physical form factors



***à la mode de* Lellouch & Lüscher (2000)**

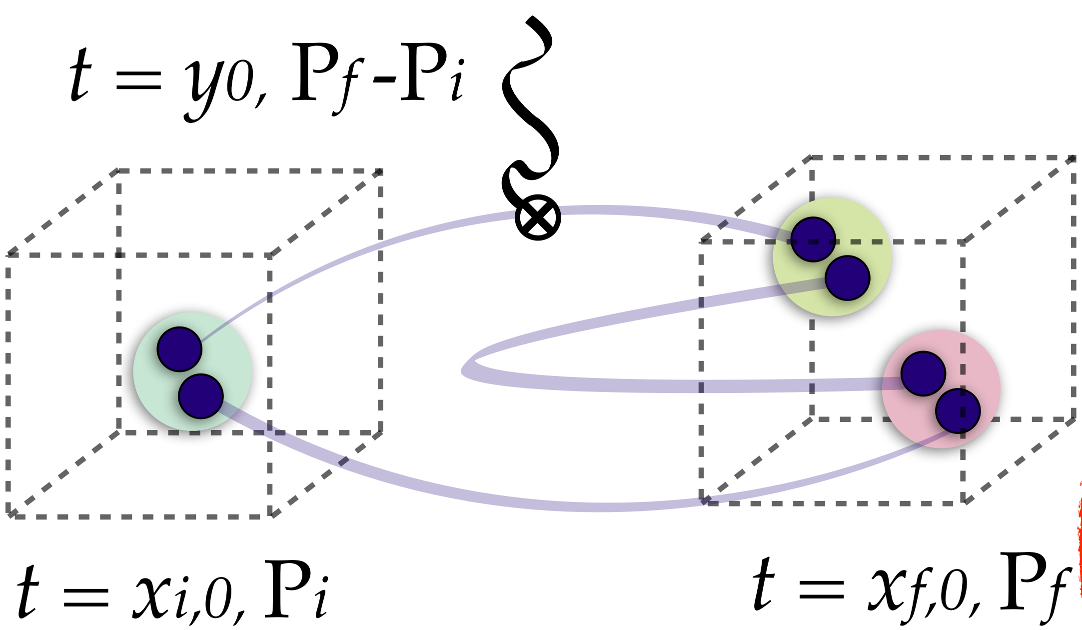
Transition form factors

$$\left|\langle E_{\Lambda_f,n_f}\mathbf{P}_f;L|\tilde{\mathcal{J}}_{\Lambda\mu}(0,\mathbf{P}_f-\mathbf{P}_i)|E_{\Lambda_i,0}\mathbf{P}_i;L\rangle\right| = \frac{1}{\sqrt{2E_{\Lambda_i,0}}}\sqrt{\left[\mathcal{A}^{\dagger}_{\Lambda_f,n_f;\Lambda\mu}\,\mathcal{R}_{\Lambda_f,n_f}\,\mathcal{A}_{\Lambda_f,n_f;\Lambda\mu}\right]}$$

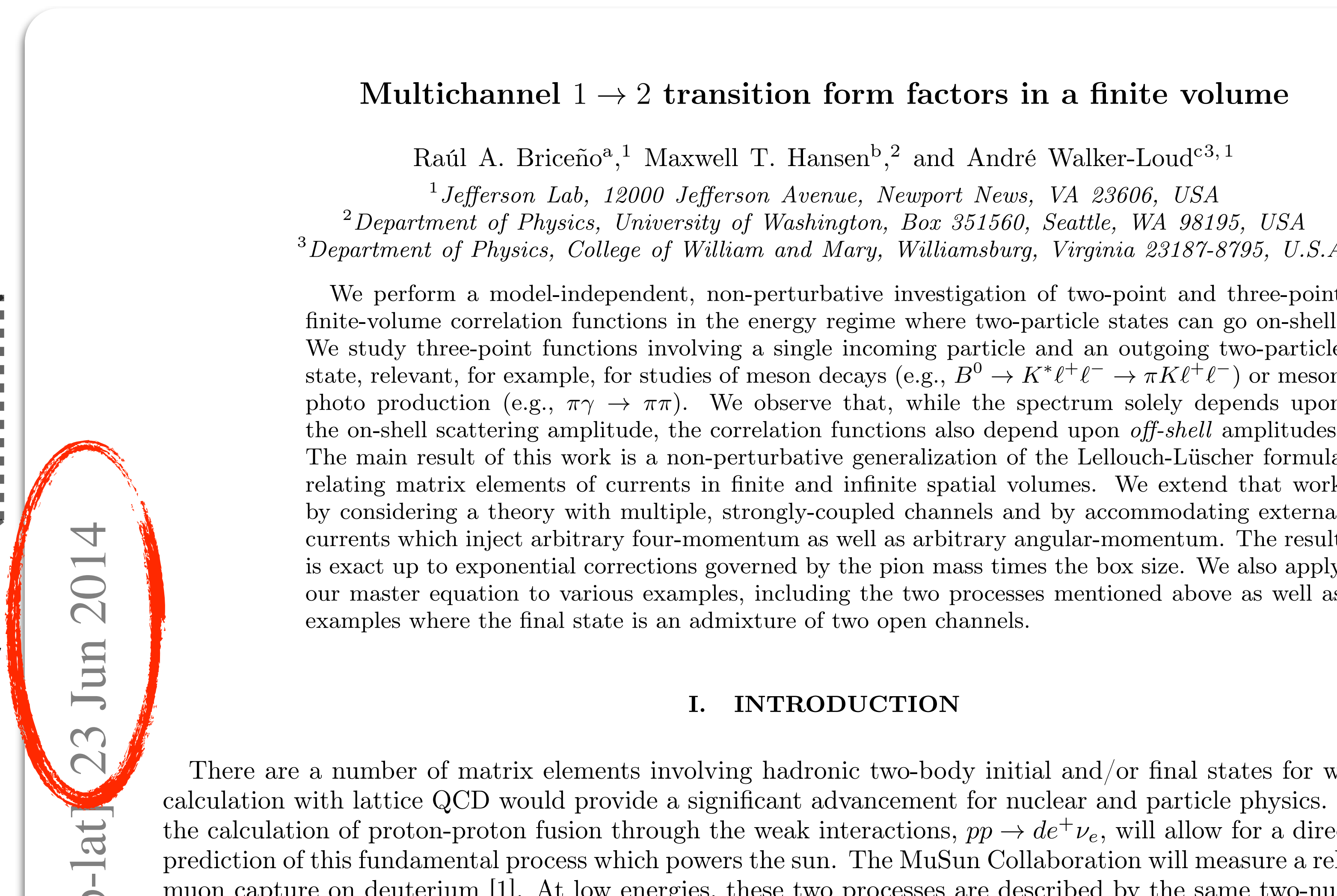
finite volume
one-to-two matrix element!

à la mode de Lellouch & Lüscher (2000)

Note: off-shellness cancels!



RB, Hansen & Walker-Loud (2014)



Multichannel $1 \to 2$ transition form factors in a finite volume

Raúl A. Briceño[a,1] Maxwell T. Hansen[b,2] and André Walker-Loud[c3,1]
[1] *Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[2] *Department of Physics, University of Washington, Box 351560, Seattle, WA 98195, USA*
[3] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187-8795, U.S.A*

We perform a model-independent, non-perturbative investigation of two-point and three-point finite-volume correlation functions in the energy regime where two-particle states can go on-shell. We study three-point functions involving a single incoming particle and an outgoing two-particle state, relevant, for example, for studies of meson decays (e.g., $B^0 \to K^*\ell^+\ell^- \to \pi K\ell^+\ell^-$) or meson photo production (e.g., $\pi\gamma \to \pi\pi$). We observe that, while the spectrum solely depends upon the on-shell scattering amplitude, the correlation functions also depend upon *off-shell* amplitudes. The main result of this work is a non-perturbative generalization of the Lellouch-Lüscher formula relating matrix elements of currents in finite and infinite spatial volumes. We extend that work by considering a theory with multiple, strongly-coupled channels and by accommodating external currents which inject arbitrary four-momentum as well as arbitrary angular-momentum. The result is exact up to exponential corrections governed by the pion mass times the box size. We also apply our master equation to various examples, including the two processes mentioned above as well as examples where the final state is an admixture of two open channels.

[hep]-lat] 23 Jun 2014

I. INTRODUCTION

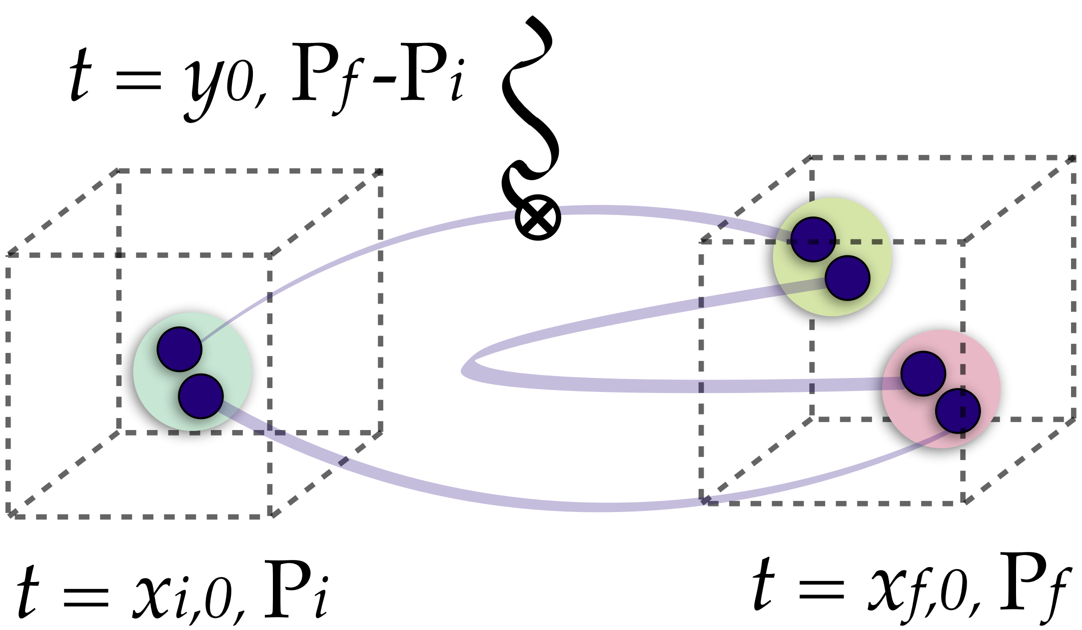
There are a number of matrix elements involving hadronic two-body initial and/or final states for w calculation with lattice QCD would provide a significant advancement for nuclear and particle physics. the calculation of proton-proton fusion through the weak interactions, $pp \to de^+\nu_e$, will allow for a dire prediction of this fundamental process which powers the sun. The MuSun Collaboration will measure a re muon capture on deuterium [1]. At low energies, these two processes are described by the same two-nu

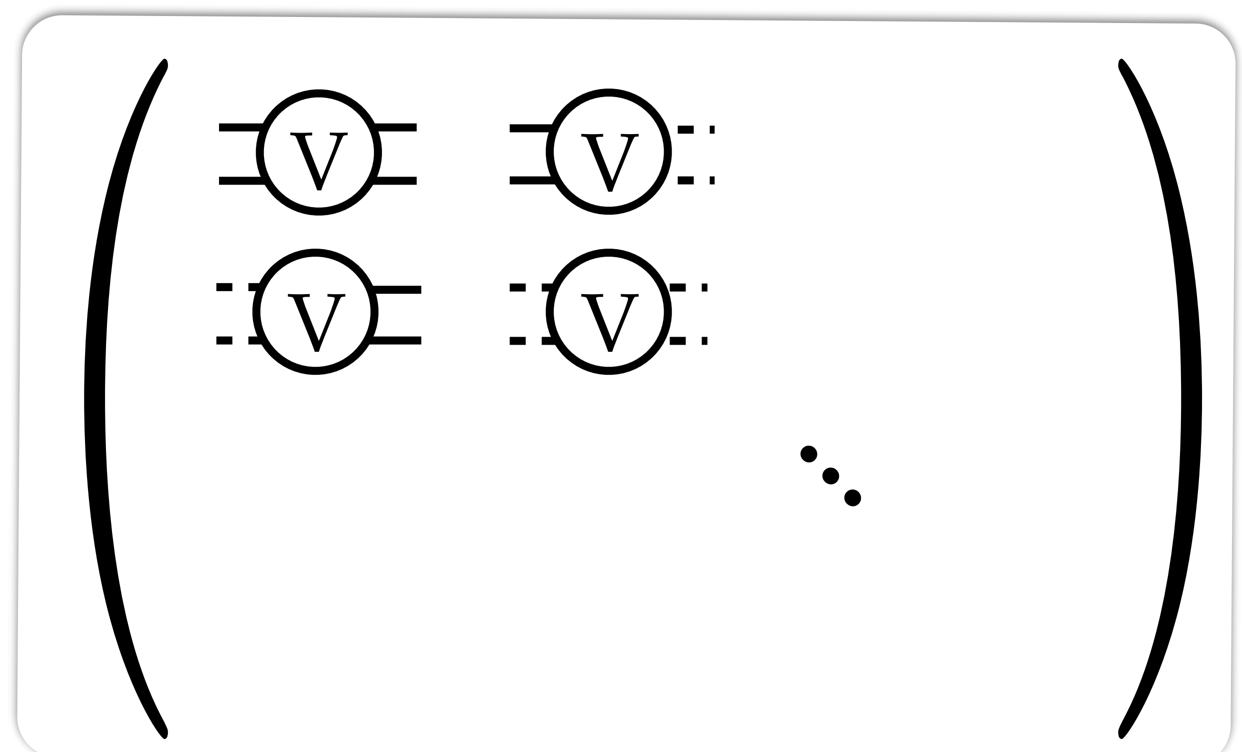
Transition form factors

$$\left|\langle E_{\Lambda_f,n_f}\mathbf{P}_f;L|\tilde{\mathcal{J}}_{\Lambda\mu}(0,\mathbf{P}_f-\mathbf{P}_i)|E_{\Lambda_i,0}\mathbf{P}_i;L\rangle\right| = \frac{1}{\sqrt{2E_{\Lambda_i,0}}}\sqrt{\left[\mathcal{A}^{\dagger}_{\Lambda_f,n_f;\Lambda\mu}\ \mathcal{R}_{\Lambda_f,n_f}\ \mathcal{A}_{\Lambda_f,n_f;\Lambda\mu}\right]}$$

two-particle propagator residue

Warning: depends on spectrum, momenta, scattering parameters and their derivatives!





a matrix in the space of open channels

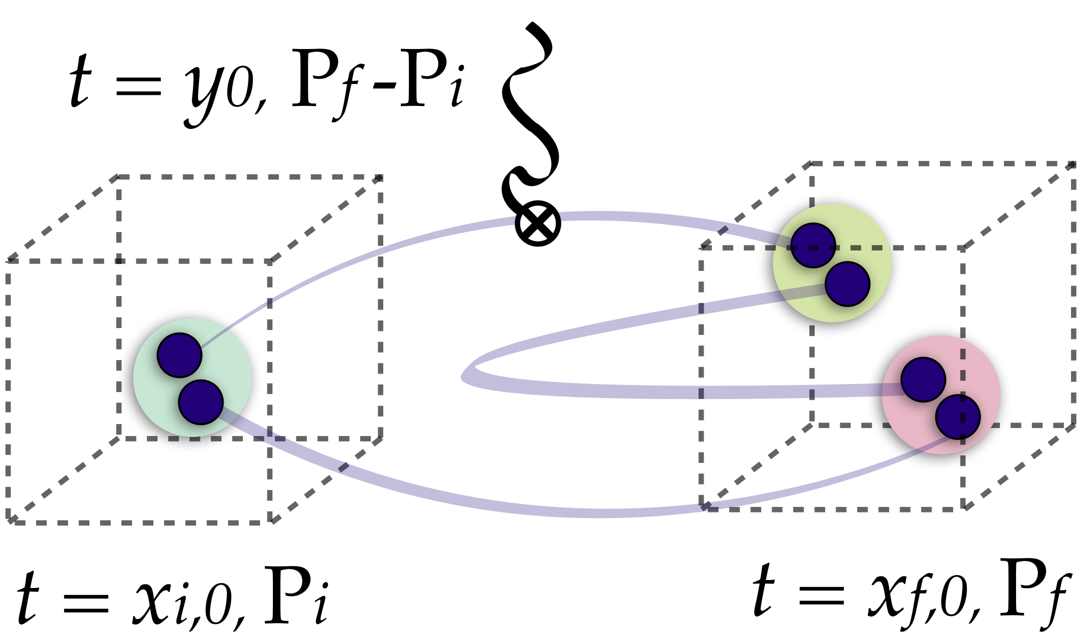
RB, Hansen & Walker-Loud (2014)

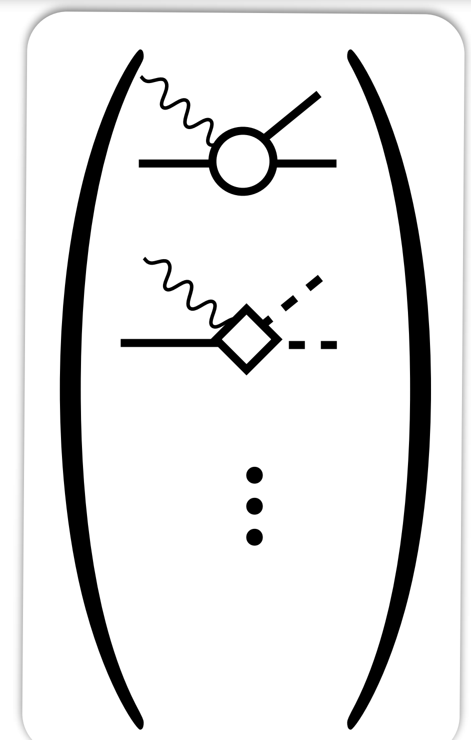
Transition form factors

$$\left|\langle E_{\Lambda_f,n_f}\mathbf{P}_f;L|\tilde{\mathcal{J}}_{\Lambda\mu}(0,\mathbf{P}_f-\mathbf{P}_i)|E_{\Lambda_i,0}\mathbf{P}_i;L\rangle\right| = \frac{1}{\sqrt{2E_{\Lambda_i,0}}}\sqrt{\left[\mathcal{A}^\dagger_{\Lambda_f,n_f;\Lambda\mu}\ \mathcal{R}_{\Lambda_f,n_f}\ \mathcal{A}_{\Lambda_f,n_f;\Lambda\mu}\right]}$$

infinite volume transition amplitude, related to infinite volume matrix elements

$$\langle a, P_f, Jm_J;\infty|\tilde{\mathcal{J}}_{\Lambda\mu}(0,\mathbf{Q};\infty)|P_i;\infty\rangle = [\mathcal{A}_{\Lambda\mu;Jm_J}]_a\ \ (2\pi)^3\ \delta^3(\mathbf{P}_f-\mathbf{P}_i-\mathbf{Q})$$



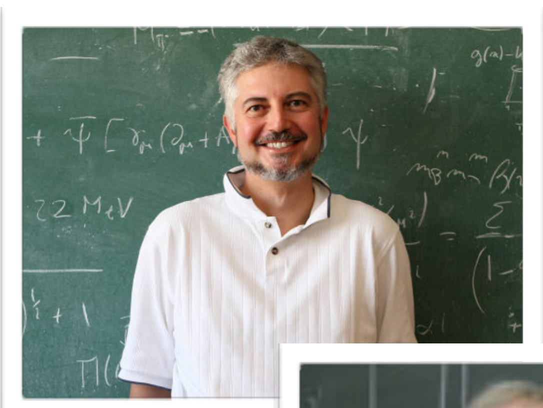


a vector in the space of open channels

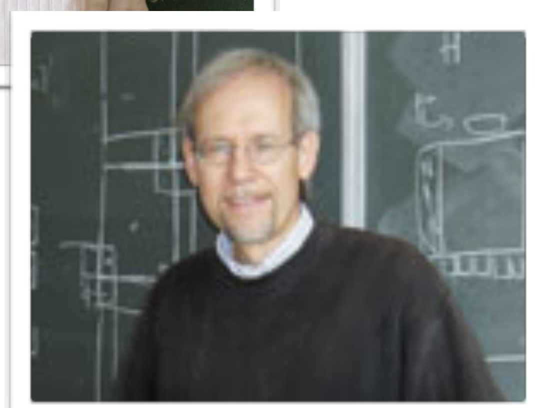
RB, Hansen & Walker-Loud (2014)

Transition form factors

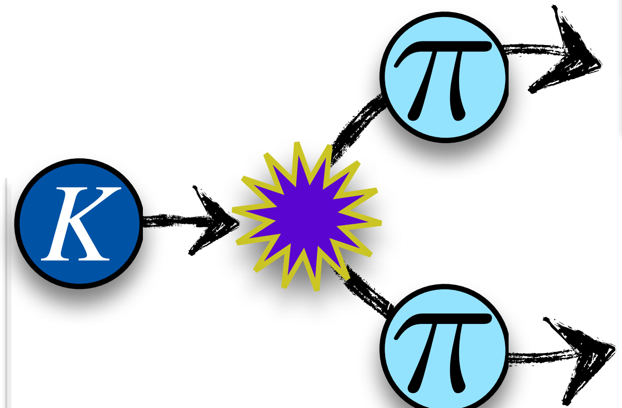
$$\left|\langle E_{\Lambda_f,n_f}\mathbf{P}_f;L|\tilde{\mathcal{J}}_{\Lambda\mu}(0,\mathbf{P}_f-\mathbf{P}_i)|E_{\Lambda_i,0}\mathbf{P}_i;L\rangle\right| = \frac{1}{\sqrt{2E_{\Lambda_i,0}}}\sqrt{\left[\mathcal{A}^{\dagger}_{\Lambda_f,n_f;\Lambda\mu}\ \mathcal{R}_{\Lambda_f,n_f}\ \mathcal{A}_{\Lambda_f,n_f;\Lambda\mu}\right]}$$



Lellouch



Lüscher



Reproduces well known K-to-$\pi\pi$ result and shows result holds even if the final and initial state are not degenerate.

RB, Hansen & Walker-Loud (2014)

Transition form factors

$$\left|\langle E_{\Lambda_f,n_f}\mathbf{P}_f;L|\tilde{\mathcal{J}}_{\Lambda\mu}(0,\mathbf{P}_f-\mathbf{P}_i)|E_{\Lambda_i,0}\mathbf{P}_i;L\rangle\right| = \frac{1}{\sqrt{2E_{\Lambda_i,0}}}\sqrt{\left[\mathcal{A}^{\dagger}_{\Lambda_f,n_f;\Lambda\mu}\ \mathcal{R}_{\Lambda_f,n_f}\ \mathcal{A}_{\Lambda_f,n_f;\Lambda\mu}\right]}$$

Relevant references:

- RB, Hansen & Walker-Loud (2014)
- Agadjanov,Bernard, Meißner & Rusetsky (2014)
- Hansen & Sharpe (2012)
- RB & Davoudi (2012)
- Meyer (2012)
- Bernard, Hoja, Meißner & Rusetsky (2012)
- Christ, Kim & Yamazaki (2005)
- Kim, Sachrajda & Sharpe (2005)
- Detmold & Savage (2004)
- Lin, Martinelli, Sachrajda, and Testa (2001)
- Lellouch & Lüscher (2000)

bosonic systems:

- *arbitrary energies, momenta*
- *arbitrary angular momentum*
- *partial-wave mixing*
- *arbitrary open channels*
- *periodic, twisted BCs*
- *generic rectangular prism*

see A. Walker-Loud's talk, today @ 17:10

baryonic systems:

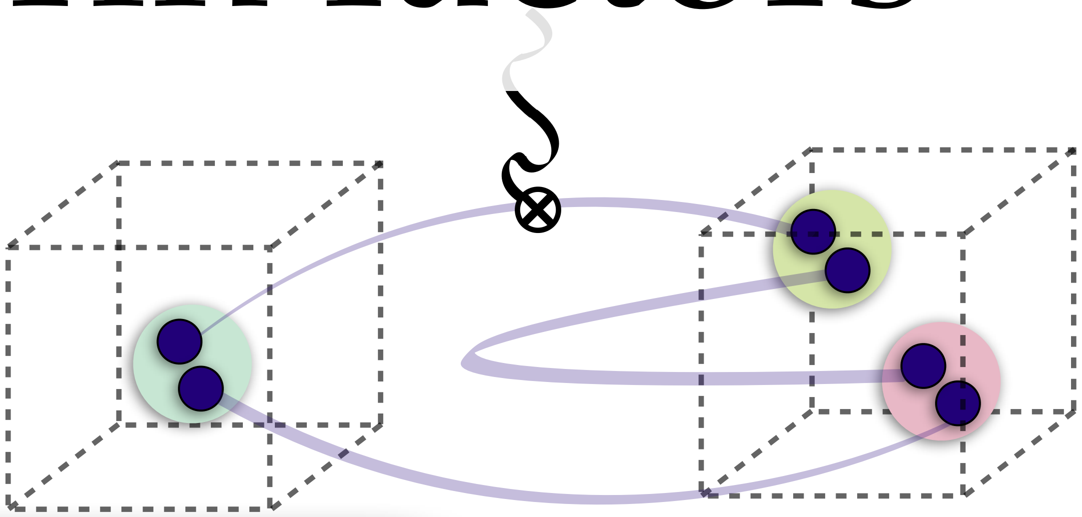
- *final state at rest*
- *no partial-wave mixing*
- *single partial wave*
- *one open channel*
- *periodic, twisted BCs*

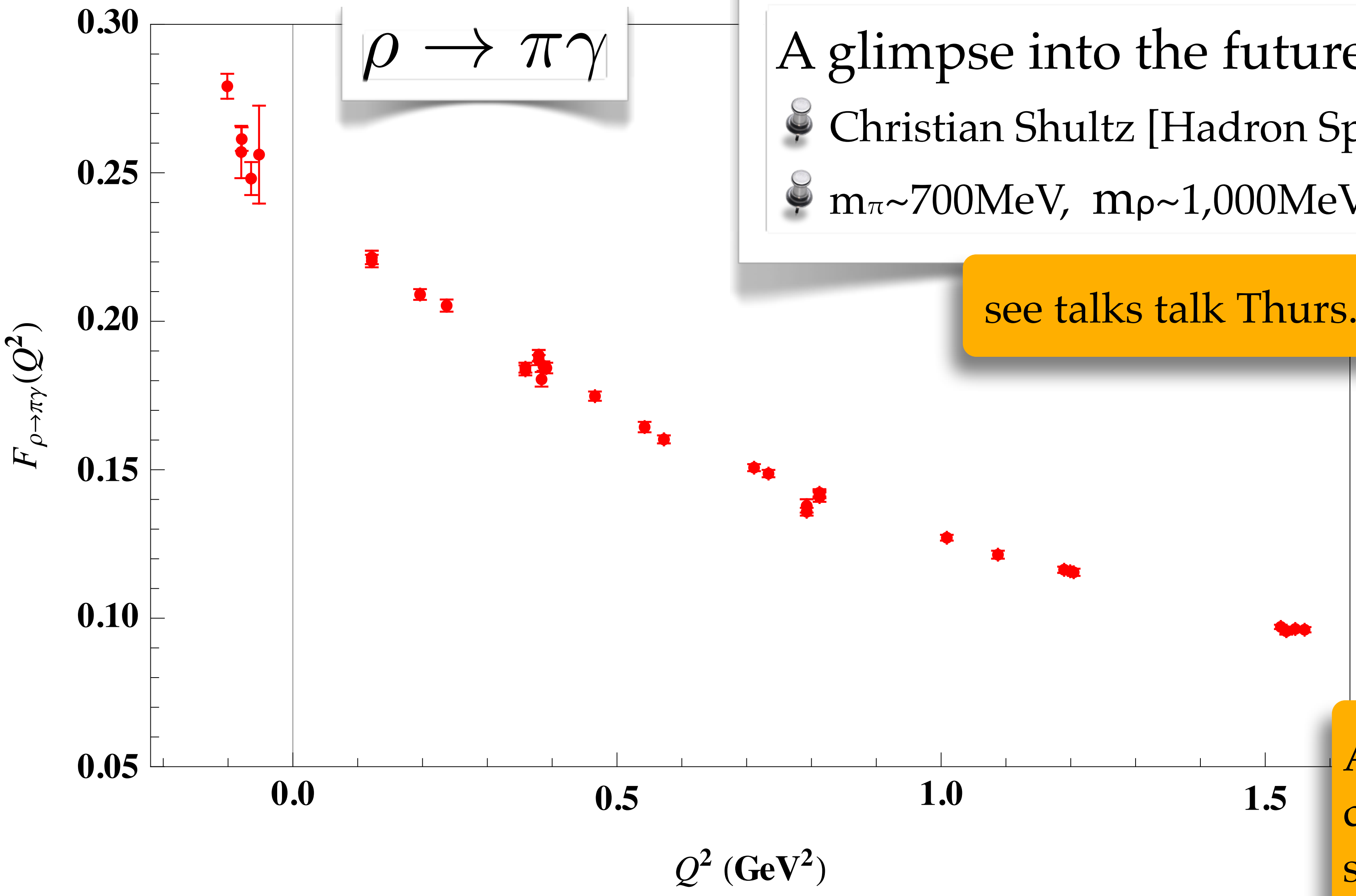
see A. Rusetsky's talk, Fri @ 17:50

Transition form factors

Best known example: $K \to \pi\pi$

see talks by N. Ishizuka, C. Kelly, D. Zhang **[yesterday!]**





A glimpse into the future:

- Christian Shultz [Hadron Spectrum Coll.]
- m_π~700MeV, m_ρ~1,000MeV, Stable ρ

see talks talk Thurs. @ 3:55pm

Also see talk by **B. Owen** on the determination of the excited nucleon state form factors, **Thurs. @ 16:15**

Status of formalism

(somewhat bias estimate)

- Spectroscopy / scattering:

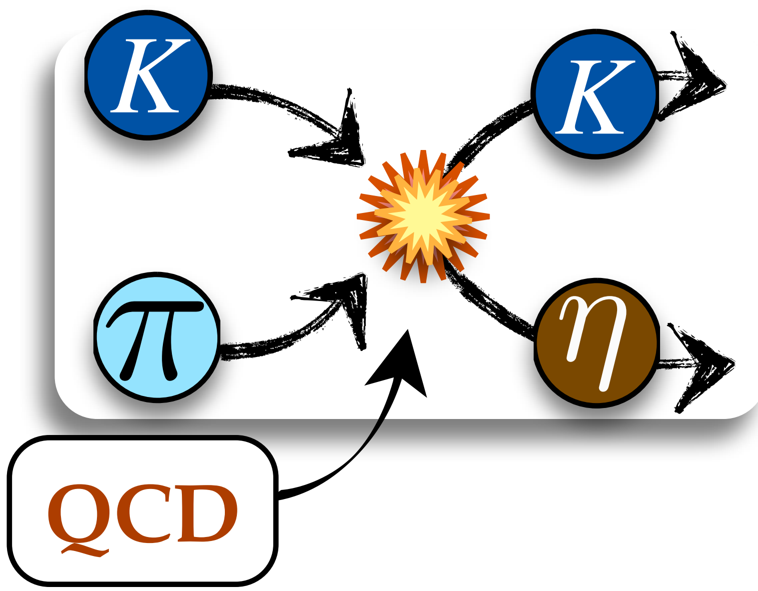
- Electromagnetic form factors:

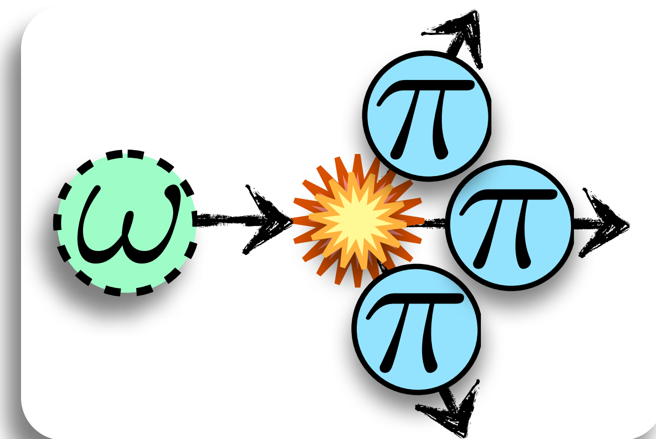
- Fundamental symmetries:

Status of formalism

(somewhat bias estimate)

- Spectroscopy / scattering:



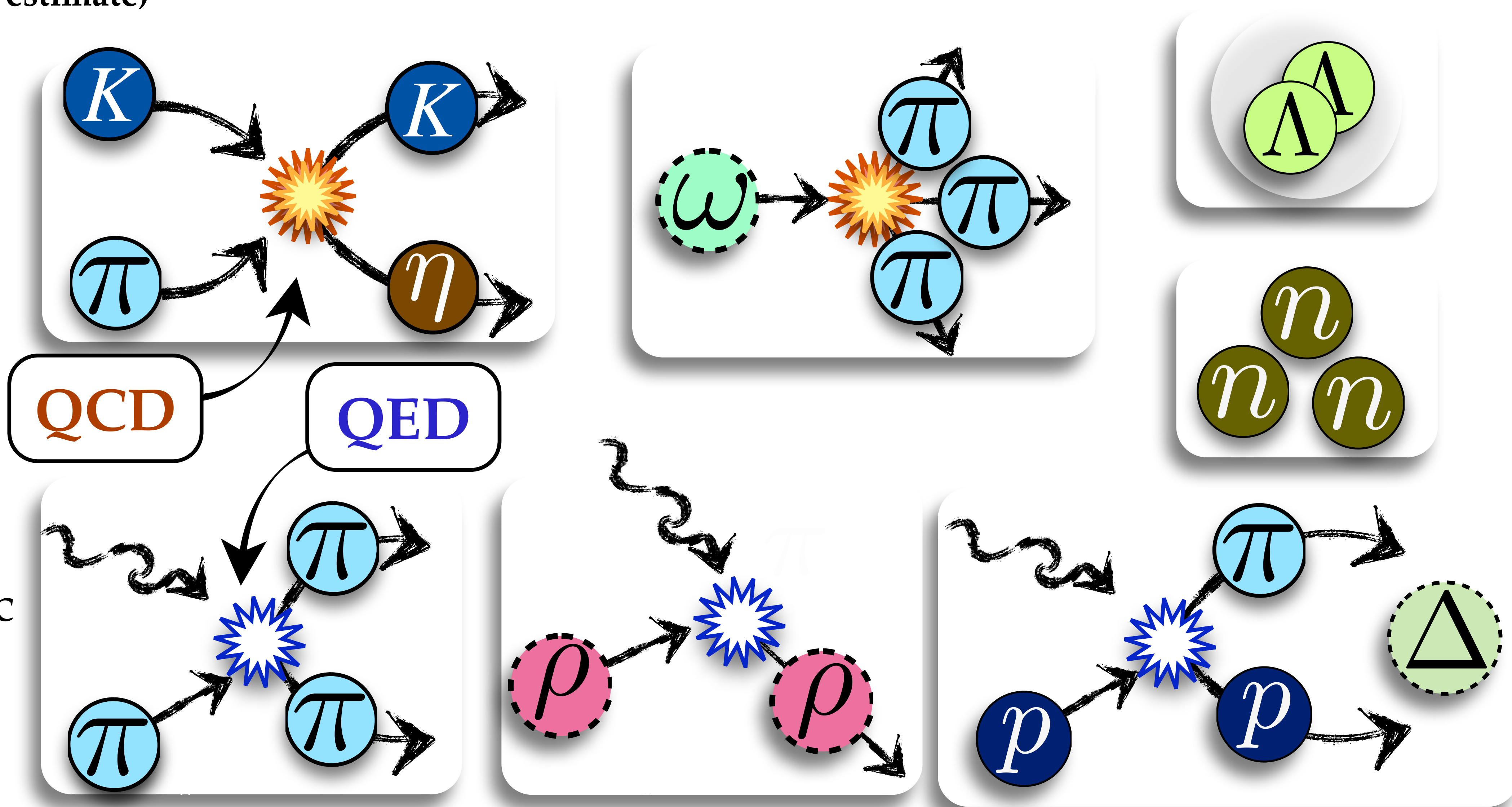


- Electromagnetic form factors:

- Fundamental symmetries:

Status of formalism

(somewhat bias estimate)



- Spectroscopy / scattering:
- Electromagnetic form factors:
- Fundamental symmetries:

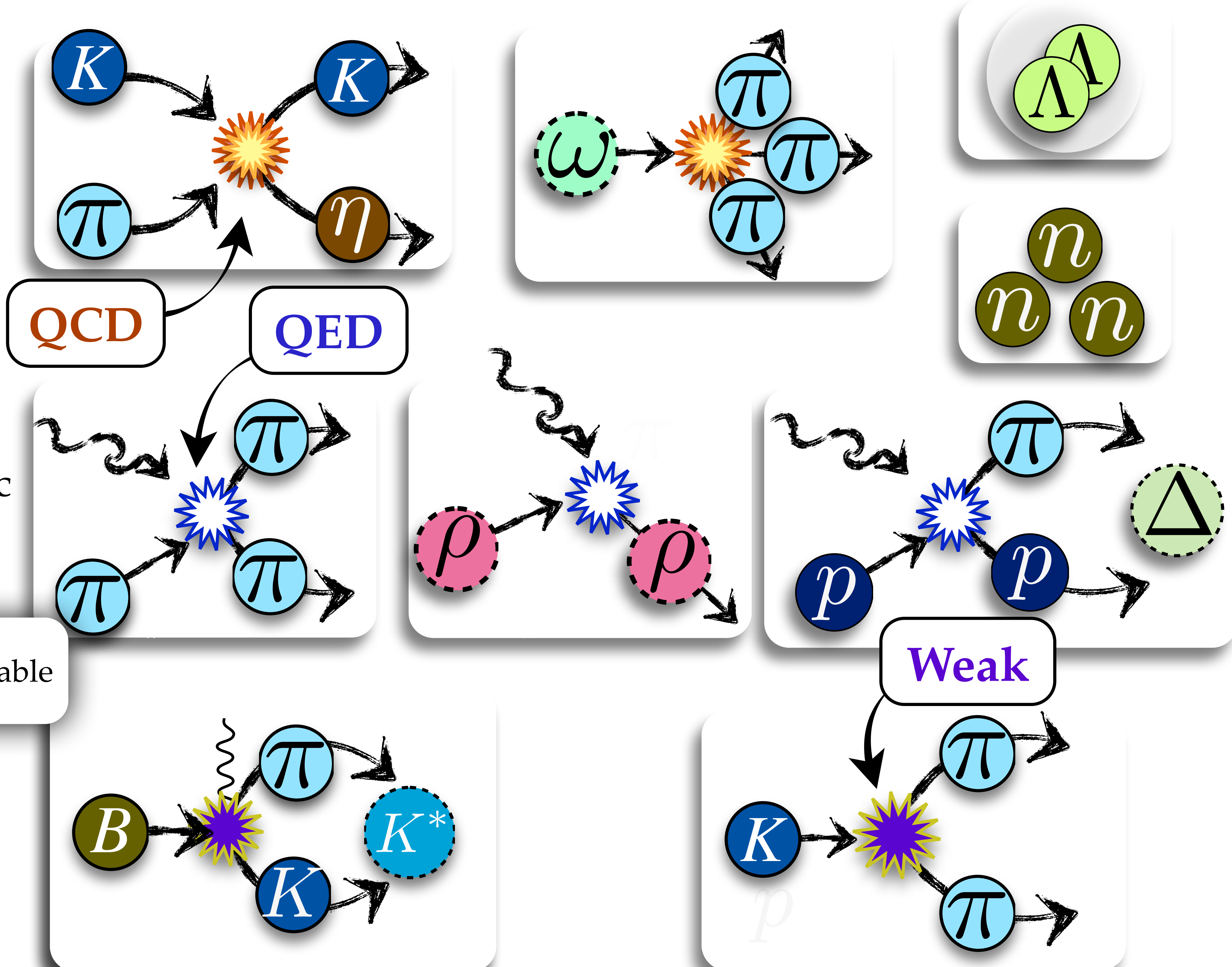
Status of formalism

(somewhat bias estimate)

- Spectroscopy / scattering:
- Electromagnetic form factors:

formally indistinguishable

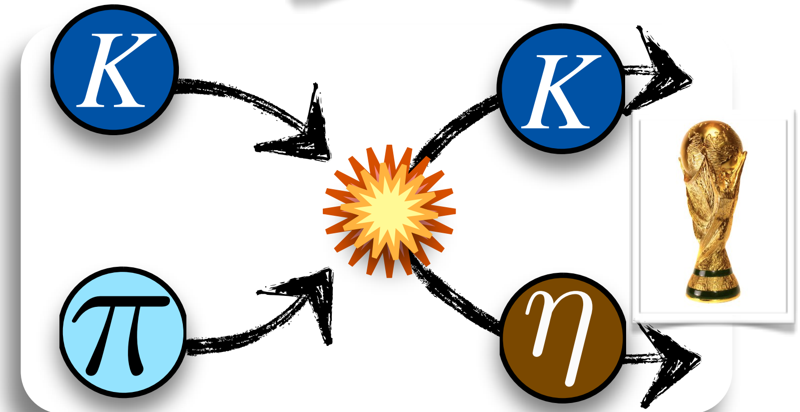
- Fundamental symmetries:

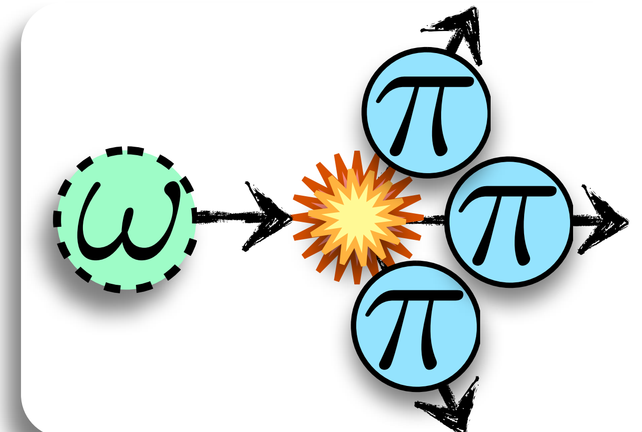


Status of formalism
(somewhat bias estimate)

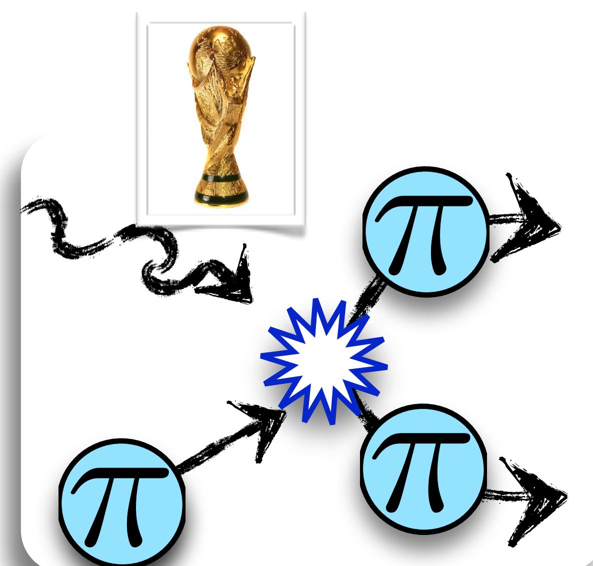
: Under control

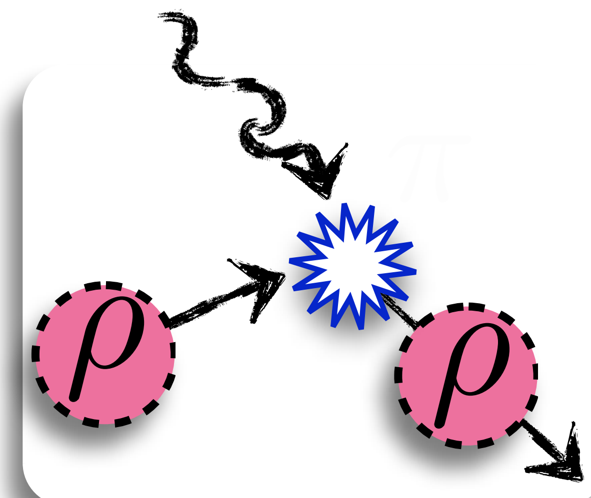
- Spectroscopy / scattering:

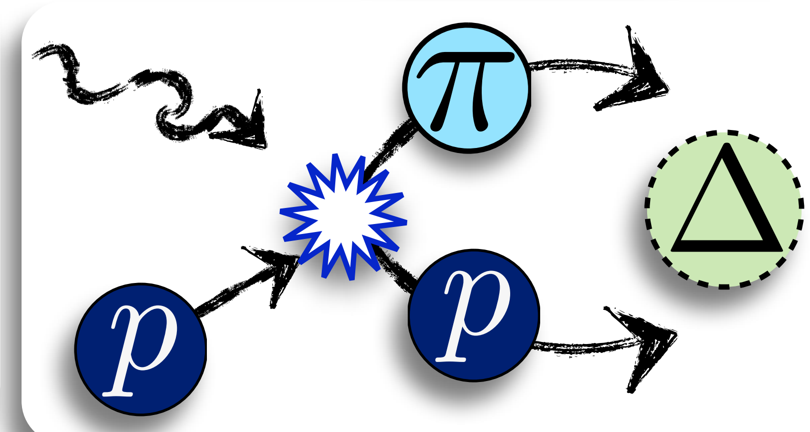




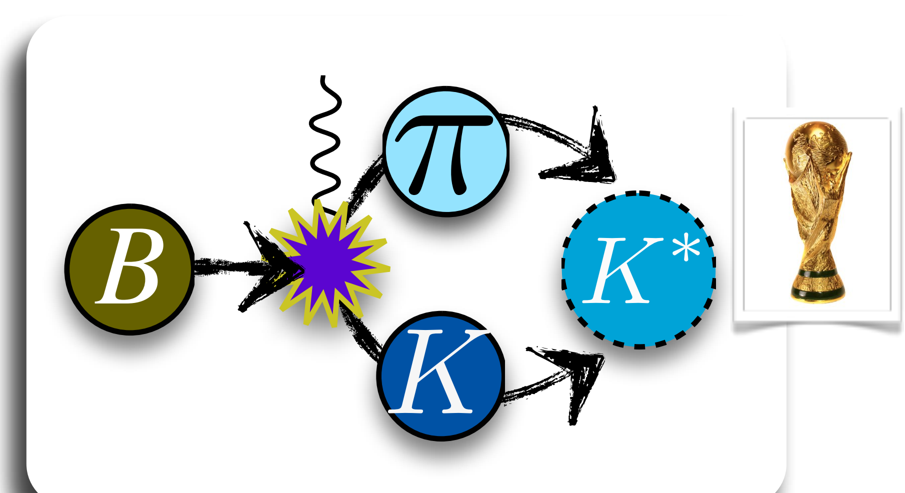
- Electromagnetic form factors:

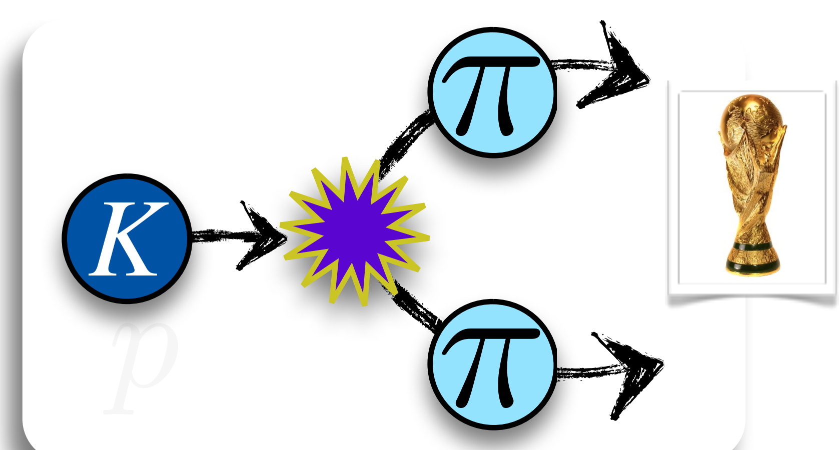






- Fundamental symmetries:





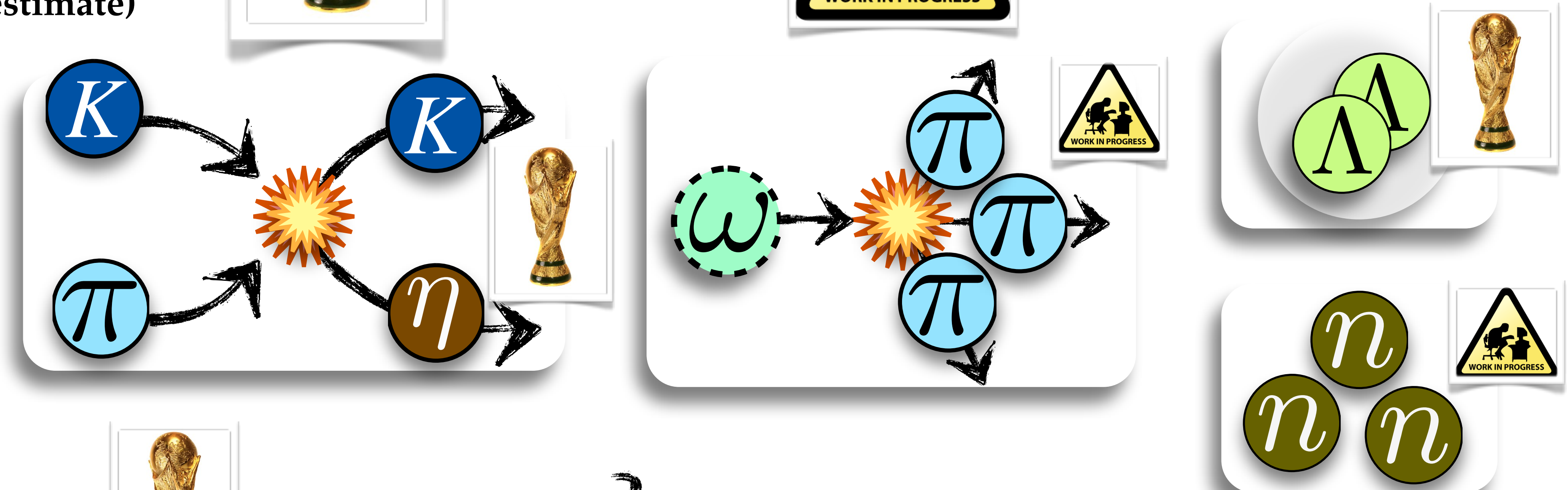
Status of formalism

(somewhat bias estimate)

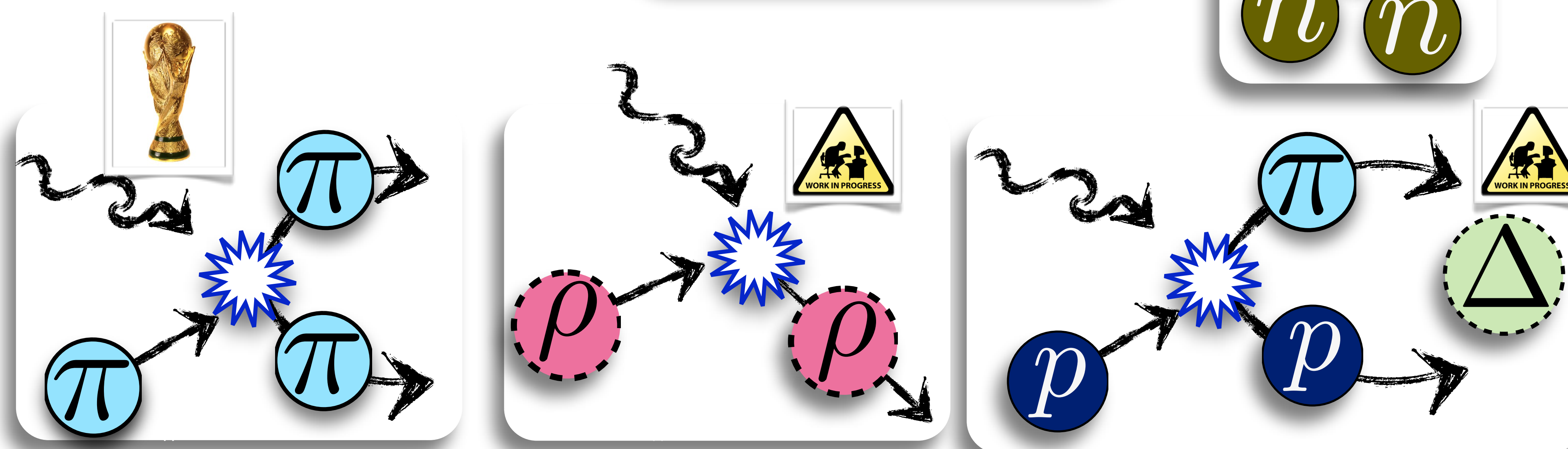
: Under control

: progress made / more to come

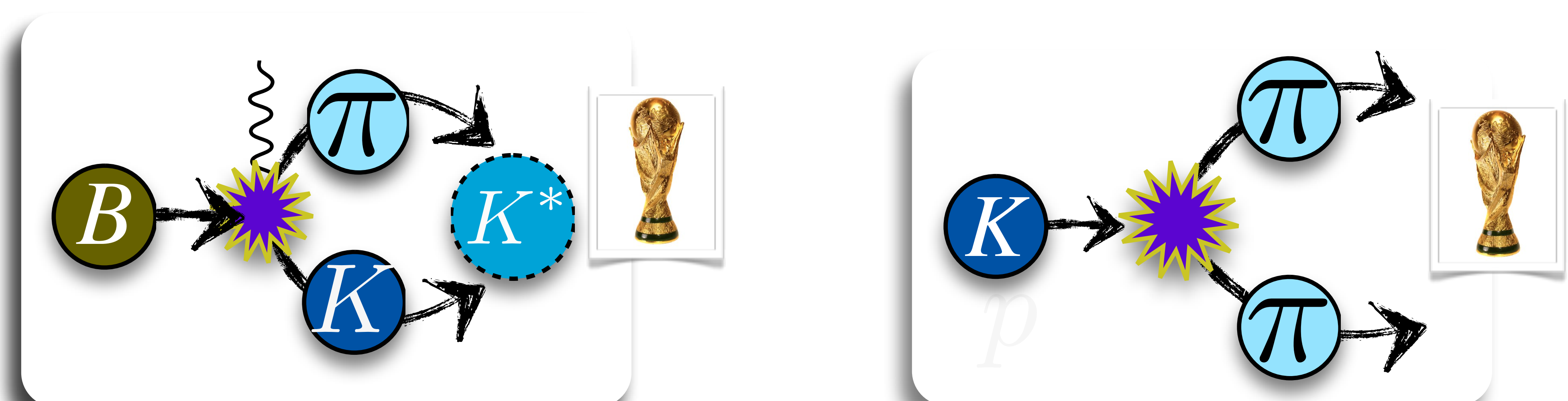
- Spectroscopy / scattering:



- Electromagnetic form factors:



- Fundamental symmetries:



Few-body talks to see

many thanks to all who sent material to share!

speakers	date / time	topic
Z. Davoudi	Wed. @ 12:50	two-baryon formalism
T. Doi	Thurs. @ 15:55	three-N force potential
M. Endres	Fri @ 18:10	noise reduction
J. Green	Wed @ 12:30	H-dibaryon
W. Kamleh	Wed. @ 09:00	five-quark operators
D. Leinweber	Today @ 16:50	the nature of the $\Lambda(1405)$
A. Rusetsky	Fri @ 17:50	Δ to $N\gamma$ transition
B. Owen	Thurs. @ 16:15	excited nucleons form factors
C. Shultz	Thurs. @ 3:55pm	radiative physics
S. Sharpe	Today @ 14:15	three-particle formalism
P. Vachaspati	Today	Poster: B decays
A. Walker-Loud	Today @ 17:10	multi-channel 1 to 2 formalism
M. Wingate	Today	Poster: B decays

Stay for T. Yamazaki's talks for more numerical results!

Thanks!