The Λ(1405) is an anti-kaon–nucleon molecule

Jonathan Hall, Waseem Kamleh, Derek Leinweber, Ben Menadue,
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The $\Lambda(1405)$

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- Even though it contains a heavy strange quark and has odd parity its mass is lower than any other excited spin-$1/2$ baryon.
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- Before the existence of quarks was confirmed, Dalitz and co-workers speculated that it might be a molecular state of an anti-kaon bound to a nucleon.
- For almost 50 years the structure of the $\Lambda(1405)$ resonance has been a subject of debate.

The $\Lambda(1405)$

- Here we'll see how a new lattice QCD simulation showing
  - The $\Lambda(1405)$ strange magnetic form factor vanishes, together with
  - A Hamiltonian effective field theory analysis of the lattice QCD energy levels,

  unambiguously establishes that the structure is dominated by a bound anti-kaon–nucleon component.

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- In both cases the strange quark is confined within a spin-0 kaon and has no preferred spin orientation.
- To conserve parity, the kaon has zero orbital angular momentum.
- Thus, the strange quark does not contribute to the magnetic form factor of the $\Lambda(1405)$ when it is in a $\overline{K}N$ molecule.

Outline

Techniques for exciting the $\Lambda(1405)$ in Lattice QCD

Quark-sector contributions to the electric form factor of the $\Lambda(1405)$

Quark-sector contributions to the magnetic form factor of the $\Lambda(1405)$

Hamiltonian effective field theory model: m_0, $\pi\Sigma$, $\overline{K}N$, $K\Xi$ and $\eta\Lambda$.

Conclusion

The $\Lambda(1405)$ and Lattice QCD

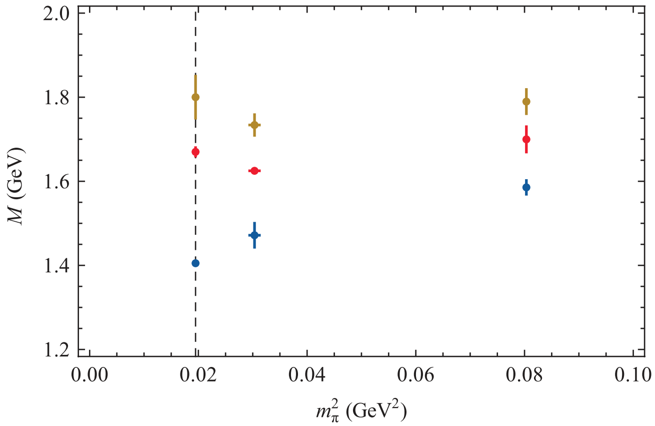
Our recent work has successfully isolated three low-lying odd-parity spin-$1/2$ states.

B. Menadue, W. Kamleh, D. B. Leinweber, M. S. Mahbub, Phys. Rev. Lett. **108**, 112001 (2012)

- An extrapolation of the trend of the lowest state reproduces the mass of the $\Lambda(1405)$.
- Subsequent studies have confirmed these results.

G. P. Engel, C. B. Lang, A. Schäfer, Phys. Rev. D **87**, 034502 (2013)

$\Lambda(1405)$ and Baryon Octet dominated states



Simulation Details

We are using the PACS-CS $(2+1)$-flavour ensembles, available through the ILDG.

S. Aoki *et al* (PACS-CS Collaboration), Phys. Rev. D **79**, 034503 (2009)

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- Single strange quark mass, with $\kappa_s = 0.13640$.
  - We use $\kappa_s = 0.13665$ for the valence strange quarks to reproduce the physical kaon mass.
- The strange quark κ_s is held fixed as the light quark masses vary.
  - Changes in the strange quark contributions are environmental effects.

The $\Lambda(1405)$ and Lattice QCD

The variational analysis is necessary to isolate the $\Lambda(1405)$.

Variational Analysis

By using multiple operators, we can isolate and analyse individual energy eigenstates:

- Construct the correlation matrix

$$G_{ij}(\mathbf{p};t) = \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}\cdot\mathbf{x}} \operatorname{tr}\left(\Gamma \, \langle\Omega|\, \chi_i(x)\, \overline{\chi}_j(0)\, |\Omega\rangle \right),$$

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  for some set $\{\,\chi_i\,\}$ operators that couple to the states of interest.
- We seek the linear combinations of the operators $\{\,\chi_i\,\}$ that perfectly isolate individual energy eigenstates, α, at momentum $\mathbf{p}$:

$$\phi^\alpha = v_i^\alpha(\mathbf{p})\,\chi_i\,, \qquad \overline{\phi}^\alpha = u_i^\alpha(\mathbf{p})\,\overline{\chi}_i\,.$$

Variational Analysis

- When successful, only state α participates in the correlation function, and one can write recurrence relations

$$G(\mathbf{p}; t + \delta t)\, \mathbf{u}^{\alpha}(\mathbf{p}) = \mathrm{e}^{-E_{\alpha}(\mathbf{p})\, \delta t}\, G(\mathbf{p}; t)\, \mathbf{u}^{\alpha}(\mathbf{p})$$

$$\mathbf{v}^{\alpha \mathsf{T}}(\mathbf{p})\, G(\mathbf{p}; t + \delta t) = \mathrm{e}^{-E_{\alpha}(\mathbf{p})\, \delta t}\, \mathbf{v}^{\alpha \mathsf{T}}(\mathbf{p})\, G(\mathbf{p}; t)$$

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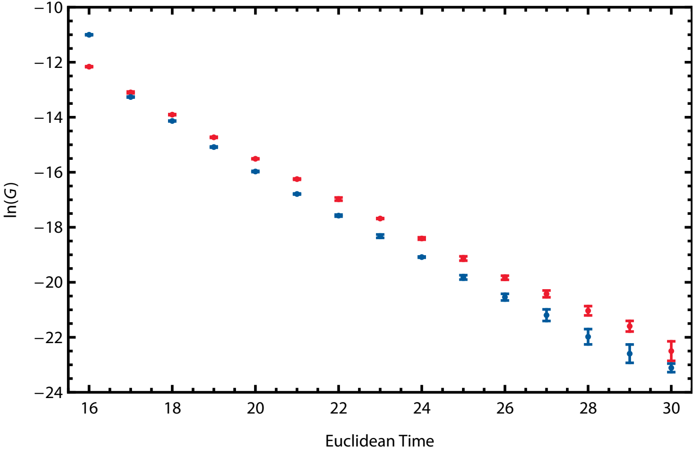
- Solve for the left, $\mathbf{v}^{\alpha}(\mathbf{p})$, and right, $\mathbf{u}^{\alpha}(\mathbf{p})$, generalised eigenvectors of $G(\mathbf{p}; t + \delta t)$ and $G(\mathbf{p}; t)$:

Eigenstate-Projected Correlation Functions

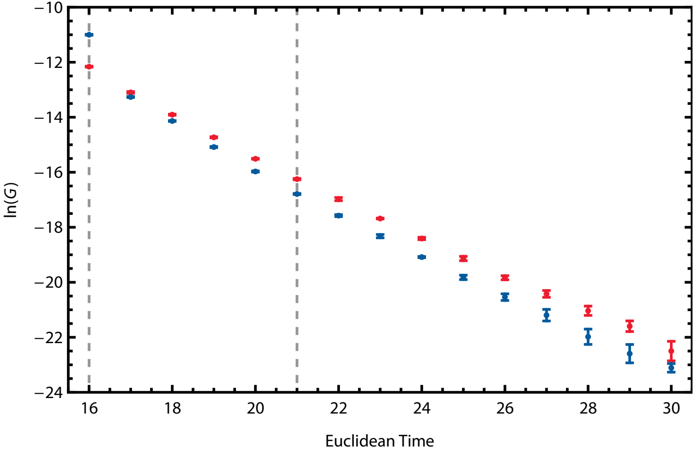
- Using these optimal operators, eigenstate-projected correlation functions are obtained

$$
\begin{aligned}
G^{\alpha}(\mathbf{p};t) &= \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}\cdot\mathbf{x}} \,\langle \Omega | \phi^{\alpha}(x)\,\overline{\phi}^{\alpha}(0) | \Omega \rangle \\
&= \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\,\mathbf{p}\cdot\mathbf{x}} \,\langle \Omega | v_i^{\alpha}(\mathbf{p})\,\chi_i(x)\,\overline{\chi}_j(0)\,u_j^{\alpha}(\mathbf{p}) | \Omega \rangle \\
&= \mathbf{v}^{\alpha\mathsf{T}}(\mathbf{p})\,G(\mathbf{p};t)\,\mathbf{u}^{\alpha}(\mathbf{p})
\end{aligned}
$$

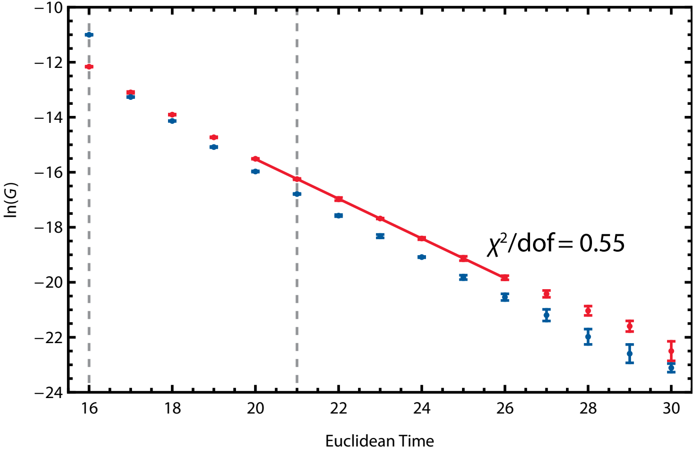
The importance of eigenstate isolation (red)



Probing with the electromagnetic current



Only the projected correlator has acceptable χ^2/dof



Operators Used in Λ(1405) Analysis

We consider local three-quark operators with the correct quantum numbers for the Λ channel, including

- Flavour-octet operators

$$\chi_1^8 = \frac{1}{\sqrt{6}}\varepsilon^{abc}\left(2(u^a C\gamma_5 d^b)s^c + (u^a C\gamma_5 s^b)d^c - (d^a C\gamma_5 s^b)u^c\right)$$
$$\chi_2^8 = \frac{1}{\sqrt{6}}\varepsilon^{abc}\left(2(u^a C d^b)\gamma_5 s^c + (u^a C s^b)\gamma_5 d^c - (d^a C s^b)\gamma_5 u^c\right)$$

- Flavour-singlet operator

$$\chi^1 = 2\varepsilon^{abc}\left((u^a C\gamma_5 d^b)s^c - (u^a C\gamma_5 s^b)d^c + (d^a C\gamma_5 s^b)u^c\right)$$

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We also use gauge-invariant Gaussian smearing to increase our operator basis.

- These results use 16 and 100 sweeps.
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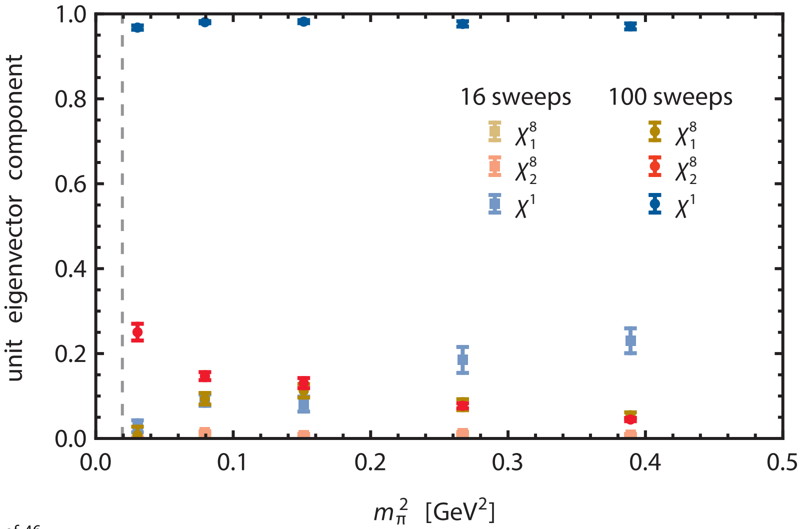
- These results use 16 and 100 sweeps.
  - Gives a 6×6 matrix.
- Also considered 35 and 100 sweeps.
  - Results are consistent with larger statistical uncertainties.

Flavour structure of the $\Lambda(1405)$



Extracting Form Factors from Lattice QCD

- To extract the form factors for a state α, we need to calculate the three-point correlation function

$$G^{\mu}_{\alpha}(\mathbf{p}', \mathbf{p}; t_2, t_1) = \sum_{\mathbf{x}_1,\, \mathbf{x}_2} \mathrm{e}^{-\mathrm{i}\, \mathbf{p}' \cdot \mathbf{x}_2} \mathrm{e}^{\mathrm{i}(\mathbf{p}' - \mathbf{p}) \cdot \mathbf{x}_1} \langle \Omega | \phi^{\alpha}(x_2)\, j^{\mu}(x_1)\, \overline{\phi}^{\alpha}(0) | \Omega \rangle$$

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- This takes the form

$$\mathrm{e}^{-E_{\alpha}(\mathbf{p}')(t_2-t_1)}\mathrm{e}^{-E_{\alpha}(\mathbf{p})t_1} \sum_{s,\, s'} \langle\Omega|\phi^{\alpha}|p', s'\rangle\, \langle p', s'|j^{\mu}|p, s\rangle\, \langle p, s|\overline{\phi}^{\alpha}|\Omega\rangle$$

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$$\mathrm{e}^{-E_{\alpha}(\mathbf{p}')(t_2-t_1)} \mathrm{e}^{-E_{\alpha}(\mathbf{p})t_1} \sum_{s,\, s'} \langle \Omega | \phi^{\alpha} | p', s' \rangle \, \langle p', s' | j^{\mu} | p, s \rangle \, \langle p, s | \overline{\phi}^{\alpha} | \Omega \rangle$$

- $\langle p', s' | j^{\mu} | p, s \rangle$ encodes the form factors of the interaction.

Current Matrix Elements for Spin-1/2 Baryons

The current matrix element for spin-1/2 baryons has the form

$$\langle p', s' | j^{\mu} | p, s \rangle = \left(\frac{m_{\alpha}^2}{E_{\alpha}(\mathbf{p}) E_{\alpha}(\mathbf{p}')} \right)^{1/2} \times$$

$$\times \, \overline{u}(\mathbf{p}') \left(F_1(q^2)\, \gamma^{\mu} + \mathrm{i}\, F_2(q^2)\, \sigma^{\mu\nu} \frac{q^{\nu}}{2m_{\alpha}} \right) u(\mathbf{p})$$

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- The Dirac and Pauli form factors are related to the Sachs form factors through

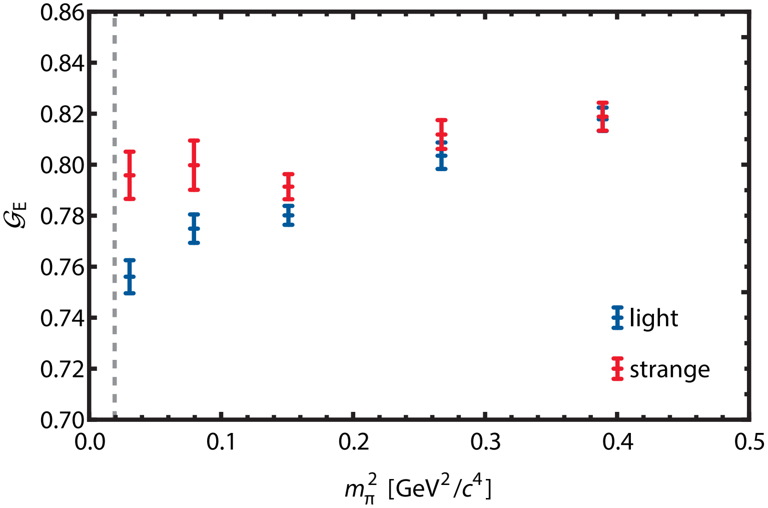
$$\mathcal{G}_{\mathrm{E}}(q^2) = F_1(q^2) - \frac{q^2}{(2m^\alpha)^2} F_2(q^2)$$
$$\mathcal{G}_{\mathrm{M}}(q^2) = F_1(q^2) + F_2(q^2)$$

Current Matrix Elements for Spin-1/2 Baryons

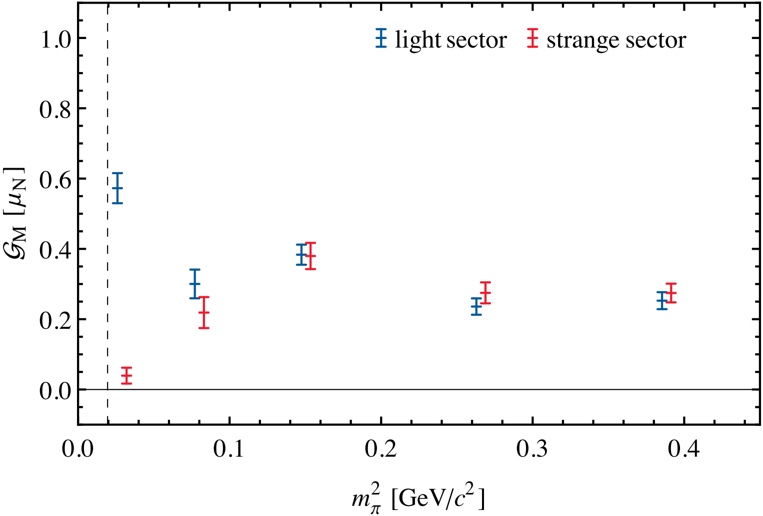
The light- and strange-quark sector contributions can be isolated.

- Eg. The strange sector is isolated by setting $q_u = q_d = 0$.
- q_s is set to unity such that we report results for single quarks of unit charge.
- Symmetry in the u-d sector provides $\mathcal{G}^u(Q^2) = \mathcal{G}^d(Q^2) \equiv \mathcal{G}^\ell(Q^2)$ for $q_u = q_d = 1$.

$\mathcal{G}_E$ for the $\Lambda(1405)$ at $Q^2 \sim 0.16\,\text{GeV}^2$



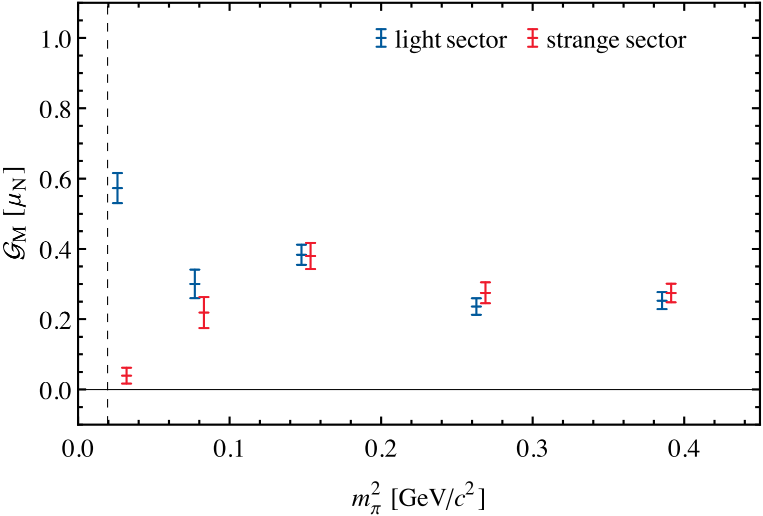
$\mathcal{G}_M$ for the $\Lambda(1405)$ at $Q^2 \sim 0.16\ \text{GeV}^2$



$\Lambda(1405)$ magnetic form factor observations

- $SU(3)$-flavour symmetry is manifest for $m_\ell \sim m_s$. All three quark flavours play a similar role.
- $\mathcal{G}_M^\ell \equiv \mathcal{G}_M^u \equiv \mathcal{G}_M^d \simeq \mathcal{G}_M^s$ for the heaviest three masses.

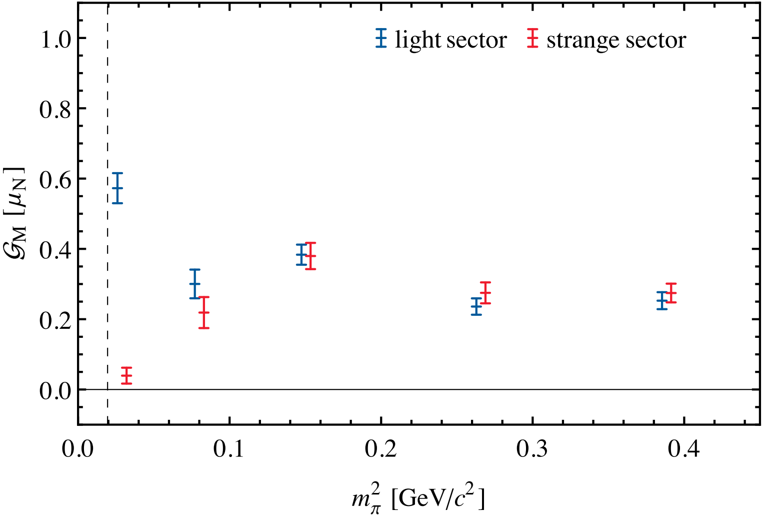
Λ(1405) magnetic form factor observations



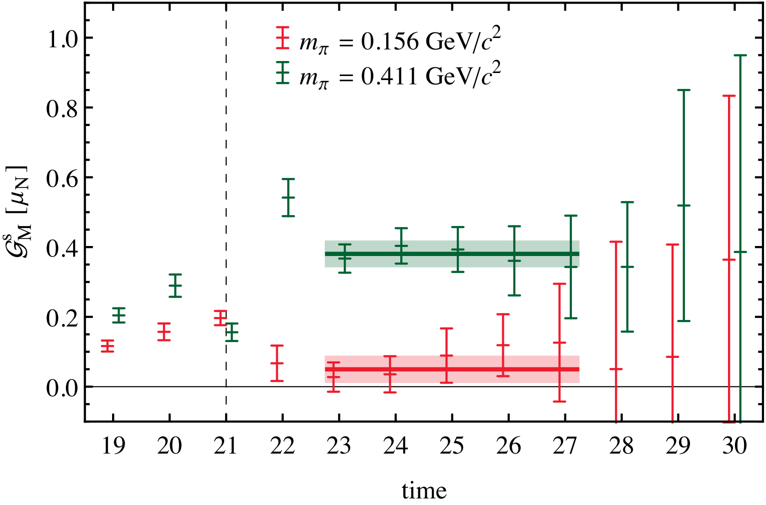
$\Lambda(1405)$ magnetic form factor observations

- The internal structure of the $\Lambda(1405)$ reorganises at the lightest quark mass.
- The strange quark contribution to the magnetic form factor of the $\Lambda(1405)$ drops by an order of magnitude and approaches zero.

Λ(1405) magnetic form factor observations



Correlation function ratio providing $\mathcal{G}_M^s(Q^2)$



$\Lambda(1405)$ magnetic form factor observations

- As the simulation parameters describing the strange quark are held fixed, this is a remarkable environmental effect of unprecedented strength.
- We observe an important rearrangement of the quark structure within the $\Lambda(1405)$ consistent with the dominance of a molecular $\overline{K}N$ bound state.

Hamiltonian Effective Field Theory Model

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- It also includes a single-particle state with bare mass, $m_0 + \alpha_0\, m_\pi^2$.
- In a finite periodic volume, momentum is quantised to $n\,(2\pi/L)$.
- Working on a cubic volume of extent L on each side, it is convenient to define the momentum magnitudes

$$k_n = \sqrt{n_x^2 + n_y^2 + n_z^2}\,\frac{2\pi}{L}\,,$$

with $n_i = 0, 1, 2, \ldots$ and integer $n = n_x^2 + n_y^2 + n_z^2$.

Hamiltonian model, H_0

Denoting each meson-baryon energy by $\omega_{MB}(k_n) = \omega_M(k_n) + \omega_B(k_n)$, with $\omega_A(k_n) \equiv \sqrt{k_n^2 + m_A^2}$, the non-interacting Hamiltonian takes the form

$$H_0 = \begin{pmatrix} m_0 + \alpha_0\, m_\pi^2 & 0 & 0 & \cdots \\ 0 & \begin{matrix} \omega_{\pi\Sigma}(k_0) & \\ & \ddots \\ & & \omega_{\eta\Lambda}(k_0) \end{matrix} & 0 & \cdots \\ 0 & 0 & \begin{matrix} \omega_{\pi\Sigma}(k_1) & \\ & \ddots \\ & & \omega_{\eta\Lambda}(k_1) \end{matrix} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} .$$

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- Interaction entries describe the coupling of the single-particle state to the two-particle meson-baryon states.

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- Each entry represents the S-wave interaction energy of the $\Lambda(1405)$ with one of the four channels at a certain value for k_n.

$$H_I = \begin{pmatrix} 0 & g_{\pi\Sigma}(k_0) & \cdots & g_{\eta\Lambda}(k_0) & g_{\pi\Sigma}(k_1) & \cdots & g_{\eta\Lambda}(k_1)\cdots \\ g_{\pi\Sigma}(k_0) & 0 & \cdots & & & & \\ \vdots & \vdots & 0 & & & & \\ & & & \ddots & & & \\ g_{\eta\Lambda}(k_0) & & & & & & \\ g_{\pi\Sigma}(k_1) & & & & & & \\ \vdots & & & & & & \\ g_{\eta\Lambda}(k_1) & & & & & & \\ \vdots & & & & & & \end{pmatrix} .$$

Eigenvalue Equation Form

- The eigenvalue equation corresponding to our Hamiltonian model is

$$\lambda = m_0 + \alpha_0\, m_\pi^2 - \sum_{M,B} \sum_{n=0}^{\infty} \frac{g_{MB}^2(k_n)}{\omega_{MB}(k_n) - \lambda}\,.$$

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- Reference to chiral effective field theory provides the form of $g_{MB}(k_n)$.

Hamiltonian model solution and fit

- The LAPACK software library routine `dgeev` is used to obtain the eigenvalues and eigenvectors of H.

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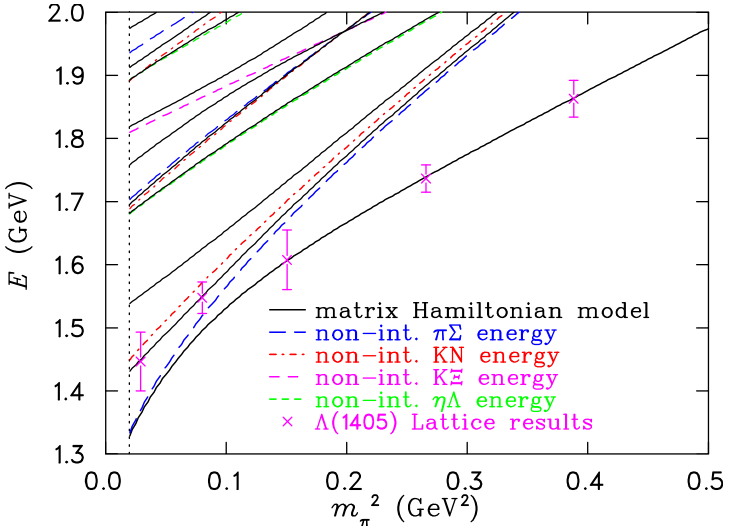
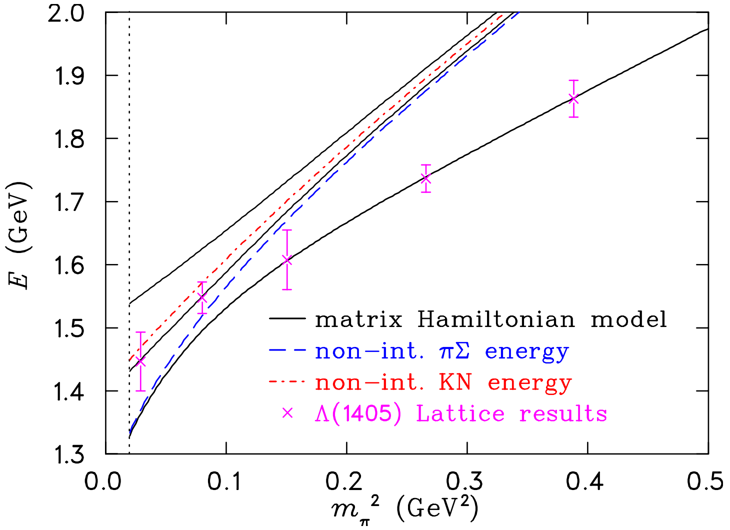
- The bare mass parameters m_0 and α_0 are determined by a fit to the lattice QCD results.

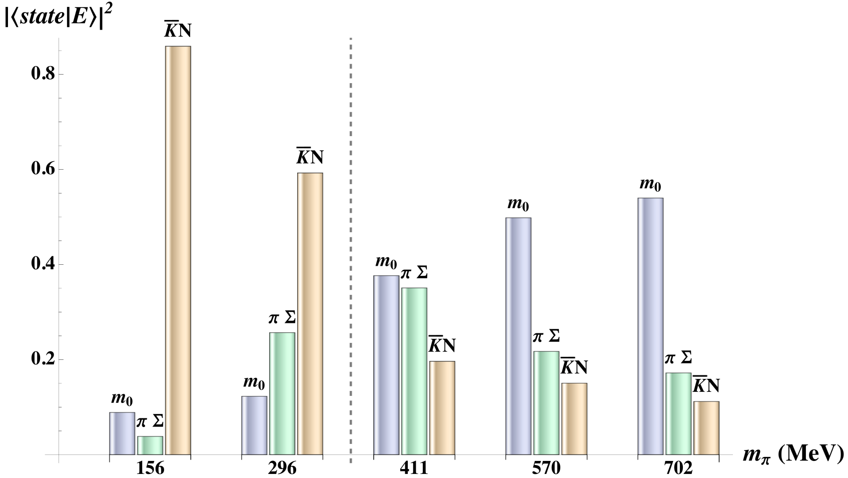
Hamiltonian model fit



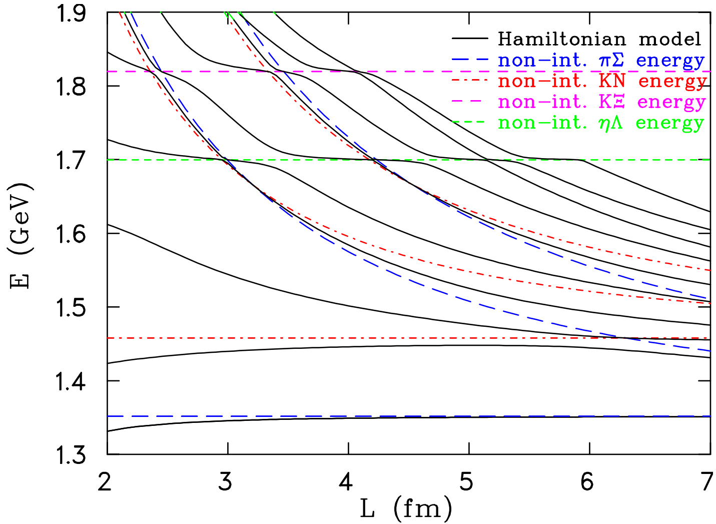
Avoided Level Crossing



Energy eigenstate, $|E\rangle$, basis $|state\rangle$ composition

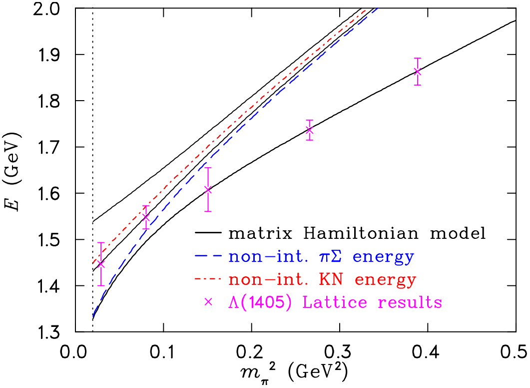


Volume dependence of the odd-parity Λ spectrum

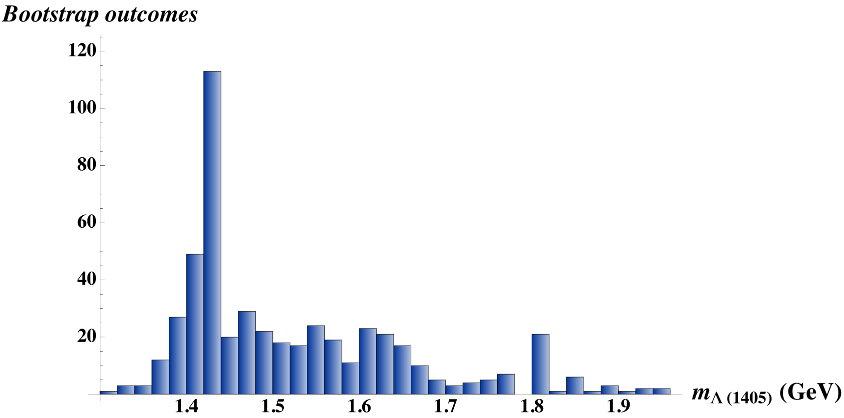


Infinite-volume reconstruction of the Λ(1405) energy

- Bootstraps are calculated by altering the value of each lattice data point by a Gaussian-distributed random number, weighted by the uncertainty.



Infinite-volume $\Lambda(1405)$ mass distribution at m_π^{phys}



Conclusions

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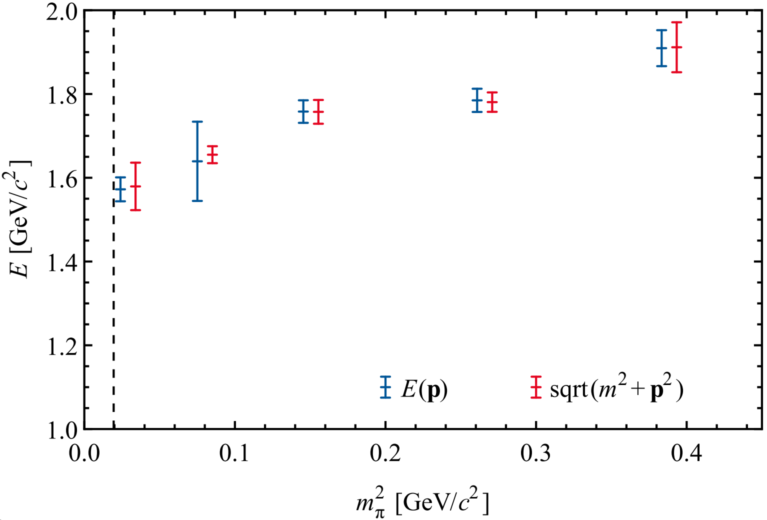
- The result ends 50 years of speculation on the structure of the $\Lambda(1405)$ resonance.

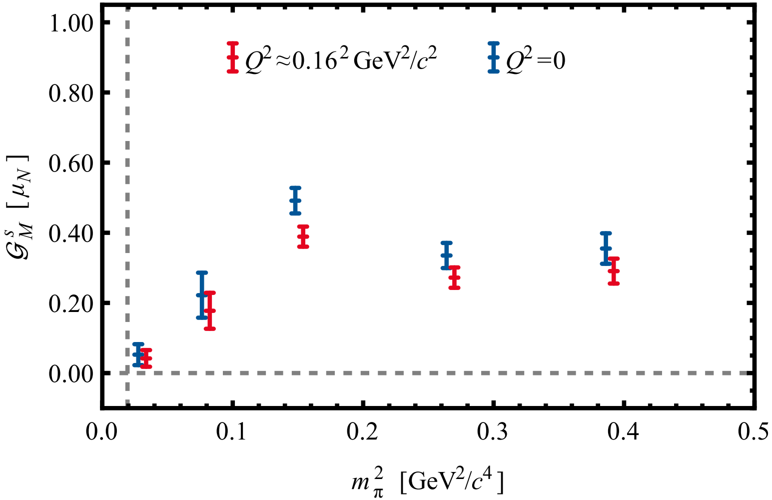
Supplementary Information

The following slides provide additional information which may be of interest.

Dispersion Relation Test for the Λ(1405)



$\mathcal{G}_\mathrm{M}^s(q^2)$ scaled to $\mathcal{G}_\mathrm{M}^s(0)$ via $\mathcal{G}_\mathrm{M}^s(q^2)/\mathcal{G}_\mathrm{E}^s(q^2)$



$\mathcal{G}_{\mathrm{E}}$ for the $\Lambda(1405)$

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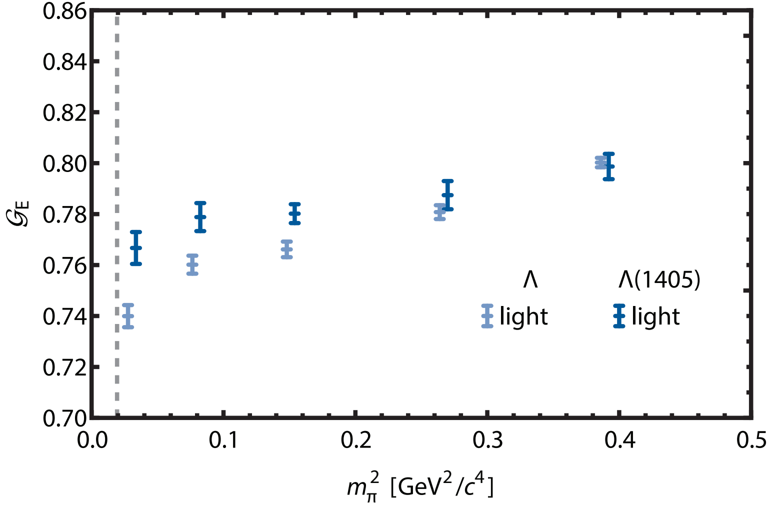

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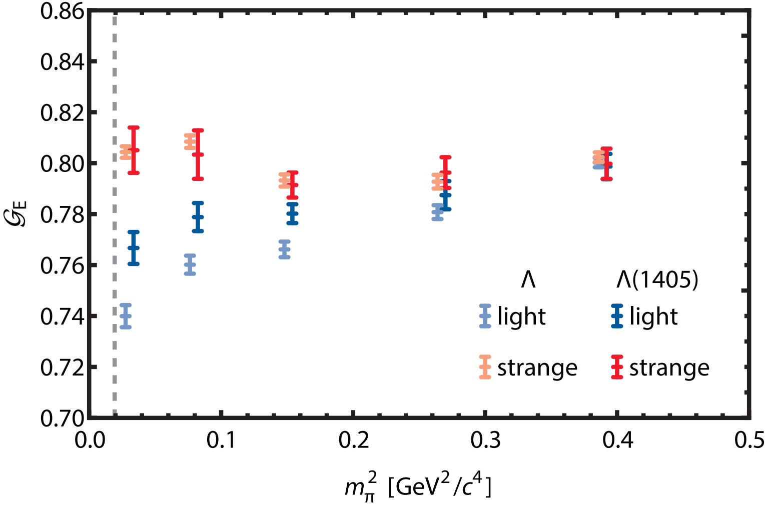
  - The strange quark may be distributed further out by the $\overline{\mathrm{K}}$ and thus have a smaller form factor.

$\mathcal{G}_E$ for the Λ(1405)



Hamiltonian model, H_I

- The form of the interaction is derived from chiral effective field theory.

$$g_{MB}(k_n) = \left(\frac{\kappa_{MB}}{16\pi^2 f_\pi^2} \frac{C_3(n)}{4\pi} \left(\frac{2\pi}{L} \right)^3 \omega_M(k_n)\, u^2(k_n) \right)^{1/2} .$$

- κ_{MB} denotes the $SU(3)$-flavour singlet couplings

$$\kappa_{\pi\Sigma} = 3\xi_0, \qquad \kappa_{\bar{K}N} = 2\xi_0, \qquad \kappa_{K\Xi} = 2\xi_0, \qquad \kappa_{\eta\Lambda} = \xi_0,$$

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- $C_3(n)$ is a combinatorial factor equal to the number of unique permutations of the momenta indices $\pm n_x$, $\pm n_y$ and $\pm n_z$.
- $u(k_n)$ is a dipole regulator, with regularization scale $\Lambda = 0.8$ GeV.

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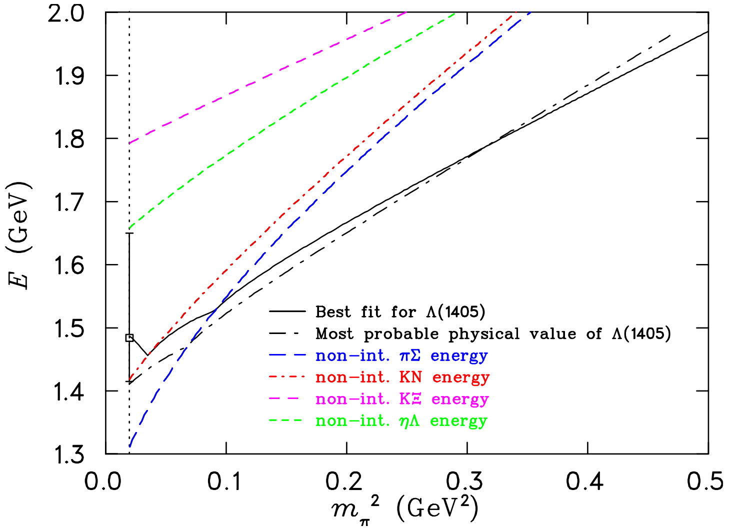

Excited State Form Factors

- The eigenstate-projected three-point correlation function is

$$G^{\mu}_{\alpha}(\mathbf{p}', \mathbf{p}; t_2, t_1) = \sum_{\mathbf{x}_1, \mathbf{x}_2} e^{-i\,\mathbf{p}'\cdot\mathbf{x}_2} e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}_1} \times \\ \times \langle \Omega | v^{\alpha}_i(\mathbf{p}')\, \chi_i(x_2)\, j^{\mu}(x_1)\, \overline{\chi}_j(0)\, u^{\alpha}_i(\mathbf{p}) | \Omega \rangle \\ = \mathbf{v}^{\alpha\,\mathsf{T}}(\mathbf{p}')\, G^{\mu}_{ij}(\mathbf{p}', \mathbf{p}; t_2, t_1)\, \mathbf{u}^{\alpha}(\mathbf{p})$$

where

$$G^{\mu}_{ij}(\mathbf{p}', \mathbf{p}; t_2, t_1) = \sum_{\mathbf{x}_1, \mathbf{x}_2} e^{-i\,\mathbf{p}'\cdot\mathbf{x}_2} e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}_1} \langle \Omega | \chi_i(x_2)\, j^{\mu}(x_1)\, \overline{\chi}_j(0) | \Omega \rangle$$

is the matrix constructed from the three-point correlation functions of the original operators $\{\, \chi_i \,\}$.

Extracting Form Factors from Lattice QCD

- To eliminate the time dependence of the three-point correlation function, we construct the ratio

$$R^{\mu}_{\alpha}(\mathbf{p}', \mathbf{p}; t_2, t_1) = \left(\frac{G^{\mu}_{\alpha}(\mathbf{p}', \mathbf{p}; t_2, t_1)\, G^{\mu}_{\alpha}(\mathbf{p}, \mathbf{p}'; t_2, t_1)}{G_{\alpha}(\mathbf{p}'; t_2)\, G_{\alpha}(\mathbf{p}; t_2)} \right)^{1/2}$$

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- To further simply things, we define the reduced ratio

$$\overline{R}^{\mu}_{\alpha} = \left(\frac{2E_{\alpha}(\mathbf{p})}{E_{\alpha}(\mathbf{p}) + m_{\alpha}} \right)^{1/2} \left(\frac{2E_{\alpha}(\mathbf{p}')}{E_{\alpha}(\mathbf{p}') + m_{\alpha}} \right)^{1/2} R^{\mu}_{\alpha}$$

Current Matrix Element for Spin-1/2 Baryons

The current matrix element for spin-1/2 baryons has the form

$$\langle p', s' | j^\mu | p, s \rangle = \left(\frac{m_\alpha^2}{E_\alpha(\mathbf{p}) E_\alpha(\mathbf{p}')} \right)^{1/2} \times$$
$$\times\, \overline{u}(\mathbf{p}') \left(F_1(q^2)\, \gamma^\mu + \mathrm{i}\, F_2(q^2)\, \sigma^{\mu\nu} \frac{q^\nu}{2m_\alpha} \right) u(\mathbf{p})$$

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- The Dirac and Pauli form factors are related to the Sachs form factors through

$$\mathcal{G}_\mathrm{E}(q^2) = F_1(q^2) - \frac{q^2}{(2m^\alpha)^2} F_2(q^2)$$
$$\mathcal{G}_\mathrm{M}(q^2) = F_1(q^2) + F_2(q^2)$$

Sachs Form Factors for Spin-1/2 Baryons

- A suitable choice of momentum ($\mathbf{q} = (q, 0, 0)$) and the (implicit) Dirac matrices allows us to directly access the Sachs form factors:
  - for $\mathcal{G}_{\mathrm{E}}$: using $\Gamma_4^{\pm}$ for both two- and three-point,

$$\mathcal{G}_{\mathrm{E}}^{\alpha}(q^2) = \overline{R}_{\alpha}^{4}(\mathbf{q}, \mathbf{0}; t_2, t_1)$$

  - for $\mathcal{G}_{\mathrm{M}}$: using $\Gamma_4^{\pm}$ for two-point and $\Gamma_j^{\pm}$ for three-point,

$$|\varepsilon_{ijk}\, q^i|\, \mathcal{G}_{\mathrm{M}}^{\alpha}(q^2) = (E_{\alpha}(\mathbf{q}) + m_{\alpha})\, \overline{R}_{\alpha}^{k}(\mathbf{q}, \mathbf{0}; t_2, t_1)$$

  - where for positive parity states,

$$\Gamma_j^{+} = \frac{1}{2}\begin{bmatrix}\sigma_j & 0 \\ 0 & 0\end{bmatrix} \qquad \Gamma_4^{+} = \frac{1}{2}\begin{bmatrix}\mathbb{I} & 0 \\ 0 & 0\end{bmatrix}$$

    and for negative parity states,

$$\Gamma_j^{-} = -\gamma_5 \Gamma_j^{+} \gamma_5 = -\frac{1}{2}\begin{bmatrix}0 & 0 \\ 0 & \sigma_j\end{bmatrix} \qquad \Gamma_4^{-} = -\gamma_5 \Gamma_4^{+} \gamma_5 = -\frac{1}{2}\begin{bmatrix}0 & 0 \\ 0 & \mathbb{I}\end{bmatrix}$$