Hadron Spectroscopy

Sasa Prelovsek

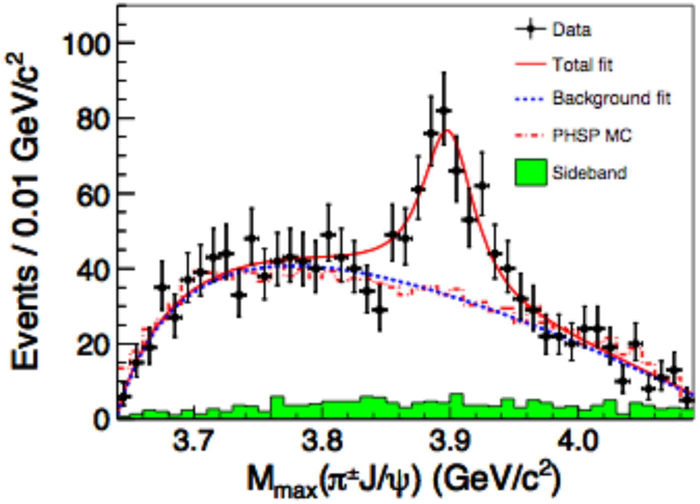
University of Ljubljana & Jozef Stefan Institute, Ljubljana, Slovenia

sasa.prelovsek@ijs.si

Lattice 2014, June 2014, Columbia University, New York City

Disclaimer

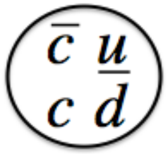
- the organizers asked me to concentrate on **mesons**, in particular to exotic quarkonium-like states (XYZ) that have been experimentally confirmed during 2013/2014



[BESIII, 2013, 1303.5949, PRL]

$Z_c^+(3900) \rightarrow \; J/\Psi \;\; \pi^+$

$\underline{c}c \;\; \underline{d}u$



such states confirmed by BeSII, Belle, LHCb, Cleo-c

- I will review also other topics in hadron spectroscopy to the extent possible

Outline

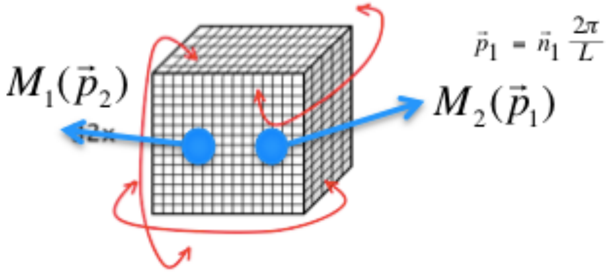
- Methods (most commonly used ones)
- ✧ States well below strong decay threshold
- ✧ Excited states: single-hadron approximation
- ✧ Near-threshold states
- ✧ Resonances
- Related studies

Methods

(most commonly used ones)

Discrete energy spectrum from correlators

Meson(like) system with given **J^{PC}** (or irrep.) is created by



$$\mathcal{O} = \bar{q}\Gamma q, \quad (\bar{q}\Gamma_1 q)_{\vec{p}_1}(\bar{q}\Gamma_2 q)_{\vec{p}_2}, \quad [\bar{q}\Gamma_3\bar{q}][q\Gamma_4 q],\ldots$$

$$\mathrm{M}_1(\vec{p}_1)\ \mathrm{M}_2(\vec{p}_2)$$

$$C_{ij}(t) = \langle 0|\, \mathcal{O}_i(t)\ \mathcal{O}_j^+(0)\,|0\rangle$$

$$= \sum_n \langle 0|\mathcal{O}_i|n\rangle\ e^{-E_n t}\ \langle n|\mathcal{O}_j^+|0\rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t} \qquad Z_i^n = \langle 0|\mathcal{O}_i|n\rangle$$

All physical states with given J^{PC} (or irrep) appear as E_n in principle (and are mixtures of) :

- "single-particle" states
- "two-particle" states: for periodic BC $\quad E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}} + \Delta E, \quad \vec{\mathrm{p}}_1 = \frac{2\pi}{L}\vec{n}_1 \quad \vec{\mathrm{p}}_2 = \frac{2\pi}{L}\vec{n}_2$
- three particle states.... (not yet in practice)

E_n and overlaps Z are extracted from variational method [C. Michael 1985, Luscher, Wolff, 1990, Blossier 2009]

$$\lambda^{(n)}(t) \propto e^{-E_n t}\,[1 + O(e^{-\Delta E t})] \qquad Z_j^{(n)}(t) = \mathrm{e}^{E_n t/2}\frac{|C_{jk}(t)u_k^{(n)}(t)|}{|C(t)^{\frac{1}{2}}u^{(n)}(t)|} \qquad C(t)u^{(n)}(t) = \lambda^{(n)}(t)C(t_0)u^{(n)}(t)$$

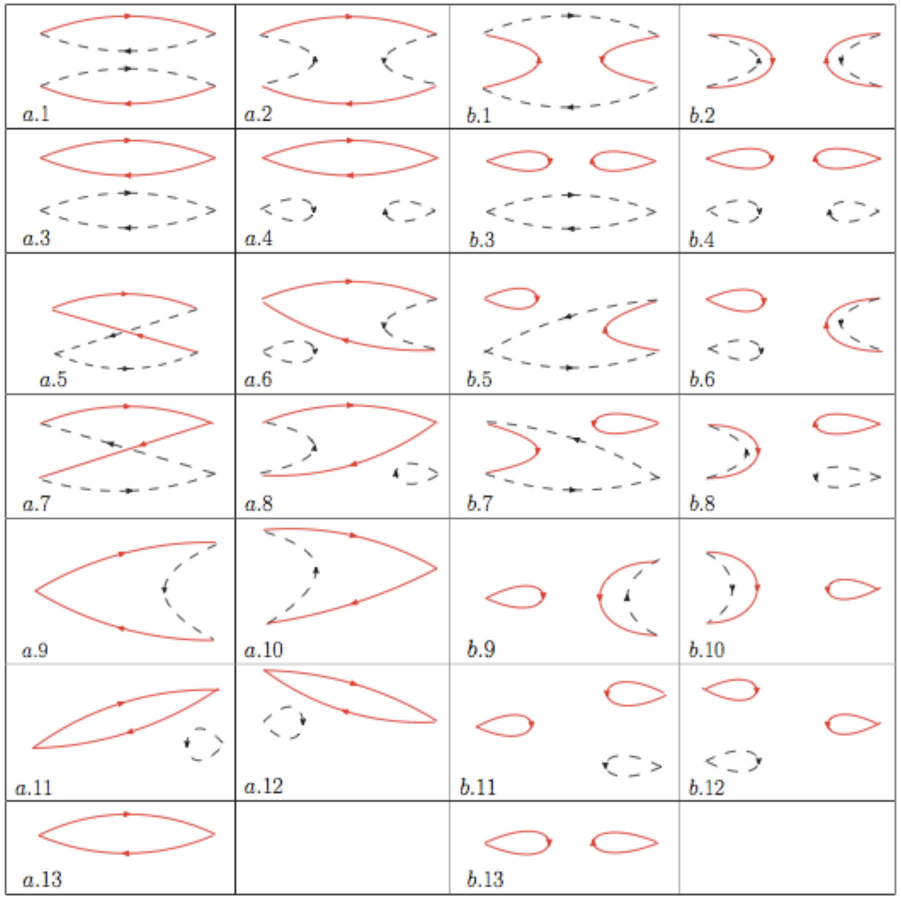
Wick contractions require all-to-all methods

- Example of Wick contractions needed for the X(3872) channel

$$\mathcal{O}:\ \bar{c}\,c,\quad (\bar{c}u)(\bar{u}c),\quad (\bar{c}c)(\bar{u}u)$$

- all-to-all methods are needed and widely used by now

- Examples:
  - distillation method [Peardon et al, JHEP 2009]
  - stochastic distillation method [Morningstar et al, PRD 2011]
  - a number of others



— c quark

- - - u,d quarks

[S.P. and L. Leskovec, Phys. Rev. Lett. 2013]

Extracting information on strong interactions

$$\tan\delta(p) = \frac{\sqrt{\pi}\; p\; L}{2\; \mathcal{Z}_{00}\left(1; (\frac{pL}{2\pi})^2\right)}$$

- Luscher-type relation : input E^{cm} [Luscher 1991]

  in favorable cases (P=0): one equation for one unknown $\delta_l(E^{cm})$

  in less favorable case (P≠0, θ≠0, coupled ch.): one equation for several unknowns $\delta_l^a(E^{cm})$

  encouraging that HSC managed to extract T-matrix for coupled Kπ, Kη [Dudek et al, HSC, 1406.4158], Wilson

  extensions and references reviewed by Briceno and Yamazaki, plenaries

$$\det\left(F^{-1} + i\mathcal{M}\right) = 0$$

$\delta_l^a(E^{cm})$

- finite-volume Hamiltonian EFT: input E [Hall et al, 1303,.4157, PRD], Leinweber

  fit E with λ of Hamiltonian EFT and extract parameters of Hamiltonian

- HALQCD method: [Ishii et al., PLB712, 437 (2012)] members of HALQCD

  determine V(r) between two mesons and extract δ(E) by solving Schrodinger eq.

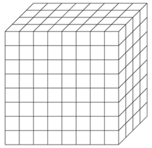
- possibility of rigorously extracting info from overlaps has not been fully explored
  so far used mostly at the intuitive level
  considerations in this direction may turn out fruitful

$$Z_i^n = \langle 0 | \mathcal{O}_i | n \rangle$$

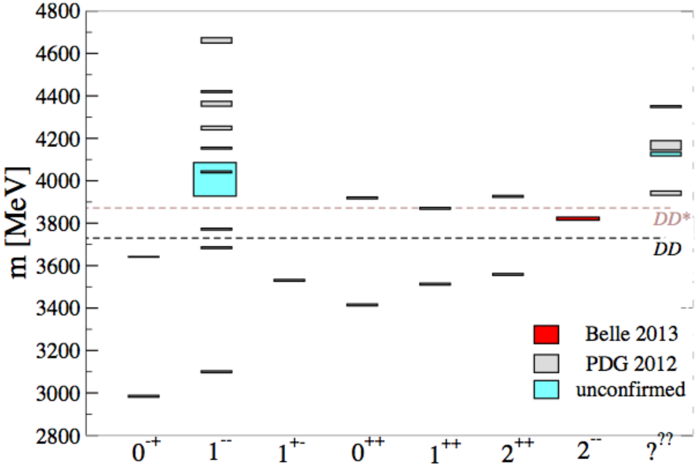
Precision spectrum:
States well below strong decay threshold

States well below threshold

- no two-particle states nearby: standard procedure applicable
- m=E (for P=0)



- extrapolation $a \to 0$, $L \to \infty$
- simulation at m_q^{phy} or extrapolation/interpolation $m_q \to m_q^{phy}$
- particular care needed for discretization errors related to am_c and am_b : complementary methods give compatible results for $a \to 0$
- many precision lattice results available !



Charmonium : $\underline{c}c$

- mixing with light hadrons is omitted for all hidden charm states that I will present !
- the rigorous treatment which goes beyond that still presents an unsolved challenge

  (i) noise in disconnected diagrams, (ii) mixing with a number of lighter states

Low lying charmonium

many results available, only few examples shown

band indicates uncertainty related to omission of charm annihilation [Levkova, DeTar]

$$M_{\overline{1S}} = \frac{1}{4}[M_{\eta_c} + 3M_{J/\psi}]$$

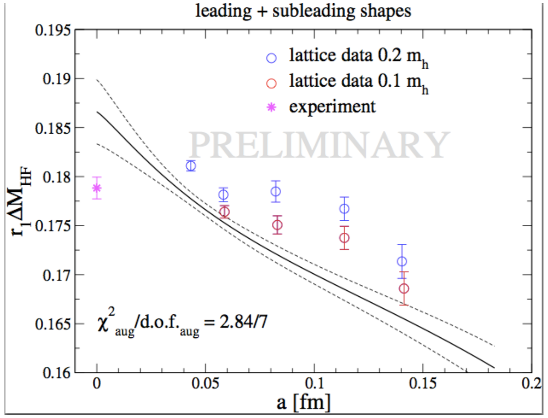
$$M_{\overline{1S}} = \frac{1}{4}[M_{\eta_{2S}} + 3M_{\psi_{2S}}]$$

$$M_{\overline{1P}} = \frac{1}{9}[M_{\chi_{c0}} + 3M_{\chi_{c1}} + 5M_{\chi_{c2}}]$$

$$[M_{J/\psi} - M_{\eta_c}]^{exp} = 113.2 \pm 0.7 \text{ MeV}$$

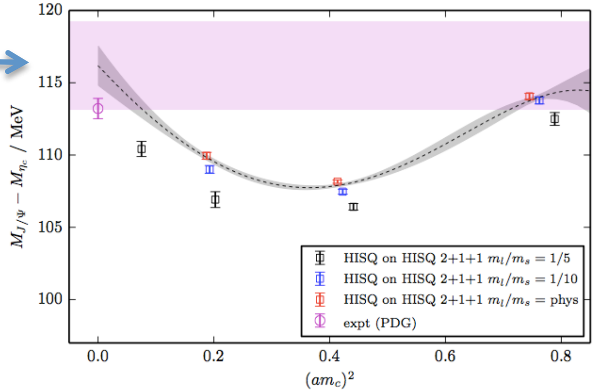
$$M_{J/\psi} - M_{\eta_c}$$

$$118.1 \pm 2.1 {}^{-1.5}_{-4.0} \text{ MeV}$$

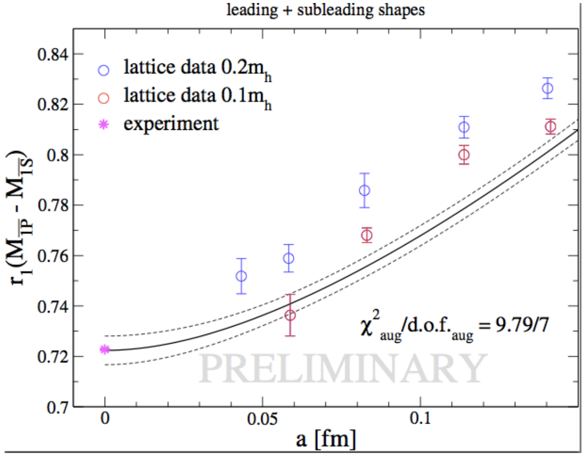


$$M_{J/\psi} - M_{\eta_c}$$

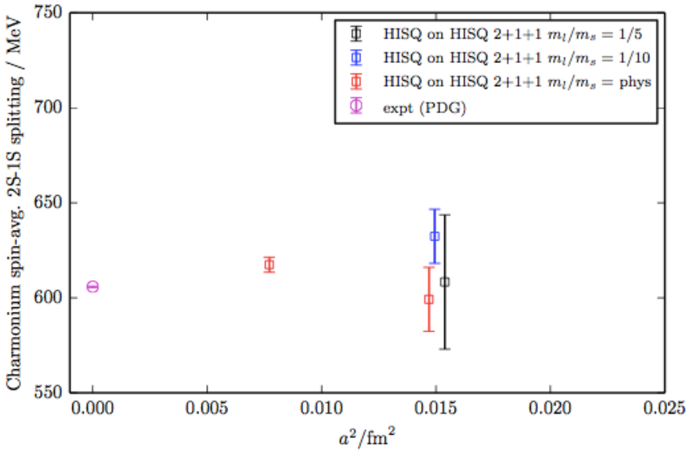
$$116.2 \pm 1.4 \pm 2.8 \text{ MeV}$$



$$M_{\overline{1P}} - M_{\overline{1S}}$$



$$M_{\overline{2S}} - M_{\overline{1S}}$$

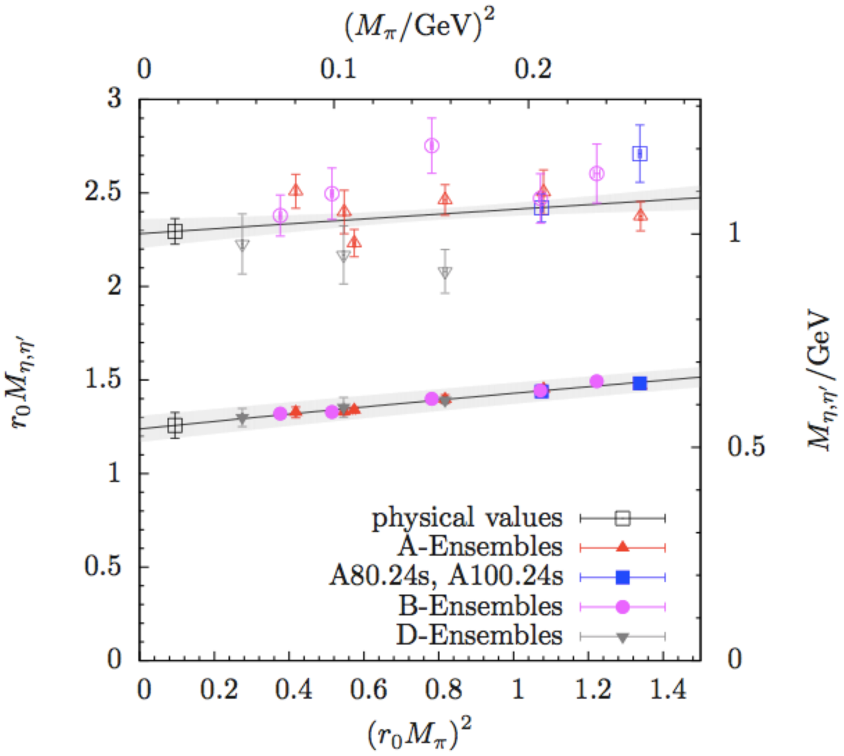


uncertainty from scale setting and disconnected diagrams not taken into account in above plots

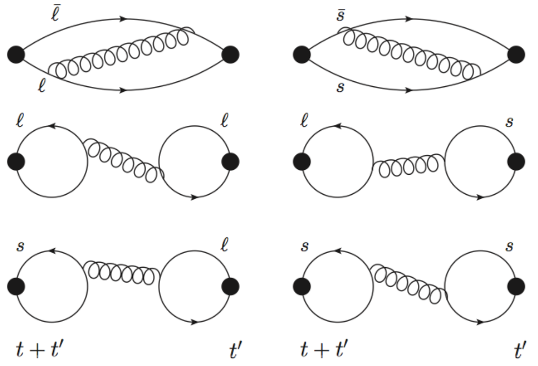
Fermilab/MILC, Daniel Mohler, Monday 17h30

HPQCD/MILC, Ben Galloway, Tuesday 15h35

η and η'



- both very narrow
- two-meson strong decay modes negligible



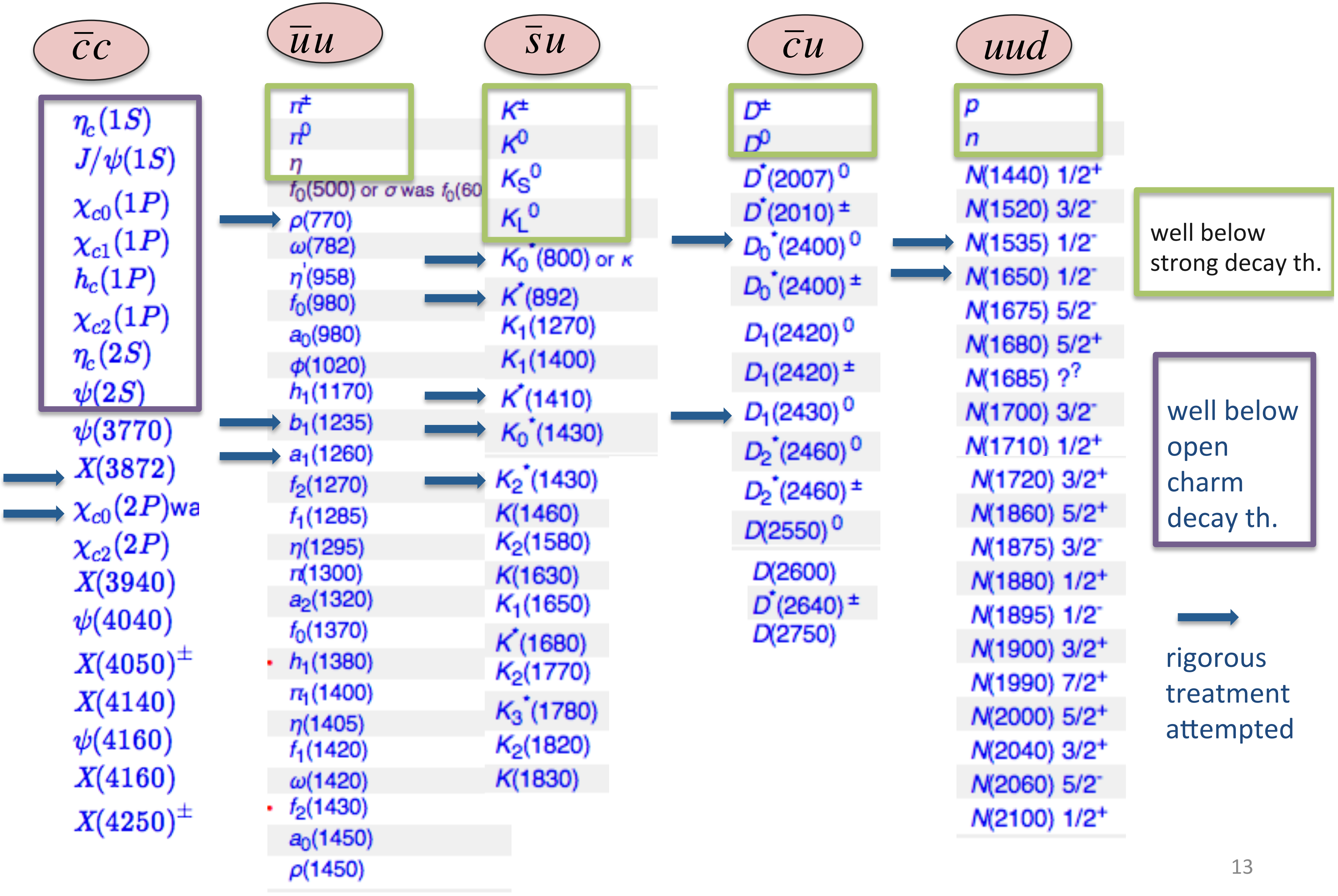
[C. Michael, K. Ottnad, C. Urbach, ETMC, 1310.1207, Phys. Rev. Lett. 2013]

"Non-precision" spectrum: states near or above threshold

only one or few a, L, $m_{u/d}$

limits $a \to 0$, $L \to \infty$, $m_{u/d} \to m_{u/d}^{phy}$ usually not performed

Almost all hadrons are near or below threshold



$\bar{c}c$

$\eta_c(1S)$
$J/\psi(1S)$
$\chi_{c0}(1P)$
$\chi_{c1}(1P)$
$h_c(1P)$
$\chi_{c2}(1P)$
$\eta_c(2S)$
$\psi(2S)$
$\psi(3770)$
$X(3872)$
$\chi_{c0}(2P)$wa
$\chi_{c2}(2P)$
$X(3940)$
$\psi(4040)$
$X(4050)^{\pm}$
$X(4140)$
$\psi(4160)$
$X(4160)$
$X(4250)^{\pm}$

$\bar{u}u$

$\pi^{\pm}$
π^0
η
$f_0(500)$ or σ was $f_0(60$
$\rho(770)$
$\omega(782)$
$\eta'(958)$
$f_0(980)$
$a_0(980)$
$\phi(1020)$
$h_1(1170)$
$b_1(1235)$
$a_1(1260)$
$f_2(1270)$
$f_1(1285)$
$\eta(1295)$
$\pi(1300)$
$a_2(1320)$
$f_0(1370)$
$h_1(1380)$
$\pi_1(1400)$
$\eta(1405)$
$f_1(1420)$
$\omega(1420)$
$f_2(1430)$
$a_0(1450)$
$\rho(1450)$

$\bar{s}u$

$K^{\pm}$
K^0
$K_S^{\ 0}$
$K_L^{\ 0}$
$K_0^{\ *}(800)$ or κ
$K^*(892)$
$K_1(1270)$
$K_1(1400)$
$K^*(1410)$
$K_0^{\ *}(1430)$
$K_2^{\ *}(1430)$
$K(1460)$
$K_2(1580)$
$K(1630)$
$K_1(1650)$
$K^*(1680)$
$K_2(1770)$
$K_3^{\ *}(1780)$
$K_2(1820)$
$K(1830)$

$\bar{c}u$

$D^{\pm}$
D^0
$D^*(2007)^0$
$D^*(2010)^{\pm}$
$D_0^{\ *}(2400)^0$
$D_0^{\ *}(2400)^{\pm}$
$D_1(2420)^0$
$D_1(2420)^{\pm}$
$D_1(2430)^0$
$D_2^{\ *}(2460)^0$
$D_2^{\ *}(2460)^{\pm}$
$D(2550)^0$
$D(2600)$
$D^*(2640)^{\pm}$
$D(2750)$

uud

p
n
$N(1440)\ 1/2^+$
$N(1520)\ 3/2^-$
$N(1535)\ 1/2^-$
$N(1650)\ 1/2^-$
$N(1675)\ 5/2^-$
$N(1680)\ 5/2^+$
$N(1685)\ ?^?$
$N(1700)\ 3/2^-$
$N(1710)\ 1/2^+$
$N(1720)\ 3/2^+$
$N(1860)\ 5/2^+$
$N(1875)\ 3/2^-$
$N(1880)\ 1/2^+$
$N(1895)\ 1/2^-$
$N(1900)\ 3/2^+$
$N(1990)\ 7/2^+$
$N(2000)\ 5/2^+$
$N(2040)\ 3/2^+$
$N(2060)\ 5/2^-$
$N(2100)\ 1/2^+$

well below strong decay th.

well below open charm decay th.

→ rigorous treatment attempted

States near or above threshold: single-hadron approximation

- only interpolating fields $\mathcal{O} \approx \bar{q}\, q$
- assumptions: all energy levels correspond to "one-particle" states

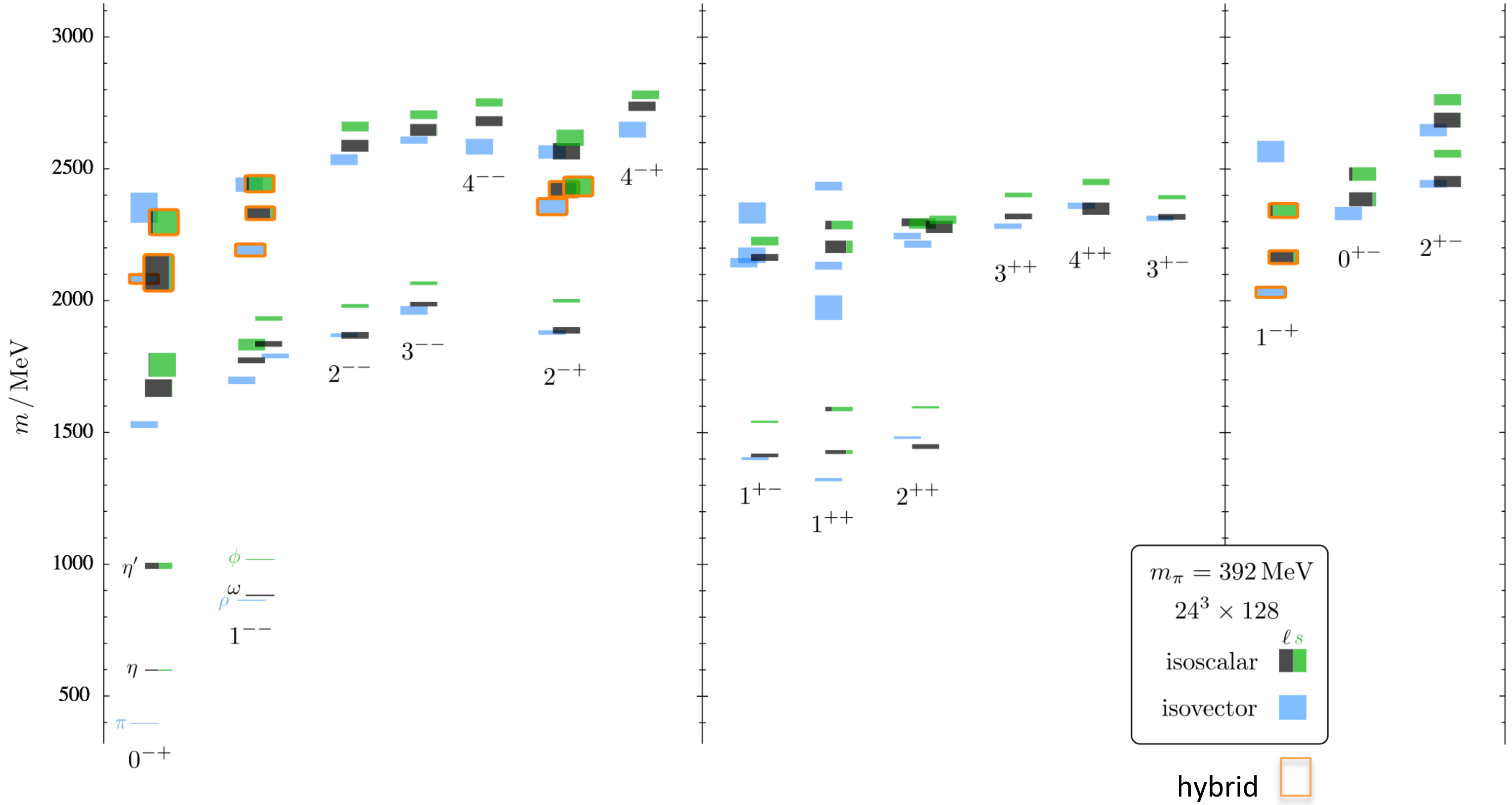
  no two-particle state is seen

  m=E (for P=0)

  these are strong assumptions ...

  but results still present valuable reference point

Isoscalar mesons : single hadron approximation



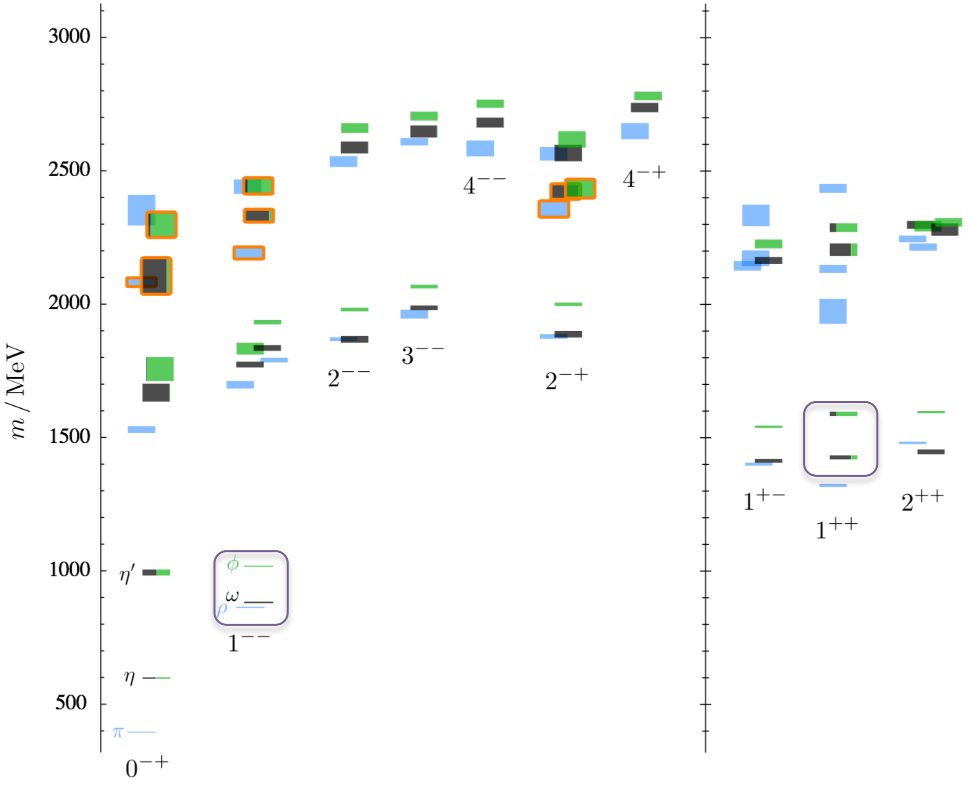
$$\begin{pmatrix} |\mathfrak{a}\rangle \\ |\mathfrak{b}\rangle \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} |\ell\rangle \\ |s\rangle \end{pmatrix}$$

$$|\ell\rangle \equiv \frac{1}{\sqrt{2}}\Big(|u\bar{u}\rangle + |d\bar{d}\rangle\Big)$$

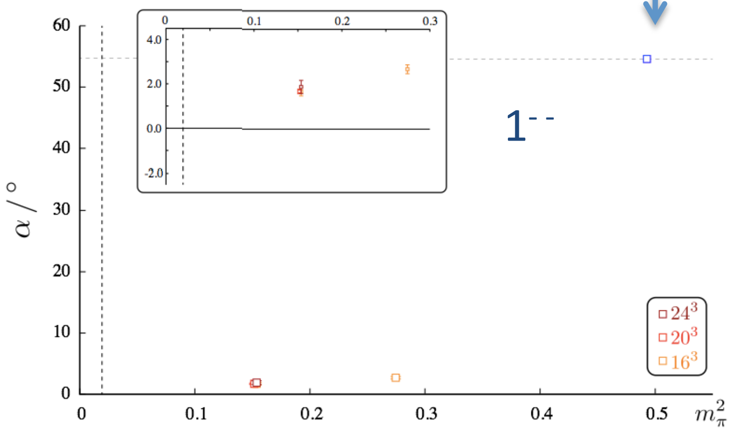
$$|s\rangle \equiv |s\bar{s}\rangle$$

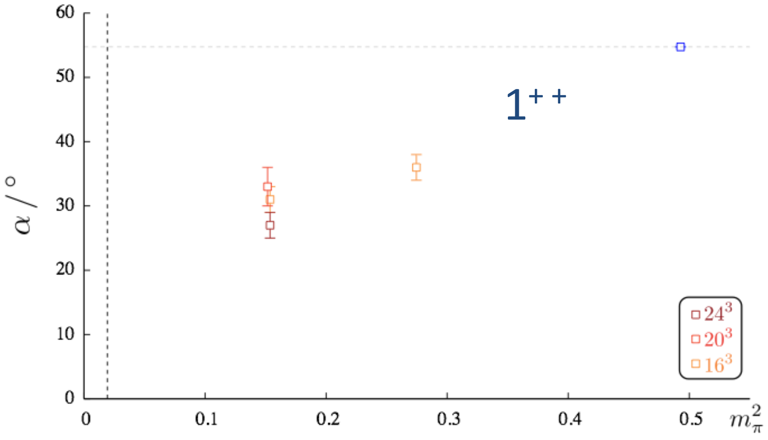
[HSC : Dudek, Edward, Guo, Thomas: 1309.2608, PRD]

Isoscalar mesons: mixing angle



SU(3)





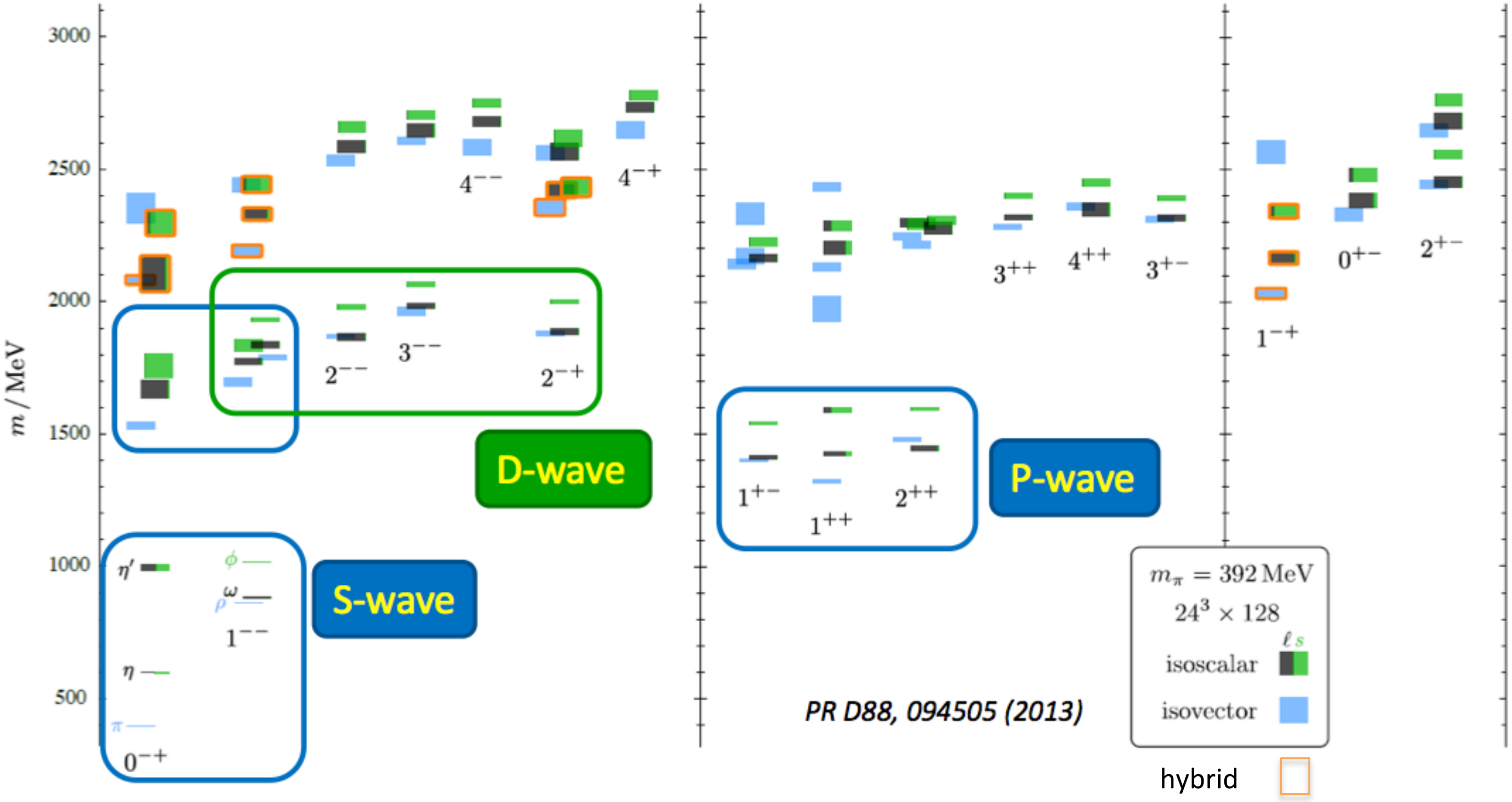
$$\begin{pmatrix} |\mathfrak{a}\rangle \\ |\mathfrak{b}\rangle \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} |\ell\rangle \\ |s\rangle \end{pmatrix}$$

$$|\ell\rangle \equiv \frac{1}{\sqrt{2}}\Big(|u\bar{u}\rangle + |d\bar{d}\rangle\Big)$$

$$|s\rangle \equiv |s\bar{s}\rangle$$

[HSC : Dudek, Edward, Guo, Thomas: 1309.2608, PRD]

Isoscalar mesons: multiplets from overlaps



S-wave

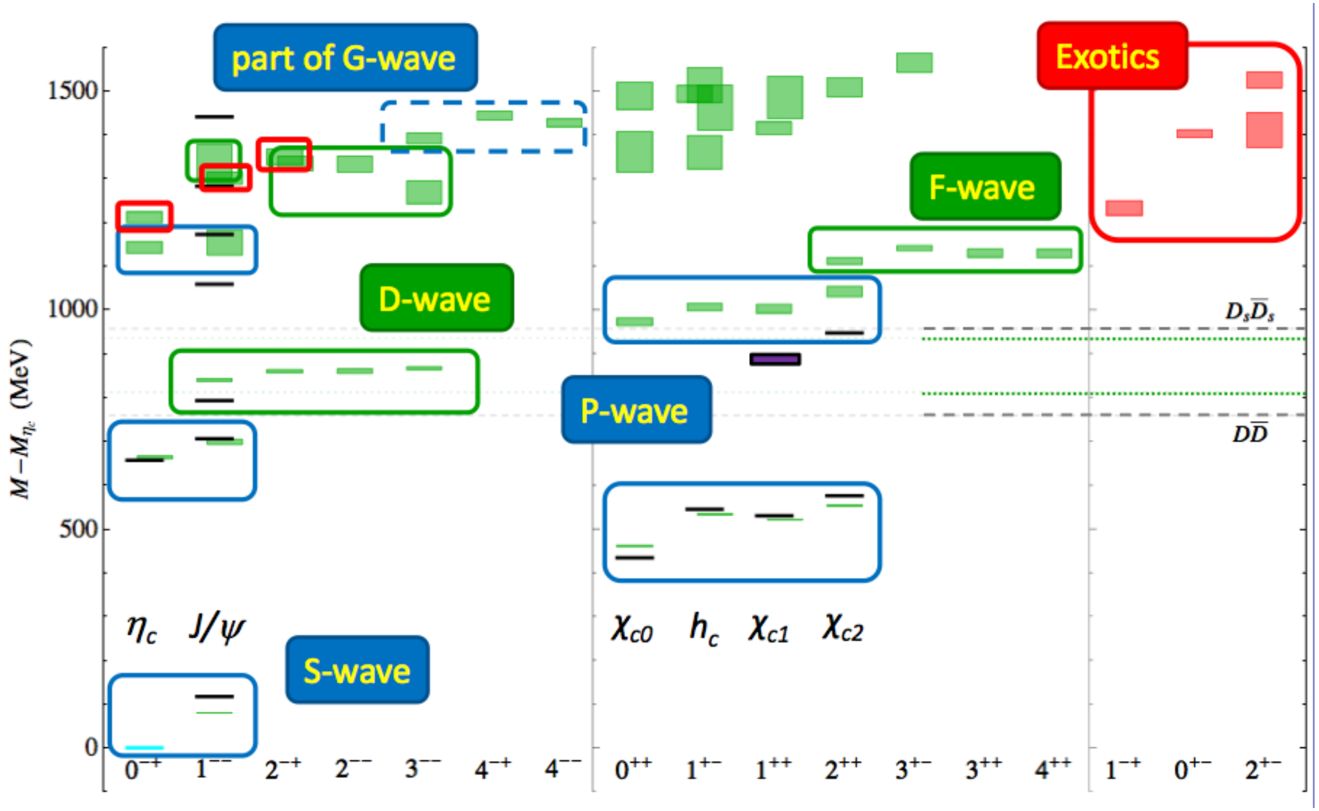
D-wave

P-wave

PR D88, 094505 (2013)

[HSC : Dudek, Edward, Guo, Thomas: 1309.2608, PRD]

$\underline{c}c$ spectrum: single hadron approximation



D and D_s mesons:

[G. Moir et al., HSC : 1301.7670, JHEP]

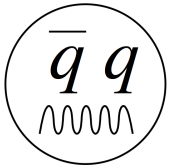
[HSC , L. Liu et al: 1204.5425, JHEP]

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, Nf=2+1
- reliable J^{PC} determination
- identification with $n\ {}^{2S+1}L_J$ multiplets using $\langle O|n\rangle$
- green: lat, black: exp

Hybrids:

some of them have exotic J^{PC}

large overlap with $O = \underline{q}\, F_{ij}\, q$



Beyond single hadron approximation

- most of the effort in this direction
- one can not expect plots with a number of multiplets soon

States near threshold

Most of interesting states are found near threshold in experiment !

Z_c^+, Z_b^+, X(3872), $D_{s0}^*(2317)$, $\Lambda(1405)$

Challenges for the lattice community: quarkonium-*like* states

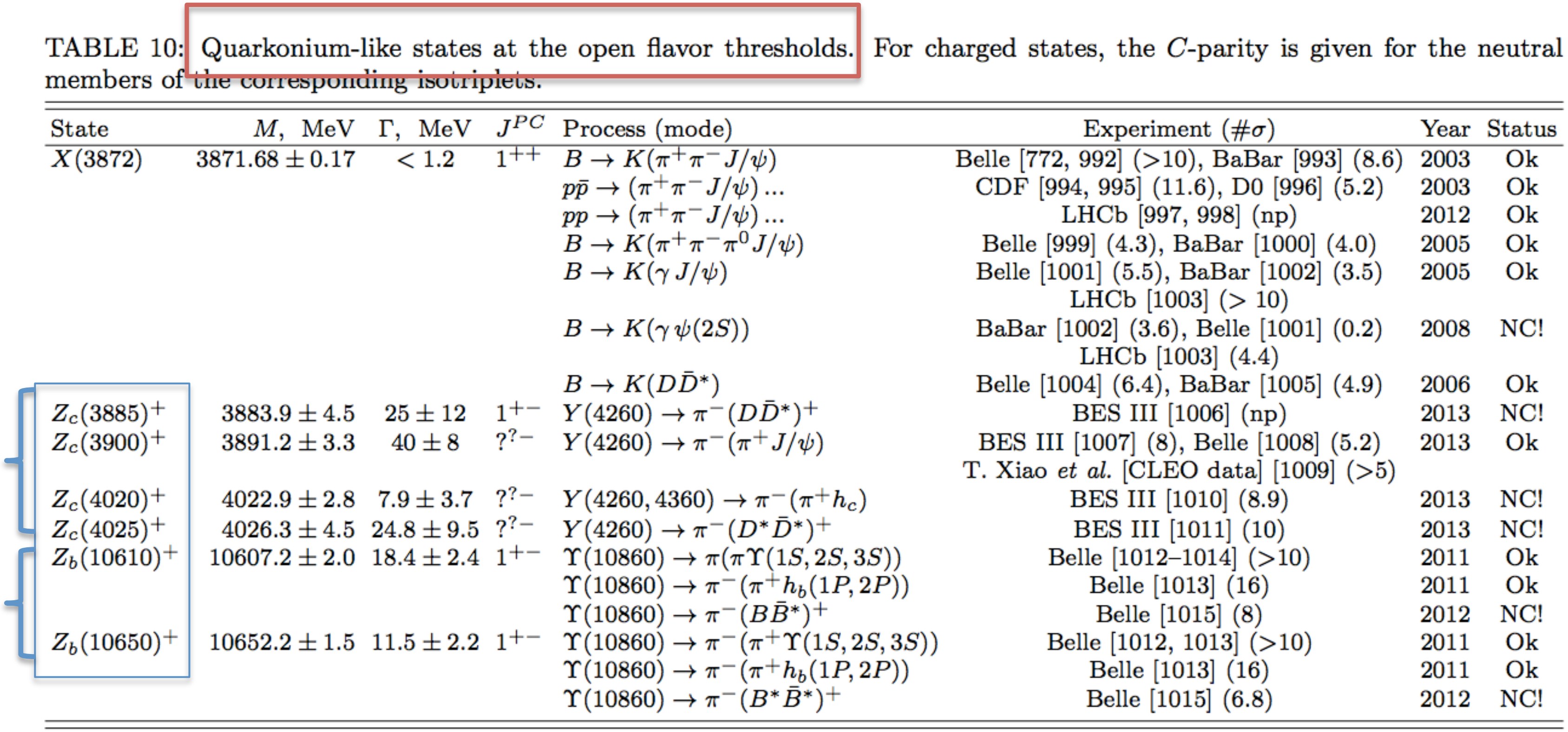
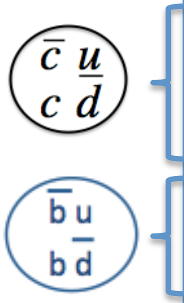
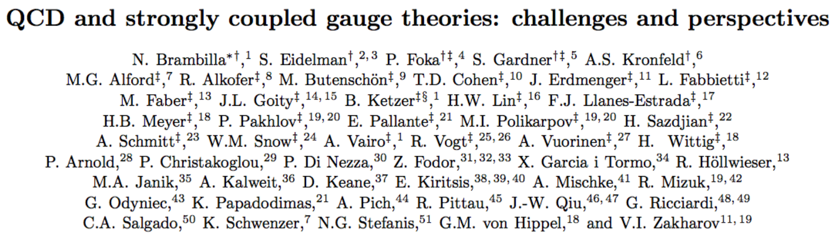


TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \to (\pi^+\pi^- J/\psi) \ldots$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \to (\pi^+\pi^- J/\psi) \ldots$	LHCb [997, 998] (np)	2012	Ok
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \to K(\gamma\, J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5) LHCb [1003] (> 10)	2005	Ok
				$B \to K(\gamma\, \psi(2S))$	BaBar [1002] (3.6), Belle [1001] (0.2) LHCb [1003] (4.4)	2008	NC!
				$B \to K(D\bar{D}^*)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
$Z_c(3885)^+$	3883.9 ± 4.5	25 ± 12	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
$Z_c(3900)^+$	3891.2 ± 3.3	40 ± 8	$?^{?-}$	$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III [1007] (8), Belle [1008] (5.2) T. Xiao *et al.* [CLEO data] [1009] (>5)	2013	Ok
$Z_c(4020)^+$	4022.9 ± 2.8	7.9 ± 3.7	$?^{?-}$	$Y(4260, 4360) \to \pi^-(\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	4026.3 ± 4.5	24.8 ± 9.5	$?^{?-}$	$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle [1012–1014] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!



[review: Brambilla et al., 1404.3723]



QCD and strongly coupled gauge theories: challenges and perspectives

N. Brambilla*†,1 S. Eidelman†,2,3 P. Foka†‡,4 S. Gardner†‡,5 A.S. Kronfeld†,6
M.G. Alford‡,7 R. Alkofer‡,8 M. Butenschön‡,9 T.D. Cohen‡,10 J. Erdmenger‡,11 L. Fabbietti‡,12
M. Faber‡,13 J.L. Goity‡,14,15 B. Ketzer‡§,1 H.W. Lin‡,16 F.J. Llanes-Estrada‡,17
H.B. Meyer‡,18 P. Pakhlov‡,19,20 E. Pallante‡,21 M.I. Polikarpov‡,19,20 H. Sazdjian‡,22
A. Schmitt‡,23 W.M. Snow‡,24 A. Vairo‡,1 R. Vogt‡,25,26 A. Vuorinen‡,27 H. Wittig‡,18
P. Arnold,28 P. Christakoglou,29 P. Di Nezza,30 Z. Fodor,31,32,33 X. Garcia i Tormo,34 R. Höllwieser,13
M.A. Janik,35 A. Kalweit,36 D. Keane,37 E. Kiritsis,38,39,40 A. Mischke,41 R. Mizuk,19,42
G. Odyniec,43 K. Papadodimas,21 A. Pich,44 R. Pittau,45 J.-W. Qiu,46,47 G. Ricciardi,48,49
C.A. Salgado,50 K. Schwenzer,7 N.G. Stefanis,51 G.M. von Hippel,18 and V.I. Zakharov11,19

More challenges: quarkonium-*like* states above threshold

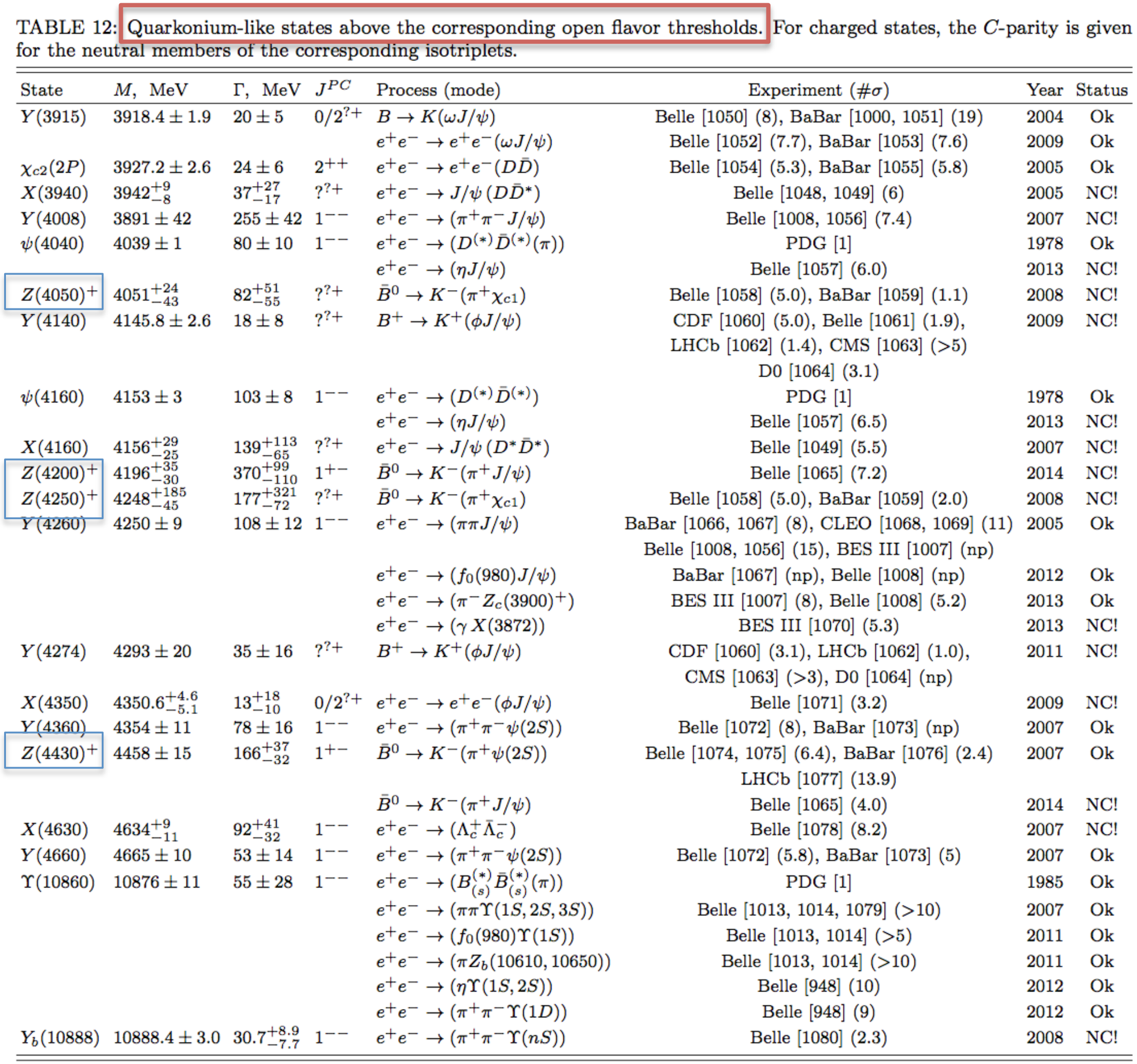


TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$Y(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle [1054] (5.3), BaBar [1055] (5.8)	2005	Ok
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle [1048, 1049] (6)	2005	NC!
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (5.0), Belle [1061] (1.9), LHCb [1062] (1.4), CMS [1063] (>5) D0 [1064] (3.1)	2009	NC!
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.5)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle [1049] (5.5)	2007	NC!
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (7.2)	2014	NC!
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11) Belle [1008, 1056] (15), BES III [1007] (np)	2005	Ok
				$e^+e^- \to (f_0(980) J/\psi)$	BaBar [1067] (np), Belle [1008] (np)	2012	Ok
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+e^- \to (\gamma\, X(3872))$	BES III [1070] (5.3)	2013	NC!
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0), CMS [1063] (>3), D0 [1064] (np)	2011	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [1071] (3.2)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4458 ± 15	166^{+37}_{-32}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4) LHCb [1077] (13.9)	2007	Ok
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (4.0)	2014	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$\Upsilon(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+e^- \to (B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \to (\pi\pi\Upsilon(1S, 2S, 3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \to (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \to (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \to (\eta\Upsilon(1S, 2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

All these believed NOT to be $Q\bar{Q}$!

[review: Brambilla et al., 1404.3723]

Charged charmonium-like $\mathbf{Z_c^+}$

(manifestly exotic)

Charged charmonium Z_c^+ : experimental status

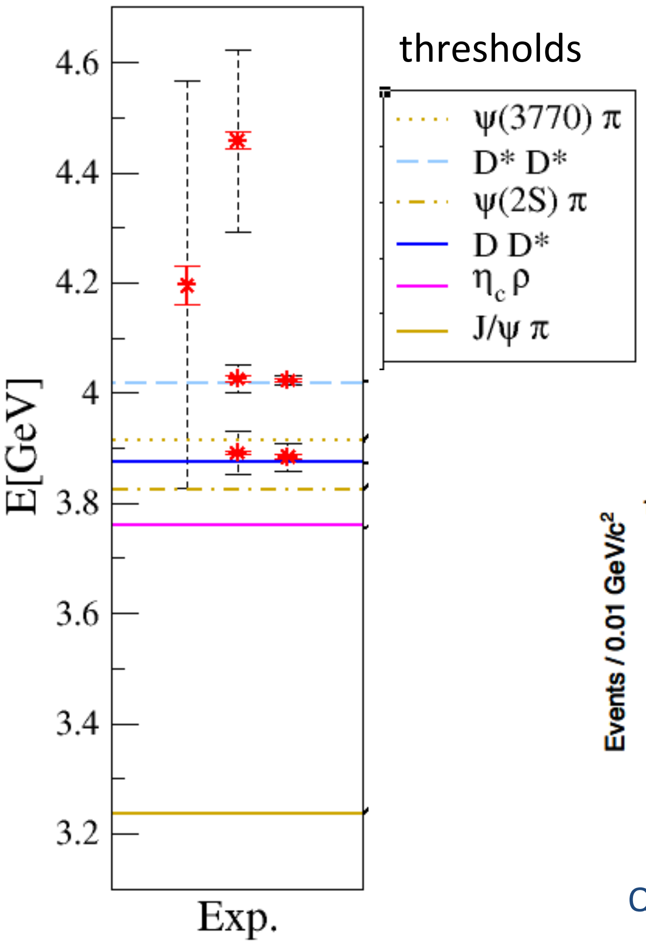
$\bar{c}\ u$
$c\ \bar{d}$

[review: Brambilla et al., 1404.3723]

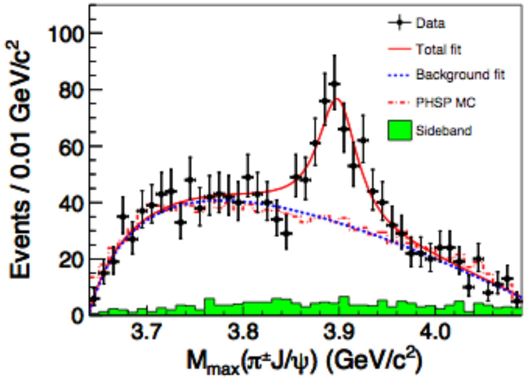
candidates with preferred

$I^G=1^+$, $J^{PC}=1^{+-}$

(C is for neutral partner)



particle	C	J^P	decay	year	coll
$Z^+(4430)$	-	1+	$\psi(2S)\ \pi^+$	2008	Belle, ~~BABAR~~, LHCb
$Z_c^+(3900)$	-	?	$J/\psi\ \ \pi^+$	2013	BESIII, Belle, CLEOc
$Z_c^+(3885)$	-	1+	$(DD^*)^+$	2013	BESIII
$Z_c^+(4020)$	-	?	$h_c(1P)\ \pi^+$	2013	BESIII
$Z_c^+(4025)$	-	?	$(D^*\ D^*)^+$	2013	BES III
$Z^+(4200)$	-	1+	$J/\psi\ \ \pi^+$	2014	Belle
$Z^+(4050)$	+	?	$\chi_{c1}\ \ \pi^+$	2008	Belle
$Z^+(4250)$	+	?	$\chi_{c1}\ \ \pi^+$	2008	Belle



[BESIII, 2013, 1303.5949, PRL]

$Z_c^+(3900)$ → $J/\Psi\ \ \pi^+$

$\underline{c}c\ \ \underline{d}u$

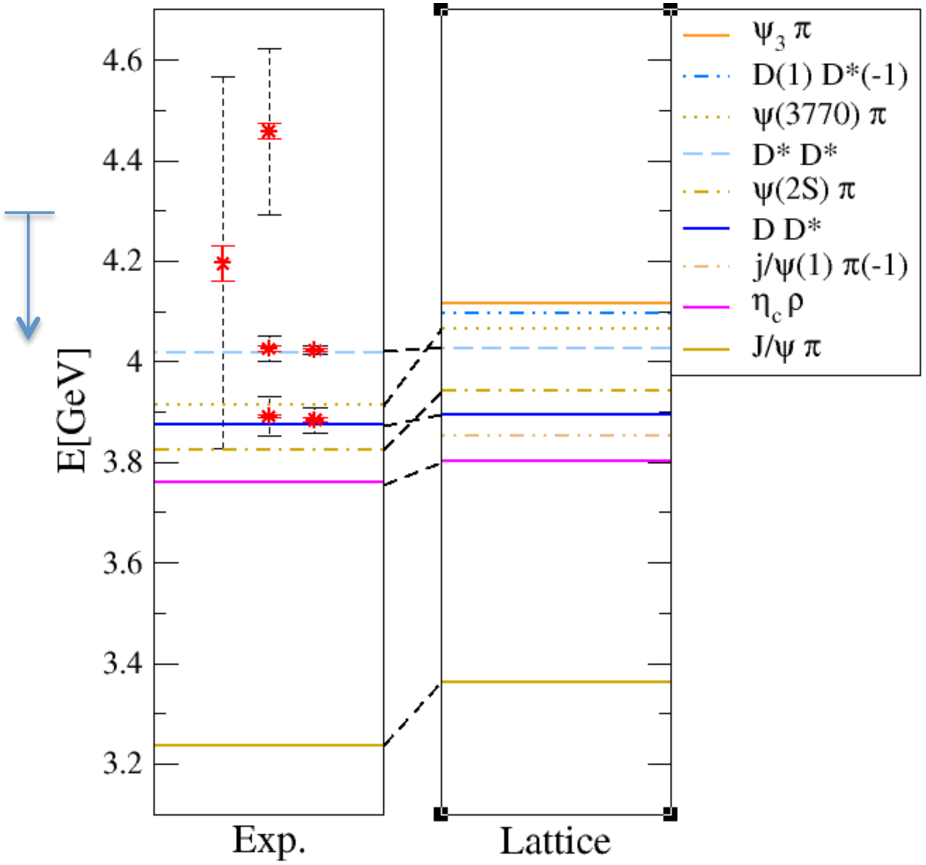
$\bar{c}\ u$
$c\ \bar{d}$

Question for our community:

Does QCD support existence of such states?

Towards evidence for Z_c^+ from lattice: $I^G=1^+$, $J^{PC}=1^{+-}$

$\bar{c}\ u$
$c\ \bar{d}$



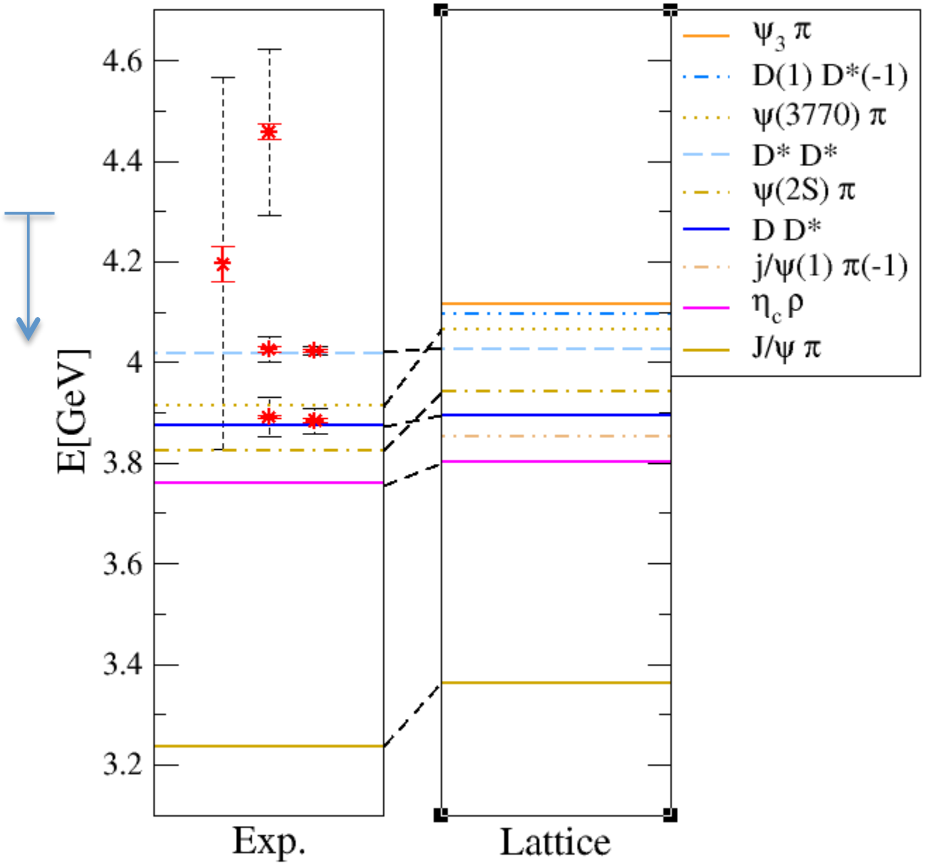
- search for Z_c^+ with E < 4.3 GeV
- horizontal lines correspond to energies of all two-particle states with E < 4.3 GeV on this lattice
  - There would be many more two-particle states for larger L !
- 9 two-particle states are expected

[S.P., Lang, Leskovec, Mohler, 1405.7623]

Wilson Clover, $m_\pi \approx 266$ MeV, L≈2 fm, $N_f=2$

Towards evidence for Z_c^+ from lattice: $I^G=1^+$, $J^{PC}=1^{+-}$

$$\begin{matrix} \bar c & u \\ c & \bar d \end{matrix}$$



[S.P., Lang, Leskovec, Mohler, 1405.7623]

Wilson Clover, $m_\pi \approx 266$ MeV, $L \approx 2$ fm, Nf=2

Meson-meson interpolators:

$$\mathcal{O}_1^{\psi(0)\pi(0)} = \bar c\gamma_i c(0)\ \bar d\gamma_5 u(0)\,,$$

$$\mathcal{O}^{\psi(1)\pi(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar c\gamma_i c(e_k)\ \bar d\gamma_5 u(-e_k)\,,$$

$$\mathcal{O}^{\eta_c(0)\rho(0)} = \bar c\gamma_5 c(0)\ \bar d\gamma_i u(0)\,,$$

$$\mathcal{O}_1^{D(0)D^*(0)} = \bar c\gamma_5 u(0)\ \bar d\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,$$

$$\mathcal{O}^{D^*(0)D^*(0)} = \epsilon_{ijk}\ \bar c\gamma_j u(0)\ \bar d\gamma_k c(0)\,,$$

and 9 others ..

Diquark antidiquark interpolators

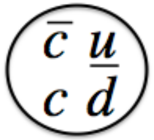
(expected to couple particularly well to exotic state but couple also to two-meson st.):

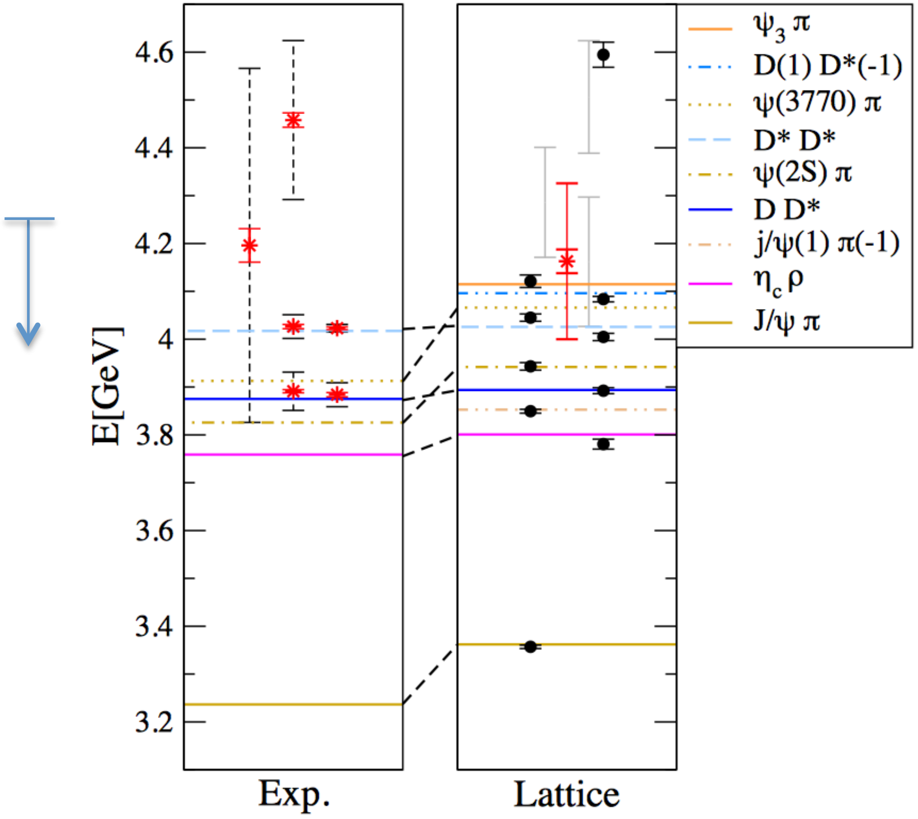
$$O_1^{4q} \approx [\bar c\ C\gamma_5\ \bar d]_{3_c}[c\ \gamma_i C\ u]_{\bar 3_c}$$

$$O_2^{4q} \approx [\bar c\ C\ \bar d]_{3_c}[c\ \gamma_i\gamma_5 C\ u]_{\bar 3_c}$$

and 2 others ..

Evidence for Z_c^+ from lattice: $I^G=1^+$, $J^{PC}=1^{+-}$





- Black circles: two-meson states
- Red asterix: candidate for Z_c^+

  (the smaller error is statistical,

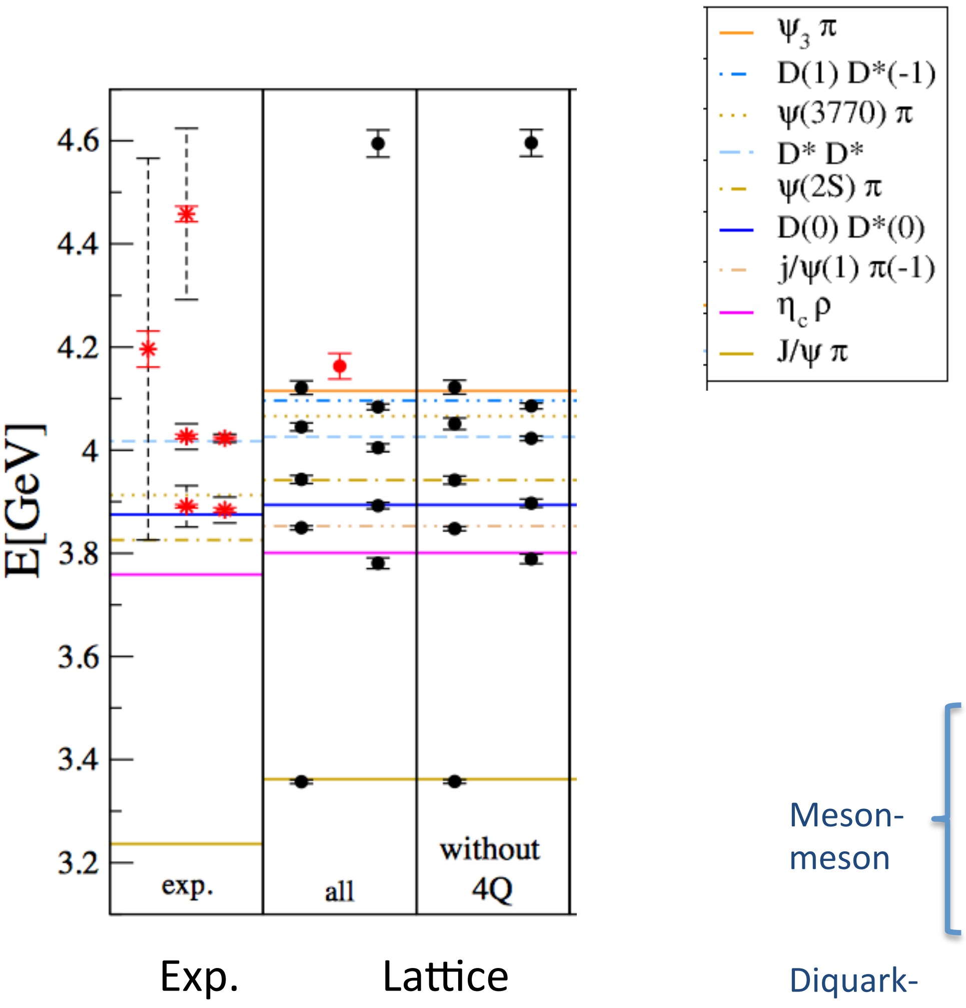
  the larger corresponds to systematics)

- 9 two meson states below 4.3 GeV
- an additional state found
- since we exhausted all two meson-states below 4.3 GeV, it is a candidate for an exotic Z_c^+.

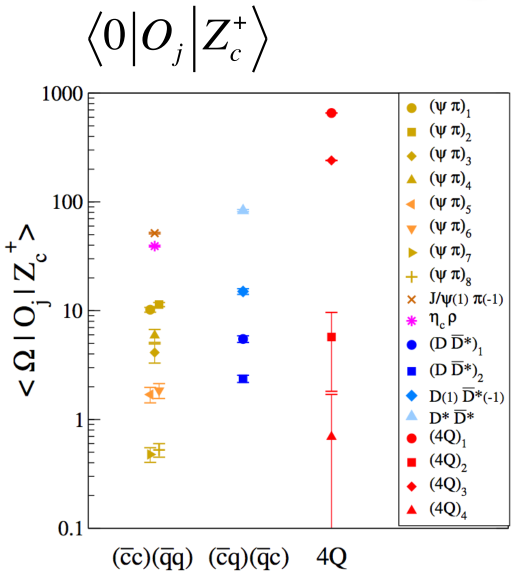
[S.P., Lang, Leskovec, Mohler, 1405.7623]
$m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

Additional supporting evidence for Z_c^+

$$\begin{matrix}\bar c & u \\ c & \bar d\end{matrix}$$



[S.P., Lang, Leskovec, Mohler, 1405.7623]



Meson-meson

$$\mathcal{O}_1^{\psi(0)\pi(0)} = \bar c\gamma_i c(0)\;\bar d\gamma_5 u(0)\,,$$

$$\mathcal{O}^{\psi(1)\pi(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar c\gamma_i c(e_k)\;\bar d\gamma_5 u(-e_k)\,,$$

$$\mathcal{O}^{\eta_c(0)\rho(0)} = \bar c\gamma_5 c(0)\;\bar d\gamma_i u(0)\,,$$

$$\mathcal{O}_1^{D(0)D^*(0)} = \bar c\gamma_5 u(0)\;\bar d\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,$$

$$\mathcal{O}^{D^*(0)D^*(0)} = \epsilon_{ijk}\;\bar c\gamma_j u(0)\;\bar d\gamma_k c(0)\,,$$

and 9 others ..

Diquark-antidiquark

4Q

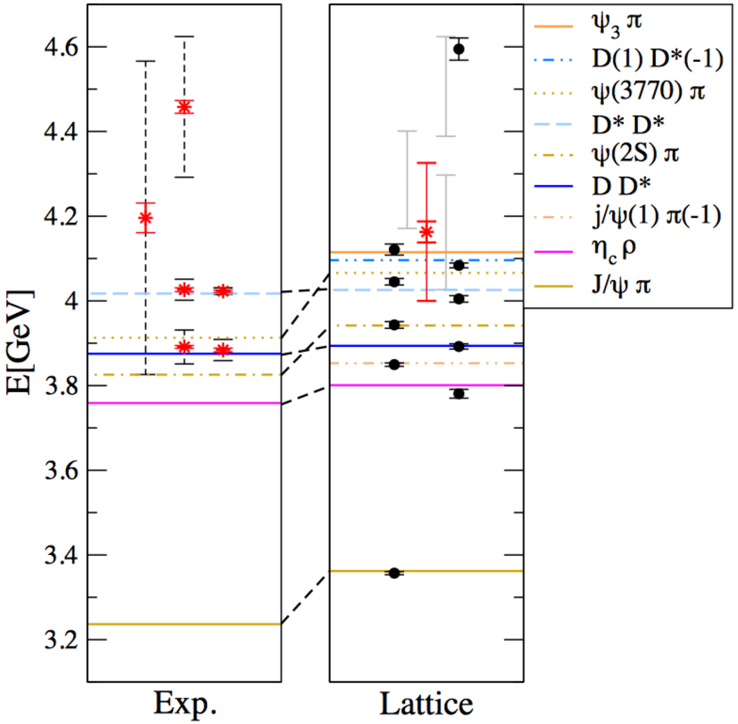
$$O_1^{4q} \approx [\bar c\; C\gamma_5\; \bar d]_{3_c}\,[c\;\gamma_i C\; u]_{\bar 3_c}$$

$$O_2^{4q} \approx [\bar c\; C\; \bar d]_{3_c}\,[c\;\gamma_i\gamma_5 C\; u]_{\bar 3_c}$$

and 2 others ..

Comparing to experimental Z_c^+ candidates

$$\begin{matrix} \bar{c} & u \\ c & \bar{d} \end{matrix}$$



Luka Leskovec, Friday 15h15

Hadron Spectroscopy

- Challenge: this problem would have to be ideally treated as 6-coupled channels, but rigorous Luscher-type treatment is not realistic in the near future ☹
- Smart ideas for improvement along these welcome!

Nearby experimental candidates:

Z_c^+(4020), Γ=7.9 ± 3.7 MeV BESIII 2013

Z_c^+(4025), Γ=24.8 ± 9.5 MeV BESIII 2013

Z_c^+(4200), Γ=370 ± 110 MeV Belle, Moriond 2014

Lattice (m_π =266 MeV, Nf=2) :

$m(Z_c^+) = 4.16$ GeV

± 0.163 GeV ± O(Γ)

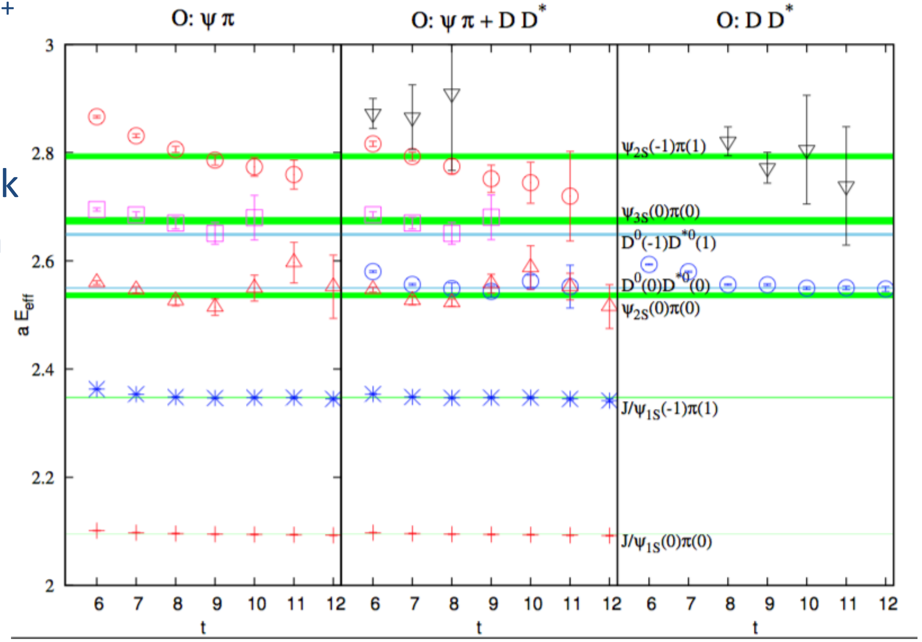
[S.P., Lang, Leskovec, Mohler, 1405.7623]

Other searches with no Z_c^+ candidate (yet)

(1) Search for $Z_c^+(4430)$ in $D^*\underline{D}_1$ scattering near threshold
3 quenched asymmetric lattices
phase shift extracted with help of asymmetric boxes
[G.Z. Meng et al, CLQCD, 0905.0752, PRD 2009]

(2) First search for $Z_c^+(3900)$
$\psi\ \pi$ and $D\ \underline{D}^*$ interpolators, no diquark antidiquark
$m_\pi \approx 266$ MeV, $L \approx 2$ fm, $N_f=2$
only two-meson states found, no candidate for Z_c^+
[S.P. & L. Leskovec, 1308.2097, Phys. Lett. B]

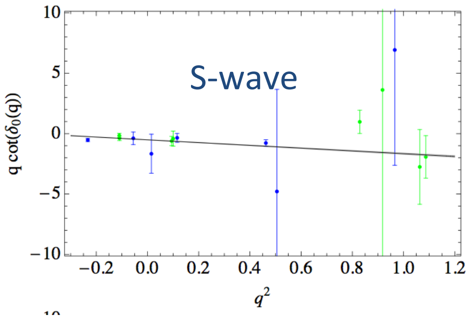
(3) $\Psi\ \pi$ and $D\ \underline{D}^*$ interpolators, no diquark antidiquark
HISQ quarks , $m_u = m_d = m_s/5$, $16^3 \times 48$, $a = 0.15$ fm
only two-meson states, no candidate for Z_c^+
[C. DeTar, Song-haeng Lee,
private communication]
C. DeTar, Poster Session

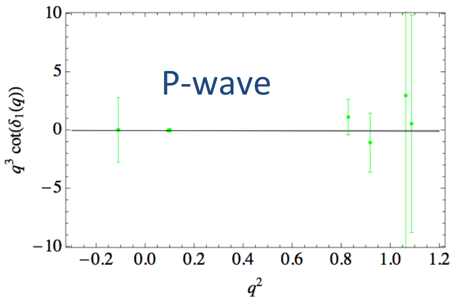


Other searches with no Z_c^+ candidate (yet)

(4) Search for resonance in $D\underline{D}^*$ scattering near threshold E ~ 3.9 GeV
- just D D* interpolators, no ψ π interpolators
- twisted mass quarks, m_π = 300, 420, 485 MeV, 32^3 x 64
- partially twisted BC for u,d (not for c) and take care about s-p mixing when present
- the authors conclude that no Z_c^+ candidate is found near DD* threshold

[Y. Chen et al, 1403.1318, CLQCD coll, Phys. Rev. D] L. Liu: Parallel, Hadron Spectrum, Friday, 14h55





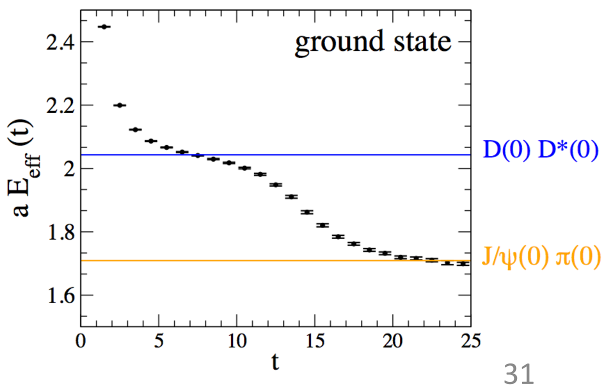
$$p\cot\delta(p)=\frac{1}{a_0}+\frac{1}{2}r_0p^2$$

	$\mu=0.003$	$\mu=0.006$	$\mu=0.008$
a_0[fm]	-0.67(1)	-2.1(1)	-0.51(7)
r_0[fm]	-0.78(3)	-0.27(7)	0.82(27)

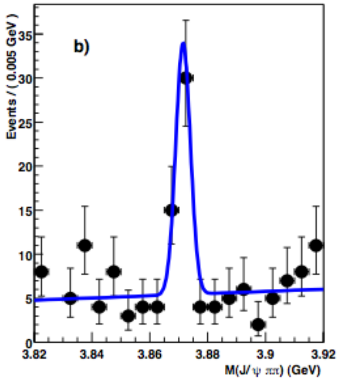
m_π = 300 MeV

- Cautionary remark and lesson based on experience from [S.P., Lang, Leskovec, Mohler, 1405.7623]
  - conclusions based on D D* interpolators may not be reliable
  - m_{eff} is dropping down to the true ground state ψ π
  - ψ π interpolators (and probably some others) needed

$\mathcal{O}$: DD^*, D^*D^*



X(3872) , $J^{PC}=1^{++}$, charmonium-like



- First charmonium-like state discovered [Belle, PRL, 2003]
- sits within 1 MeV of $D^0\underline{D}^{0*}$ threshold
  8 MeV below D^+D^{*-} threshold

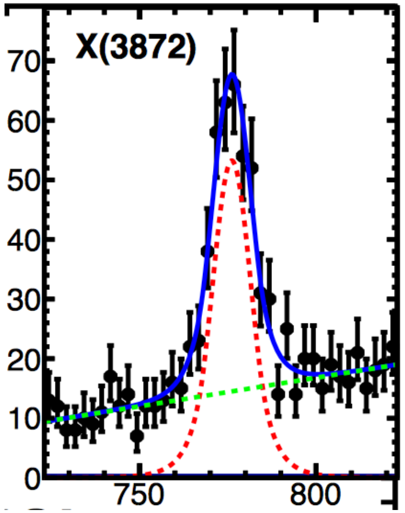
isospin breaking effects my be important

- believed to have a large molecular $D^0\underline{D}^{0*}$ Fock component
- $\Gamma < 1.2$ MeV
- decays to I=0, 1 equally important

X(3872) → J/Ψ ω (I=0)

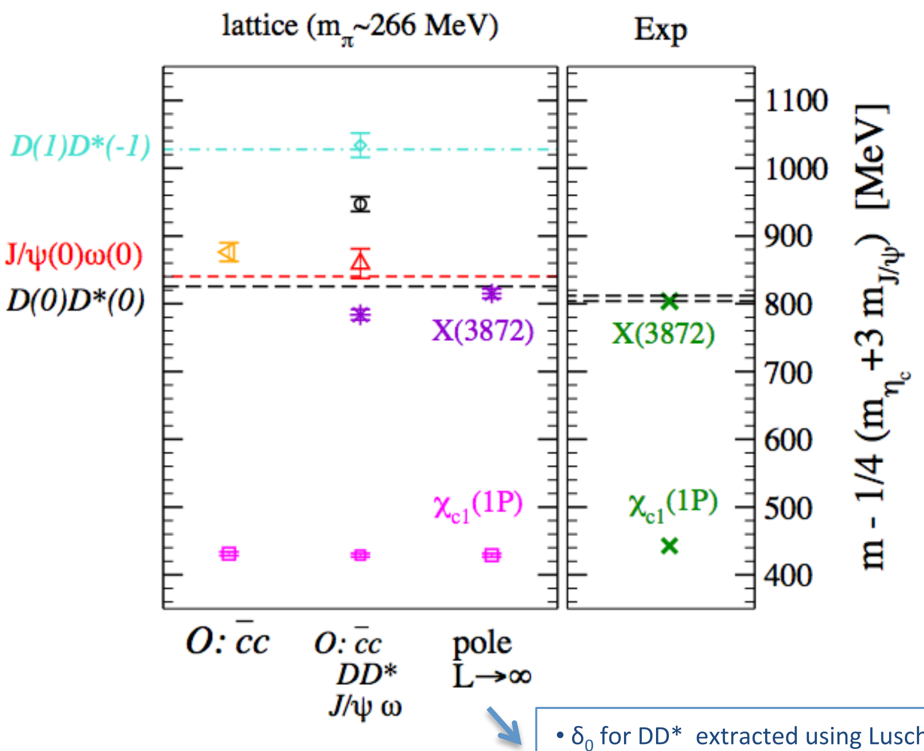
X(3872) → J/Ψ ρ (I=1)

[LHCb, PRL 2013]



Evidence for X(3872) : $J^{PC}=1^{++}$, I=0

$$\mathcal{O}:\ \bar{c}\,c,\quad DD^*,\quad J/\psi\,\omega$$

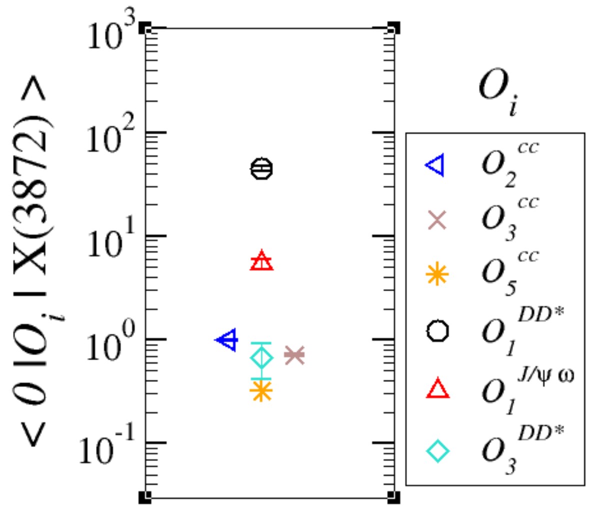


- δ_0 for DD* extracted using Luscher's rel. and interpolated near threshold
- pole in T-matrix $T\propto[\cot\delta - i]^{-1}=\infty$ found just below DD* threshold.

[S.P. and L. Leskovec : 1307.5172, Phys. Rev. Lett.]

$m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

$$\langle 0|O_j|X(3872)\rangle$$



Overlaps normalized to $\langle 0|O_1^{cc}|X(3872)\rangle$

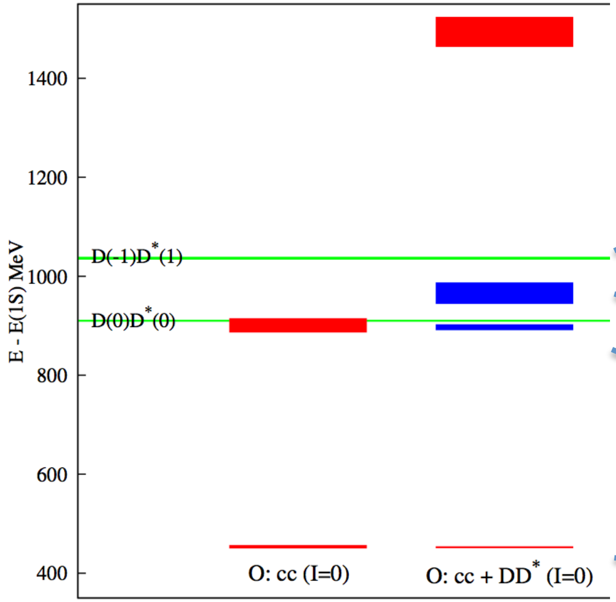
X(3872)	m - $(m_{D0}+m_{D0^*})$
lat	- 11 ± 7 MeV
exp	- 0.14 ± 0.22 MeV

New evidence for X(3872) : $J^{PC}=1^{++}$, I=0

$\mathcal{O}$: $\bar{c}\, c$, DD^*

HISQ quarks , $m_u = m_d = m_s/5$, $16^3 \times 48$, a =0.15 fm

[C. DeTar, Song-haeng Lee] C. DeTar, Poster Session



Interpretation

They argue D(1)D*(-1) state is not found although O incorporated

D(0)D*(0)

X(3872)

χ_{c1}

X(3872)	m - $(m_{D0}+m_{D0^*})$
lat	- 13 ± 6 MeV
exp	- 0.14 ± 0.22 MeV

Possible direction to improve on X(3872):

- larger volumes since molecule may be of considerable size
- isospin breaking on the lattice

remember: sits 1 MeV of $D^0\underline{D}^{0*}$ threshold and 8 MeV below D^+D^{*-} threshold, decays to I=0,1

Related analytical studies

- *Light quark mass dependence of the X(3872) in XEFT*
  [M. Jansen, H.-W. Hammer, Yu Jia , 1310.6937, Phys. Rev. D]

- *Strategies for an accurate determination of the X(3872) energy from QCD lattice simulations*
  [E. J. Garzon, R. Molina, A. Hosaka, E. Oset, 1310.0972, Phys. Rev. D]

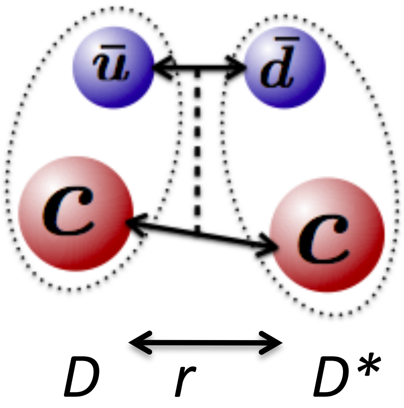
- *Hidden charm molecules in a Finite Volume*
  [M. Albaladejo, C. Hidalgo-Duque, J. Nieves, E. Oset, 1312.5339]

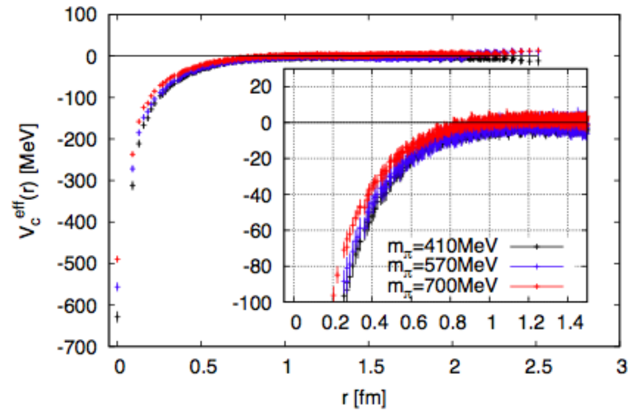
Searches for double charm tetraquark with $J^P=1^+$, I=0

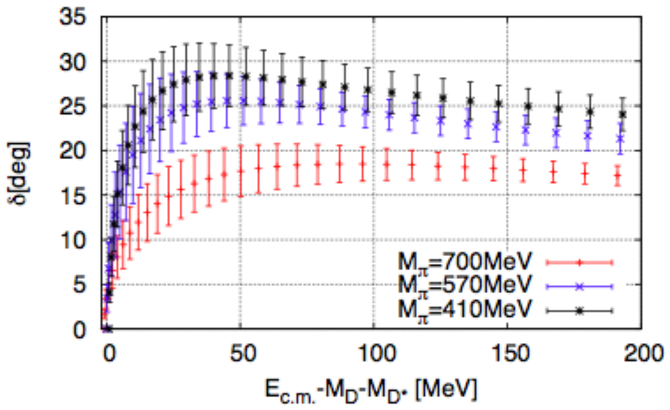
(1) HALQCD method [Ishii et al., PLB712, 437 (2012)]

- potential between D and D* , and corresponding phase shift
- $m_\pi \approx 410-700$ MeV, $L \approx 2.9$ fm, Nf=2+1
- potential is attractive, no bound tetraquark state found

[Y. Ikeda, HALQCD coll, , 1311.6214, Phys. Lett. B 2014]







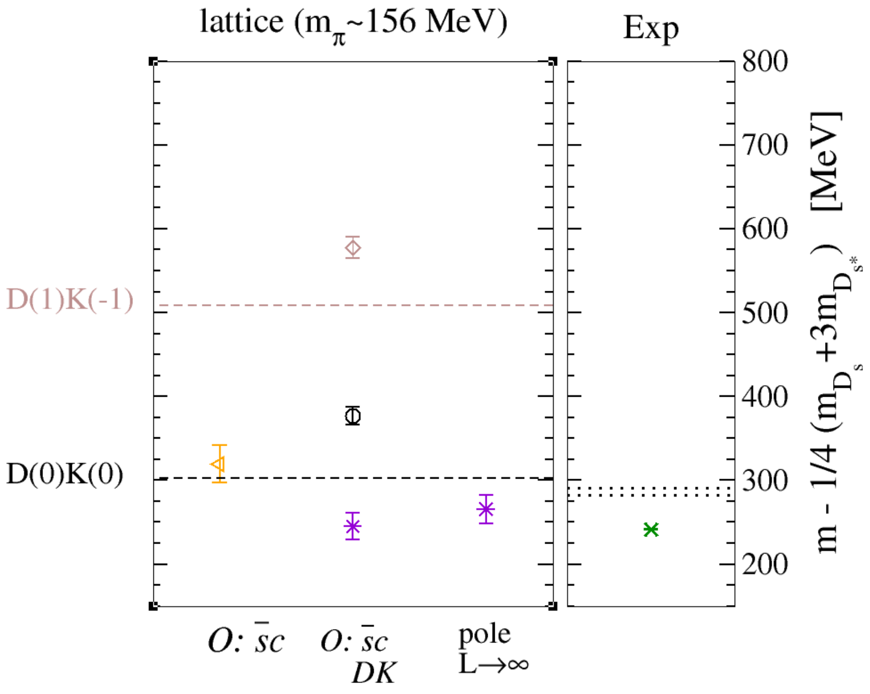
(2) variational method with DD*, D*D* and tetraquark interpolators

preliminary results do not lead yet to the conclusion on existence of these states

Andrea Guerrieri, Wednesday 12h10, Hadron Spectrum

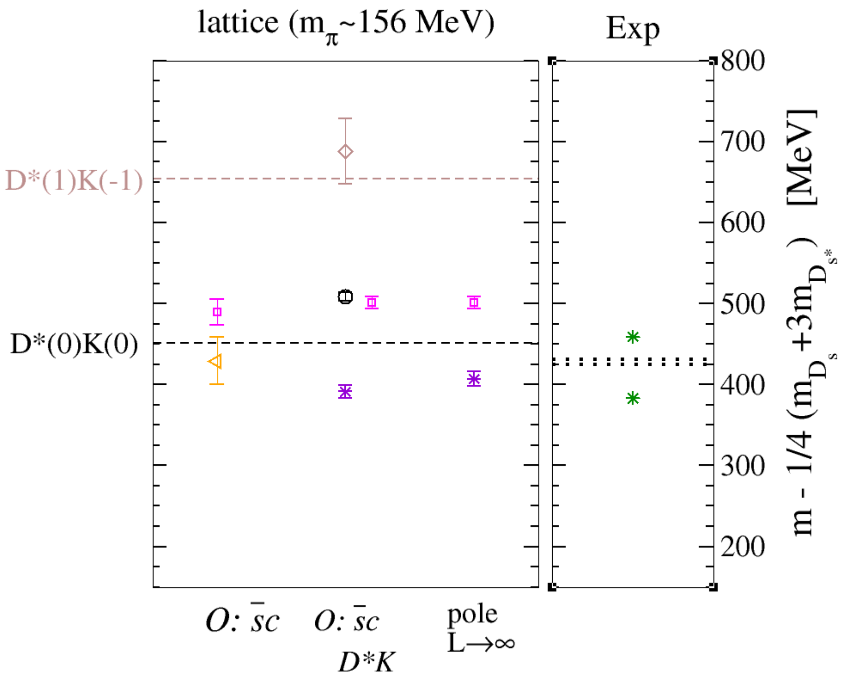
D_s states near DK and D*K thresholds

DK in s-wave and $D_{s0}^*(2317)$ bound state



[D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn, 1308.3175, Phys. Rev. Lett.]

D*K s-wave and $D_{s1}(2460)$, $D_{s1}(2536)$



[C. Lang, L. Leskovec, D. Mohler, S.P. , R. Woloshyn, 1403.8103]

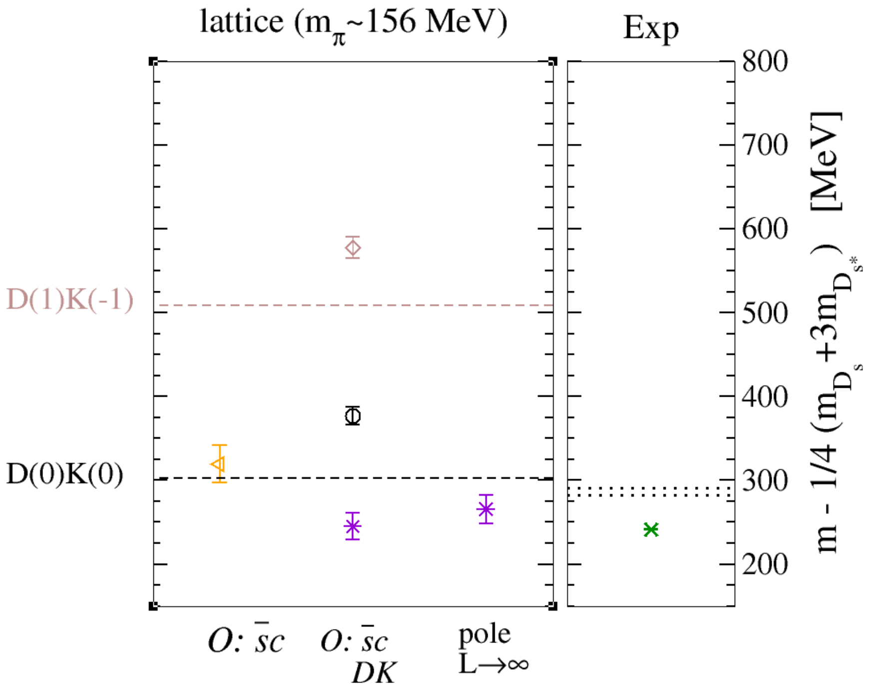
Wilson Clover, m_π≈156 MeV, L≈2.9 fm, Nf=2+1

$a_0 = -1.33 \pm 0.20$ fm
$r_0 = 0.27 \pm 0.17$ fm

- phase shift for DK or D*K $\tan\delta(p) = \frac{\sqrt{\pi}\, p\, L}{2\, \mathcal{Z}_{00}\left(1; (\frac{pL}{2\pi})^2\right)}$
- parametrization of phase shift near th. $p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$
- poles in S-matrix correspond do bound states $T \propto \frac{1}{\cot\delta - i} = \infty$

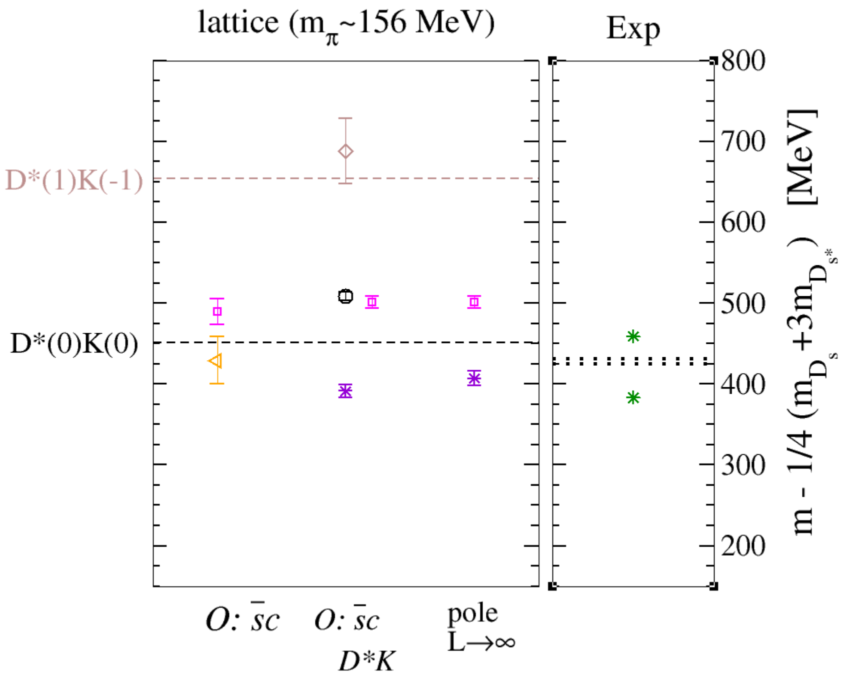
$a_0 = -1.11 \pm 0.11$ fm
$r_0 = 0.10 \pm 0.10$ fm

DK s-wave and D_{s0}^*(2317) bound state



[D. Mohler, C. Lang, L. Leskovec, S.P. ,
R. Woloshyn, 1308.3175, Phys. Rev. Lett.]

D*K s-wave and D_{s1}(2460), D_{s1}(2536)

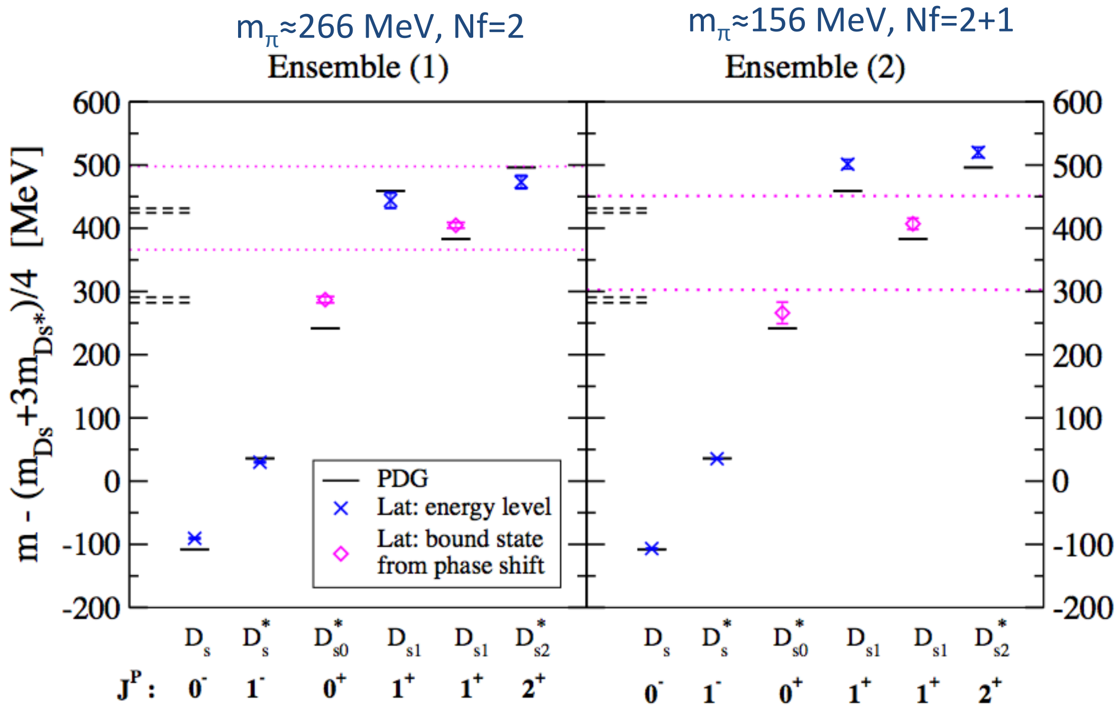


for mc=∞ : D_{s1}(2536) does not couple to s-wave
[Isgur Wise 1991]

[C. Lang, L. Leskovec, D. Mohler, S.P. ,
R. Woloshyn, 1403.8103]

m_π≈156 MeV, L≈2.9 fm, Nf=2+1

D_s and D spectrum



D_s mesons (near-threshold)

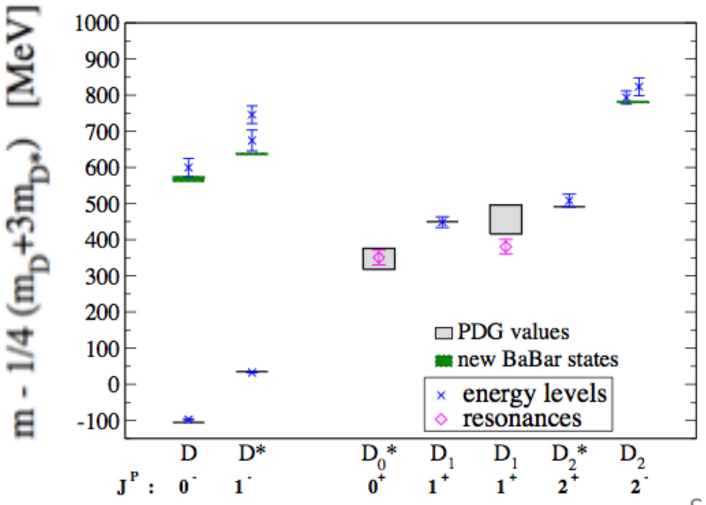
[C. Lang, L. Leskovec, D. Mohler, S.P. , R. Woloshyn: PRL 2013, 1403.8103]

these results

C. B. Lang, Monday, 17h50

preliminary results for DK and Dπ

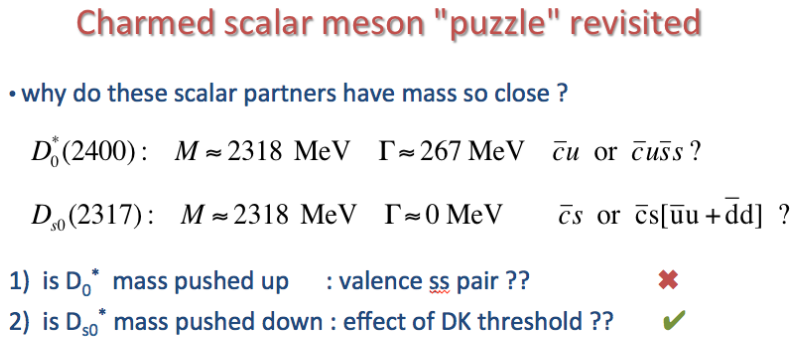
S. Ryan, Thursday 16h15



D mesons

(resonances)

[D. Mohler, S.P. , R. Woloshyn: PRD 2013]



Charmed scalar meson "puzzle" revisited

- why do these scalar partners have mass so close ?

$D_0^*(2400): \quad M \approx 2318\ \text{MeV} \quad \Gamma \approx 267\ \text{MeV} \quad \bar{c}u$ or $\bar{c}u\bar{s}s$?

$D_{s0}(2317): \quad M \approx 2318\ \text{MeV} \quad \Gamma \approx 0\ \text{MeV} \quad \bar{c}s$ or $\bar{c}s[\bar{u}u+\bar{d}d]$?

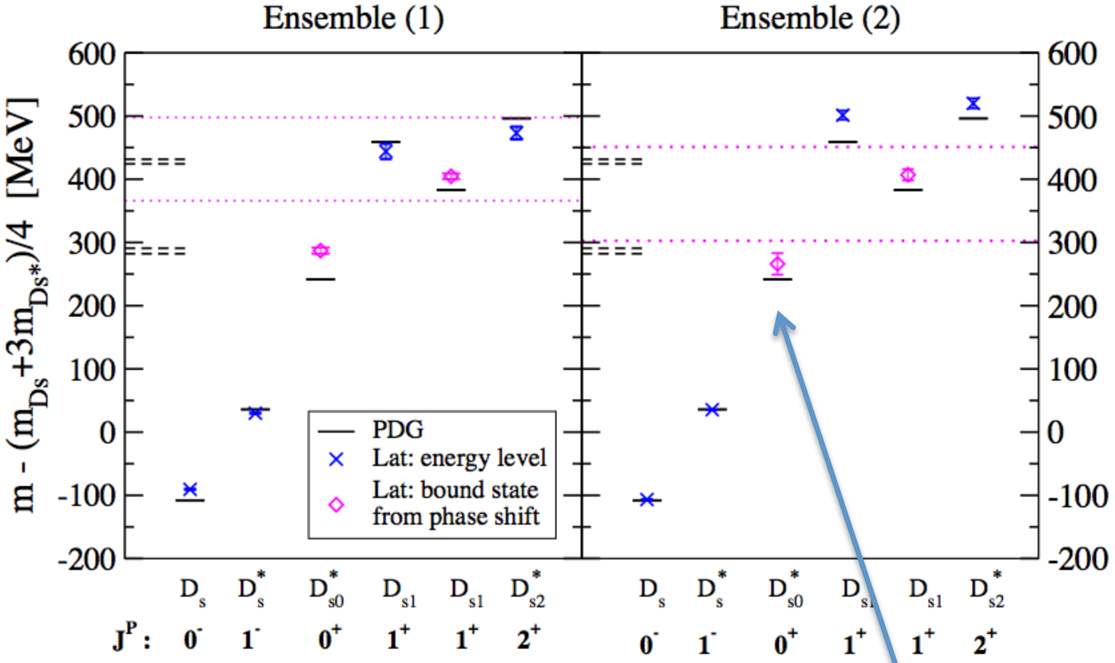
1) is D_0^* mass pushed up : valence ss pair ?? ✖

2) is D_{s0}^* mass pushed down : effect of DK threshold ?? ✔

D_s and D spectrum

m_π≈266 MeV, Nf=2

m_π≈156 MeV, Nf=2+1



D_s mesons (near-threshold)

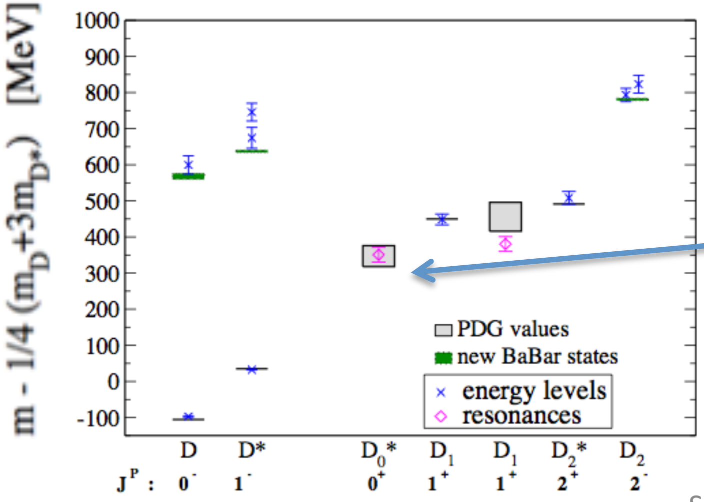
[C. Lang, L. Leskovec, D. Mohler, S.P. , R. Woloshyn: PRL 2013, 1403.8103]

these results

C. B. Lang, Monday, 17h50

preliminary results for DK and Dπ

S. Ryan, Thursday 16h15



D mesons

(resonances)

[D. Mohler, S.P. ,

R. Woloshyn:

PRD 2013]

Charmed scalar meson "puzzle" revisited

- why do these scalar partners have mass so close ?

$D_0^*(2400): \quad M \approx 2318\ \mathrm{MeV} \quad \Gamma \approx 267\ \mathrm{MeV} \quad \bar{c}u \ \text{or} \ \bar{c}u\bar{s}s\ ?$

$D_{s0}(2317): \quad M \approx 2318\ \mathrm{MeV} \quad \Gamma \approx 0\ \mathrm{MeV} \quad \bar{c}s \ \text{or} \ \bar{c}s[\bar{u}u+\bar{d}d]\ ?$

1) is D_0^* mass pushed up : valence ss pair ?? ✖

2) is D_{s0}^* mass pushed down : effect of DK threshold ?? ✔

Composition of Λ(1405), $J^P=½^-$

Derek Leinweber, Wednesday, Hadron Spectroscopy

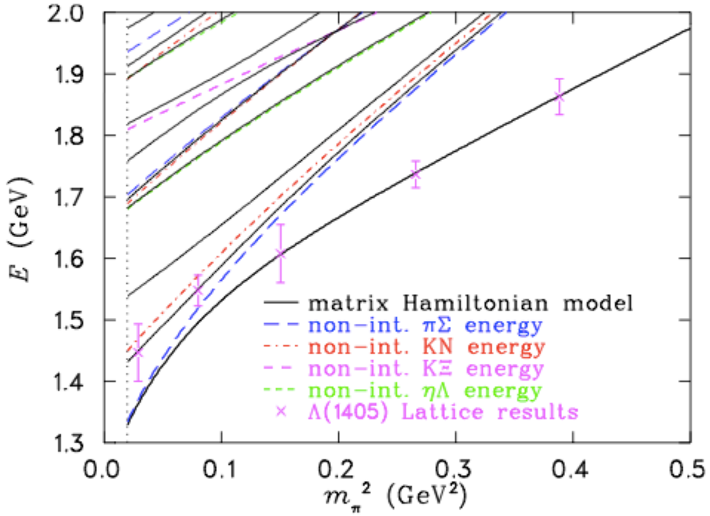
- exp: resonance in πΣ located below KN th.
- ground state energy determined from lat. using

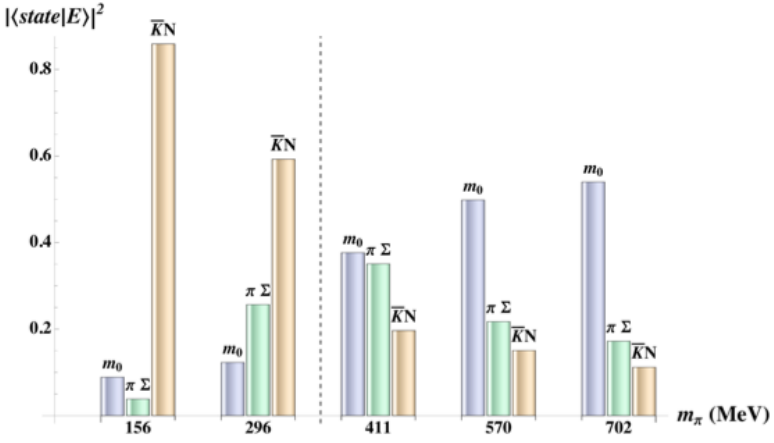
  O = uds and represented by pink crosses
- ground state E fitted with the eigenvalue of finite volume Hamiltonian EFT and parameters extracted [Hall et al, 1303,.4157, PRD]
- Hamiltonian EFT describes interactions between

  uds, πΣ, $\underline{K}$N, KΞ, ηΛ
- composition of eigenstate is extracted from EFT
- authors conclude that Λ(1405) is dominated by $\underline{K}$N at the physical quark mass
- PACS-CS conf, m_π=150-700 MeV, L=2.9 fm

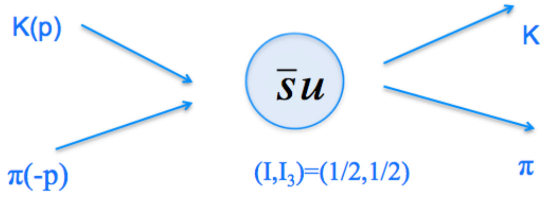
Effect of including Nπ interpolators for other channels
Waseem Kamleh, Wednesday, Hadron Sepctrum

Valentina Verduci, Poster Session





Mesons above threshold – resonances



(generalized) Luscher formulae

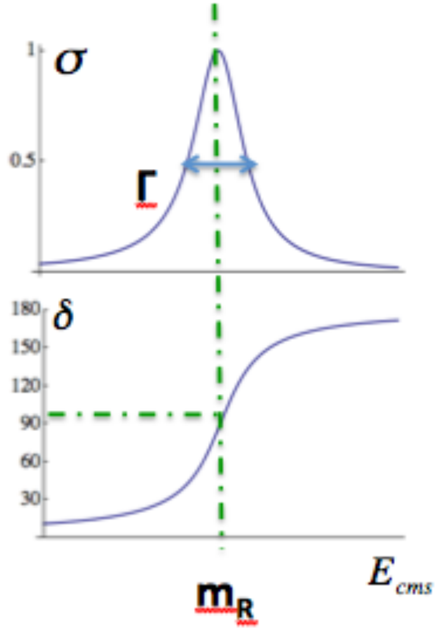
Breit Wigner fit

$$E_n(L) \;\rightarrow\; \delta \;\rightarrow\; m\,,\Gamma \ \text{or coupling}$$

Spectrum for L~ few fm

phase shift for elastic scat.

Resonance parameters



$$\mathcal{O} = \bar{q}\Gamma q,$$

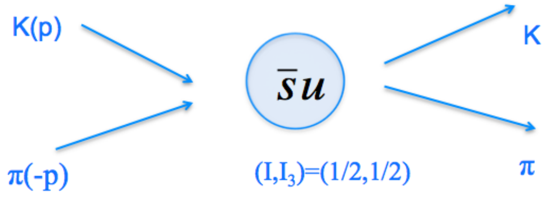
$$(\bar{q}\Gamma_1 q)_{\vec{p}_1}\,(\bar{q}\Gamma_2 q)_{\vec{p}_2} = M_1(\vec{p}_1)\; M_2(\vec{p}_2)$$

ρ resonance in ππ

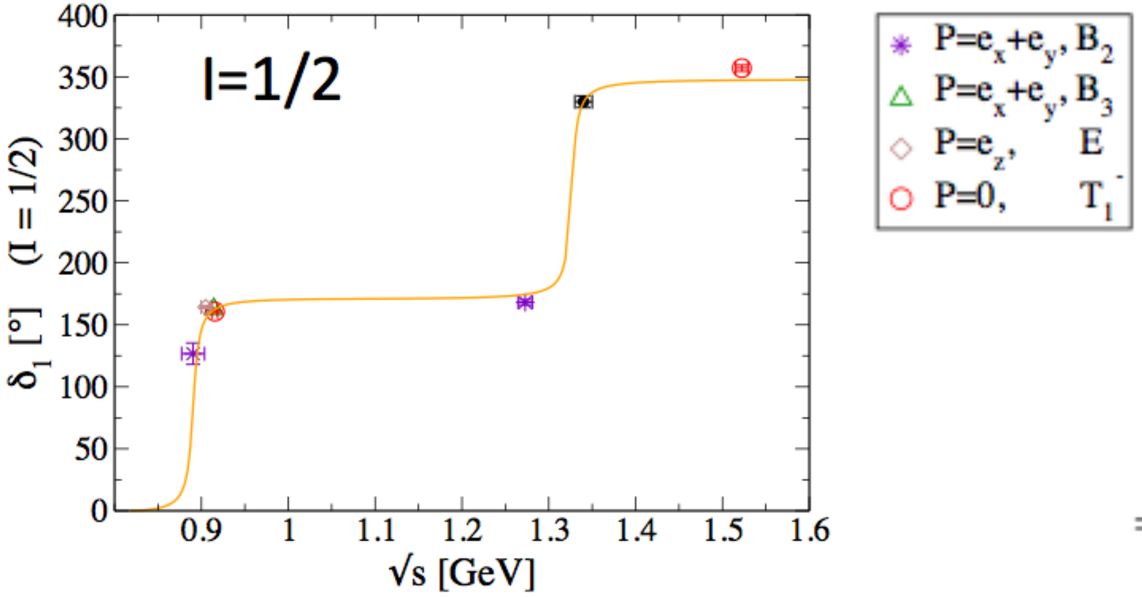
reviewed by Takeshi Yamazaki, plenary talk

K*(892) resonance Kπ

reviewed by Takeshi Yamazaki, plenary talk



Kπ , I=1/2: p-wave phase shift



$$\text{BW}: \quad \delta = \text{acot}\,\frac{m_R^2 - E_{cms}^2}{m_R\,\Gamma}$$

$$\Gamma[K^* \to K\pi] = \frac{g^2}{6\pi}\frac{p^{*3}}{s}$$

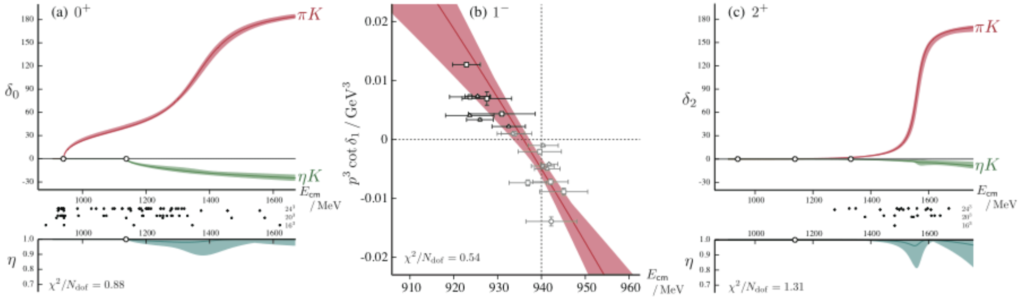
	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6
exp	891.66 ± 0.26	5.72 ± 0.06

[S.P. ,Lang, Leskovec, Mohler, 1307.0736, PRD 2013]

$m_\pi \approx 266$ MeV

Resonances in Kπ, Kη coupled channels

discussed by Yamazaki, Briceno, Wilson at Lat 14

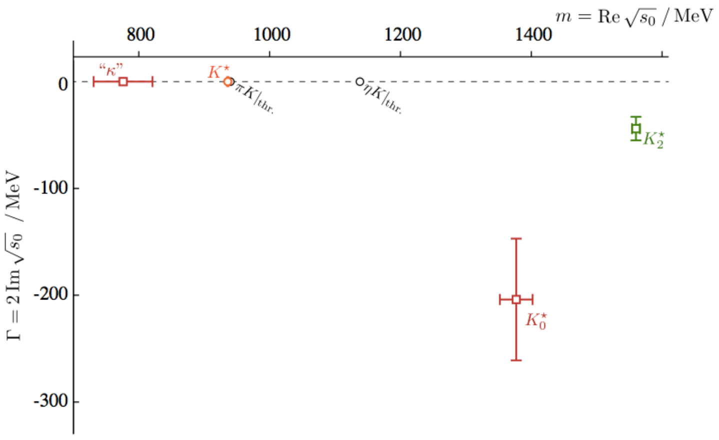


- $\underline{q}q$, Kπ, Kη interpolators
- a number of different $0<P\leq 2$
- for each E_n: one determinant equation for many unknowns
- T-matrix parametrized to get around this problem

$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, \; t_{ij} = \frac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}}$$

- the location of poles of T-matrix in complex plain is given below
- K*(892) and κ are below threshold for this m_π
- K_0^*, K_2^* are resonances
- m_π=391 MeV, N_L=16, 20, 24

[Dudek, Edwards, Thomas, Wilson, HSC, 1406.4158]



$$\det\left[\delta_{ij}\delta_{JJ'} + i\rho_i\, t_{ij}^{(J)}(E_{\sf cm})\left(\delta_{JJ'} + i\mathcal{M}^{\vec{P}\Lambda}_{JJ'}(p_i L)\right)\right] = 0,$$

location of poles in T matrix in complex plane

D-meson resonances in Dπ and D*π

$$\Gamma(E) \equiv g^2 \frac{p}{E^2}$$

discussed by Daniel Mohler, plenary at Lattice 2012

g is compared to exp instead of Γ (Γ depends on phase sp. and m_π)

$J^P=0^+$: D π

$D_0^*(2400)$	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

$J^P=1^+$: D* π

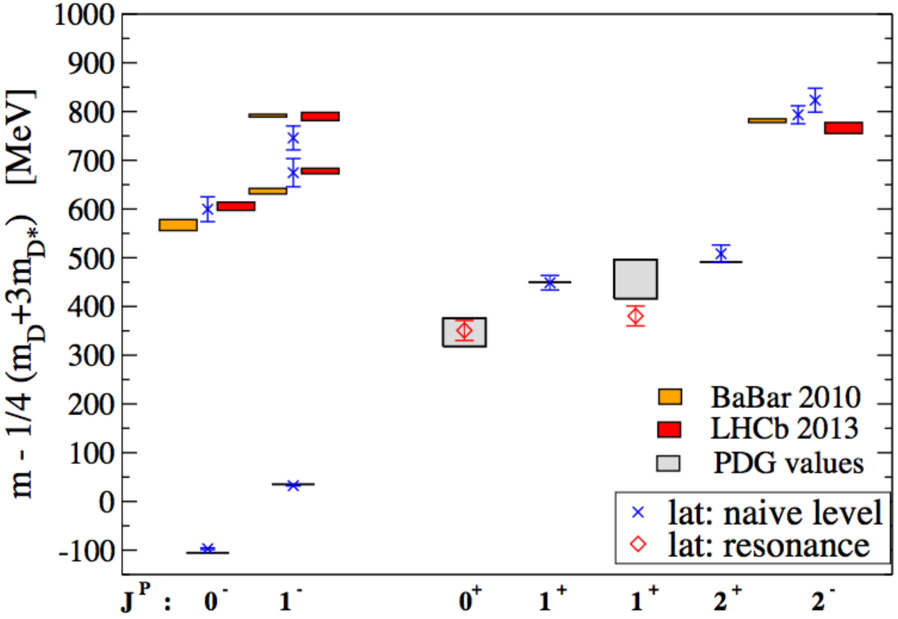
(analysis of spectrum in this
case is based on an assumption
given in paper below)

$D_1(2430)$	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV

first lattice result for strong decay width of a hadron containing charm quark

[D. Mohler, S.P. , R. Woloshyn: 1208.4059, PRD]

- $m_\pi \approx 266$ MeV, L≈2 fm, Nf=2



Lightest axial resonances a_1(1260) and b_1(1235)

- Simulating scattering:

  $\rho\ \pi$ in 1^{++} channel to extract a_1

  $\omega\ \pi$ in 1^{+-} channel to extract b_1

- $m_\pi \approx 266$ MeV, L≈2 fm, Nf=2 , P=0 [Lang, Leskovec, Mohler, S.P. , 1401.2088, JHEP]

resonance	$a_1(1260)$			$b_1(1235)$	
quantity	$m_{a_1}^{\rm res}$ [GeV]	$g_{a_1\rho\pi}$ [GeV]	$a_{l=0}^{\rho\pi}$ [fm]	$m_{b_1}^{\rm res}$ [GeV]	$g_{b_1\omega\pi}$ [GeV]
lat	$1.435(53)(^{+0}_{-109})$	1.71(39)	0.62(28)	$1.414(36)(^{+0}_{-83})$	input
exp	1.230(40)	1.35(30)	-	1.2295(32)	0.787(25)

$$\Gamma(E) \equiv g^2 \frac{p}{E^2}$$

- ρ and ω assumed to be stable which is a good approximation for given simulation parameters
- going beyond that aprpoximation will be very challenging
- analytical study of a_1 for unstable ρ: [Roca, Oset, 1201.0438]
- analytical studies of 3-particles:

  [Hansen, Sharpe 1311.4848; Polejaeva, Rusetsky, 1203.1241; Briceno, Davoudi, 1212.3398]

Isovectors including meson-meson interpolators

- anisotropic, m_π=240, 390 MeV, N_L=24,32
- a number of $\underline{q}q$ and MM interpolators

  for a number of u,d,s channels
- stochastic distillation
- results for ρ channel shown

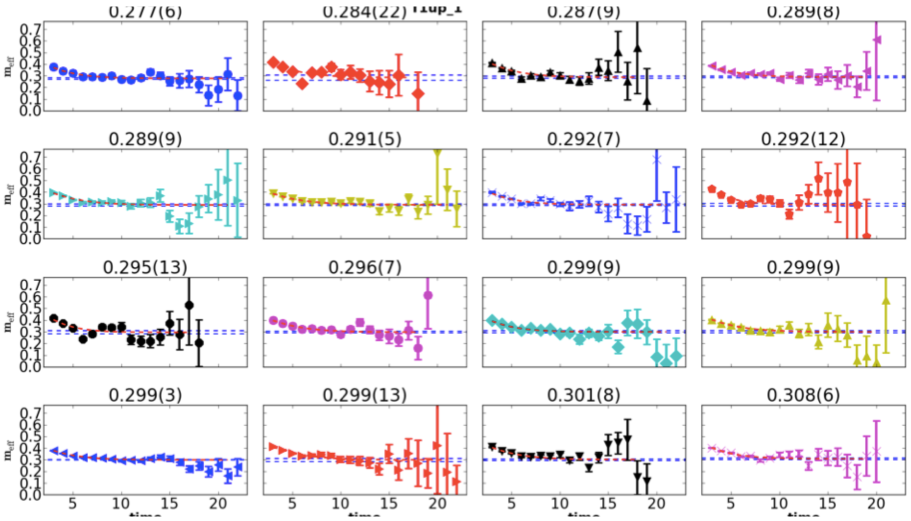
Morningstar, Wednesday, Hadron Spectroscopy

- numbers of operators for $I=1,\ S=0,\ P=(0,0,0)$ on 24^3 lattice

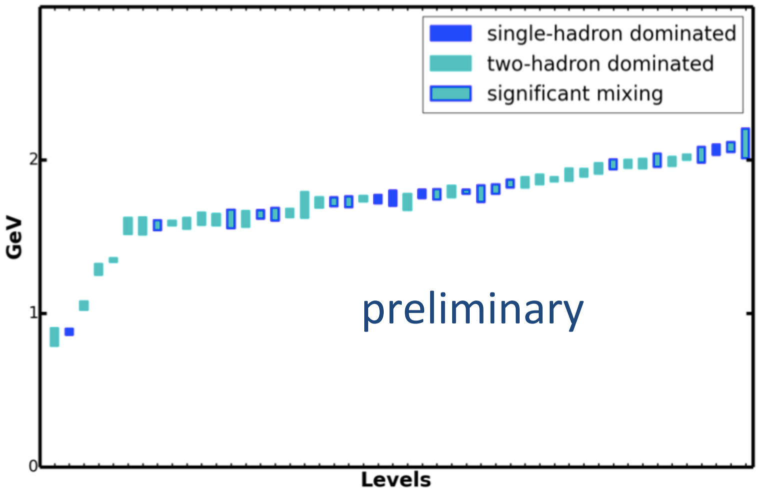
$(24^2\|390)$	A_{1g}^+	A_{1u}^+	A_{2g}^+	A_{2u}^+	E_g^+	E_u^+	T_{1g}^+	T_{1u}^+	T_{2g}^+	T_{2u}^+
SH	9	7	13	13	9	9	14	23	15	16
"$\pi\pi$"	6	12	2	6	8	9	15	17	10	12
"$\eta\pi$"	2	10	8	4	8	11	21	14	14	13
"$\phi\pi$"	2	10	8	4	8	11	23	3	14	13
"$K\overline{K}$"	0	4	1	4	1	4	8	10	4	6
Total	19	43	32	31	34	44	81	67	57	60

effective masses $\widetilde{m}^{\rm eff}(t)$ for levels 16 to 31

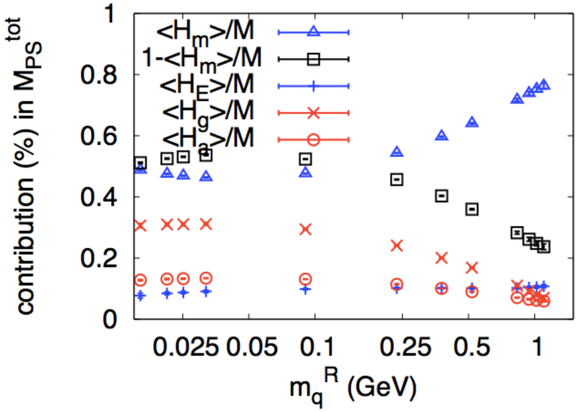
$32^3 \times 256$ lattice for $m_\pi \sim 240$ MeV



E_n in ρ channel for approx. 50 levels



Related topics



- Meson mass decomposition

  Yi-Bo Yang et al, XQCD coll., 1405.4440, Yang, Tuesday

$$M = -\langle T_{44} \rangle = \langle H_E \rangle + \langle H_m \rangle + \langle H_g \rangle + \langle H_a \rangle$$

$$\boxed{H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(D \cdot \gamma)\psi} \quad \boxed{H_m = \sum_{u,d,s\cdots} \int d^3x\, m \overline{\psi}\psi} \quad \boxed{H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2)}$$

- Extended QCD

  David Kaplan, 1306.5818

$$\mathcal{D} = \not{D} + \not{v} + i\not{a}\gamma_5 + 2\left(\Phi P_+ + \Phi^\dagger P_-\right)$$

$$S_{\rm XQCD} = N_c \int d^4x \left[\overline{\psi}\left(\mathcal{D} + m\right)\psi \;+ S_{\rm YM} + \lambda^2\left(\mathrm{Tr}\,\Phi^\dagger\Phi + \frac{1}{2}\mathrm{Tr}\left[\mathbf{v}_\mu^2 + \mathbf{a}_\mu^2\right]\right)\right]$$

$$\int e^{-N_c\lambda^2 \int d^4x \left[\mathrm{Tr}\,\Phi^\dagger\Phi + \frac{1}{2}\mathrm{Tr}\,(\mathbf{v}_\mu\mathbf{v}_\mu + \mathbf{a}_\mu\mathbf{a}_\mu)\right]} \det\left[\mathcal{D} + m\right] = \mathcal{N} \det\left[\not{D} + m\right]$$

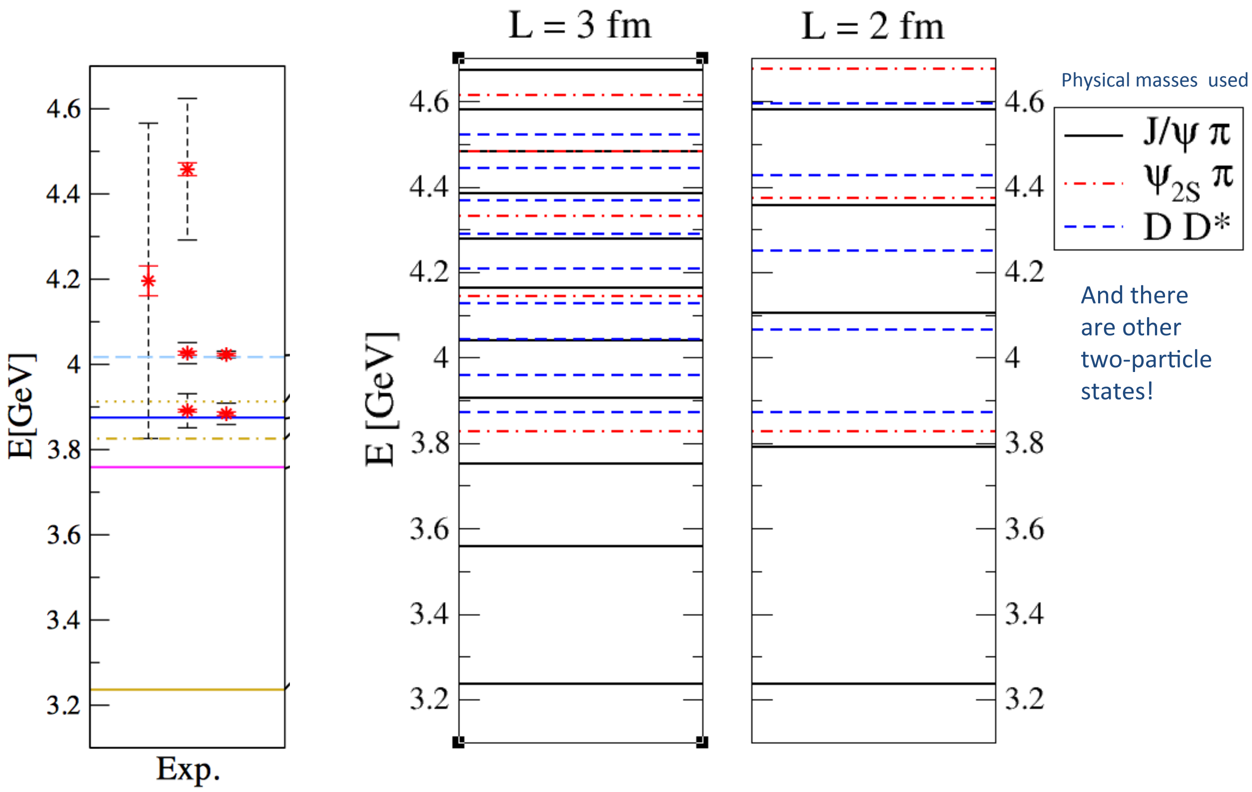
Exactly equivalent to QCD

limit Nc=∞ : direct connection with nonrelativistic quark models with constituent quark mass
respects chiral symmetry giving massless pion in chiral limit

Challenges: two examples

Challenge : precision simulation of Z_c^+

On larger volume: more two particle states



Physical masses used

And there are other two-particle states!

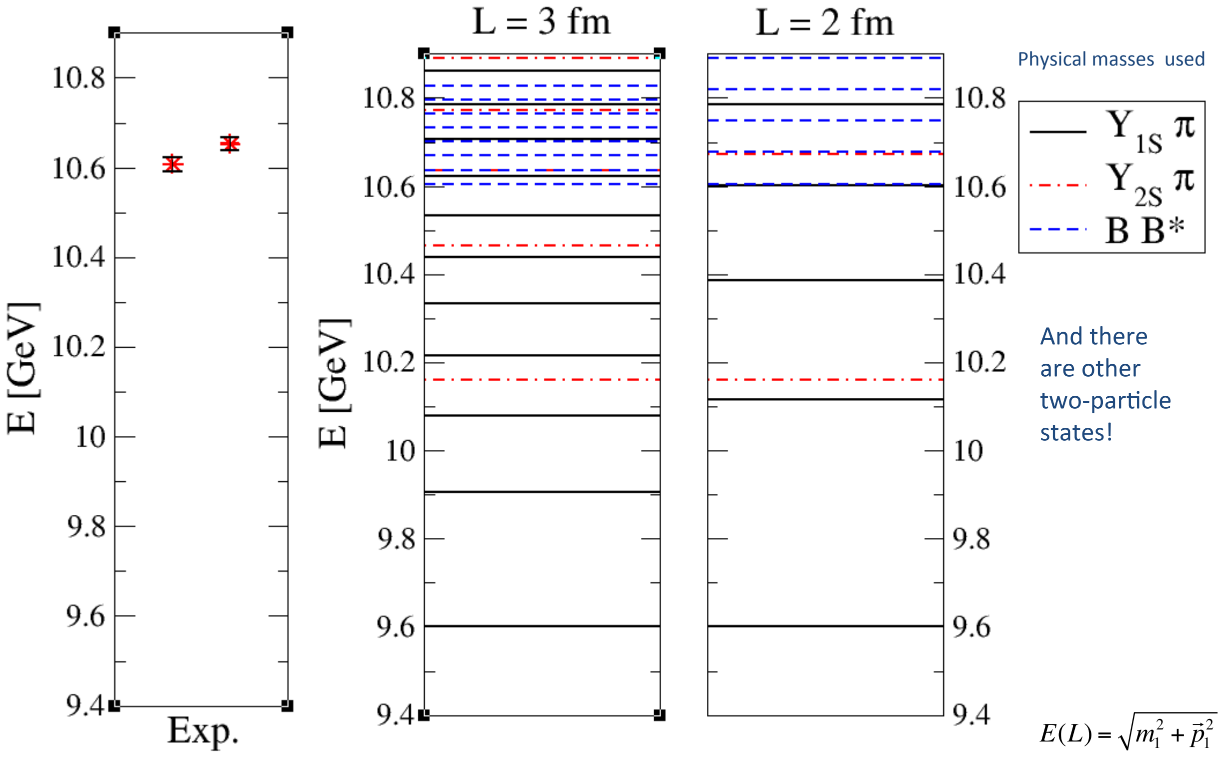
$$E(L) = \sqrt{m_1^2 + \vec{p}_1^{\,2}} + \sqrt{m_2^2 + \vec{p}_2^{\,2}} + \Delta E$$

$$\vec{p}_1 = \tfrac{2\pi}{L}\vec{n}_1 \qquad \vec{p}_2 = \tfrac{2\pi}{L}\vec{n}_2$$

Rigorous treatment very challenging: at least 6 two-particle channels coupled !!

Another challenge: Z_b^+

On larger volume: more two-particle states



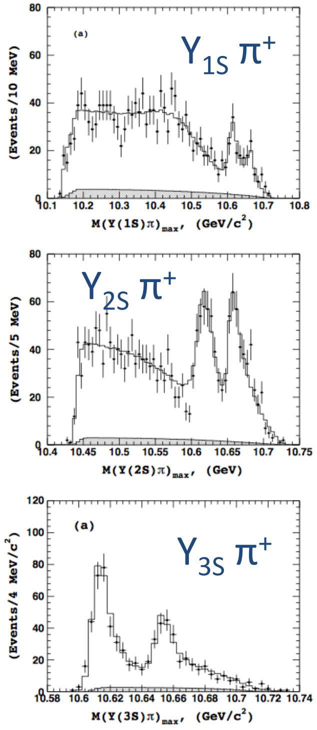
Physical masses used

And there are other two-particle states!

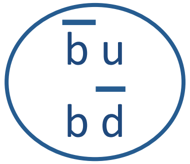
$$E(L)=\sqrt{m_1^2+\vec{p}_1^{\,2}}+\sqrt{m_2^2+\vec{p}_2^{\,2}}+\Delta E$$

$$\vec{p}_1=\tfrac{2\pi}{L}\vec{n}_1 \qquad \vec{p}_2=\tfrac{2\pi}{L}\vec{n}_2$$

Belle 2011



$Z_b^+ \to \Upsilon_{nS}\ \pi^+$



Rigorous treatment very challenging: at least 6 two-particle channels coupled !!

Conclusions

Recent developments in hadron spectroscopy (with emphasis on mesons):

- below threshold states treated with unprecedented accuracy
- extensive results for multiplets within single-hadron approximation
- first rigorous treatments of near-threshold states:

  evidences for Z_c^+, X(3872), $D_{s0}^*(2317)$, Λ(1405)
- a number of resonances studied rigorously:

  ρ, κ, K*, K_0^* , K_2^*, D_0^*, D_1 , a_1 , b_1
- coupled inelastic problem treated in QCD for the first time (to my knowledge):

  Kπ, Kη : κ, K*, K_0^* , K_2^*

Conclusions

Recent developments in hadron spectroscopy (with emphasis on mesons):

- below threshold states treated with unprecedented accuracy
- extensive results for multiplets within single-hadron approximation
- first rigorous treatments of near-threshold states:

  evidences for Z_c^+, X(3872), $D_{s0}^*(2317)$, Λ(1405)
- a number of resonances studied rigorously:

  ρ, κ, K*, K_0^* , K_2^*, D_0^*, D_1 , a_1 , b_1
- coupled inelastic problem treated in QCD for the first time (to my knowledge):

  Kπ, Kη : κ, K*, K_0^* , K_2^*

Conclusions

Many exciting challenges remain !

✧ An urgent example: quarkonium-like states

- can one afford to study them on larger volumes given the increasing number of two particle states?
- how about interesting states that lie even higher above threshold(s) ?
- rigorous treatment near multiple thresholds?

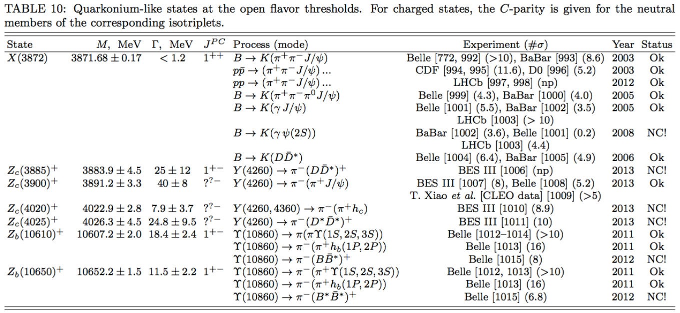


TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \to (\pi^+\pi^- J/\psi) \ldots$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \to (\pi^+\pi^- J/\psi) \ldots$	LHCb [997, 998] (np)	2012	Ok
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \to K(\gamma J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5)	2005	Ok
					LHCb [1003] (> 10)		
				$B \to K(\gamma\, \psi(2S))$	BaBar [1002] (3.6), Belle [1001] (0.2)	2008	NC!
					LHCb [1003] (4.4)		
				$B \to K(D\bar{D}^*)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
$Z_c(3885)^+$	3883.9 ± 4.5	25 ± 12	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
$Z_c(3900)^+$	3891.2 ± 3.3	40 ± 8	$?^{?-}$	$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
					T. Xiao *et al.* [CLEO data] [1009] (>5)		
$Z_c(4020)^+$	4022.9 ± 2.8	7.9 ± 3.7	$?^{?-}$	$Y(4260, 4360) \to \pi^-(\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	4026.3 ± 4.5	24.8 ± 9.5	$?^{?-}$	$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle [1012–1014] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!

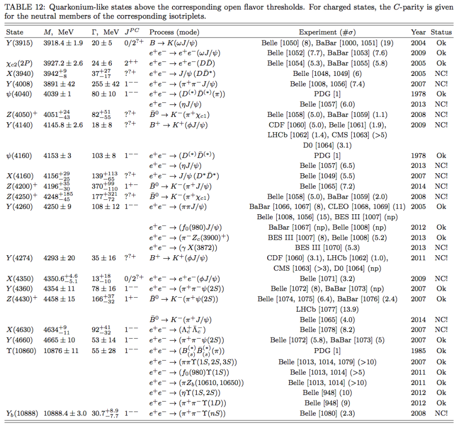
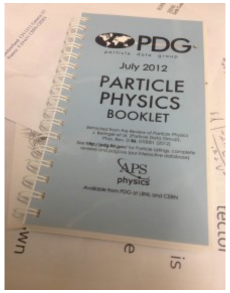


TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$Y(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle [1054] (5.3), BaBar [1055] (5.8)	2005	Ok
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle [1048, 1049] (6)	2005	NC!
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (5.0), Belle [1061] (1.9),	2009	NC!
					LHCb [1062] (1.4), CMS [1063] (>5)		
					D0 [1064] (3.1)		
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.5)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle [1049] (5.5)	2007	NC!
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (7.2)	2014	NC!
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11)	2005	Ok
					Belle [1008, 1056] (15), BES III [1007] (np)		
				$e^+e^- \to (f_0(980) J/\psi)$	BaBar [1067] (np), Belle [1008] (np)	2012	Ok
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+e^- \to (\gamma\, X(3872))$	BES III [1070] (5.3)	2013	NC!
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0),	2011	NC!
					CMS [1063] (>3), D0 [1064] (np)		
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [1071] (3.2)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4458 ± 15	166^{+37}_{-32}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4)	2007	Ok
					LHCb [1077] (13.9)		
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (4.0)	2014	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$\Upsilon(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+e^- \to (B^{(*)}_{(s)}\bar{B}^{(*)}_{(s)}(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \to (\pi\pi\Upsilon(1S, 2S, 3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \to (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \to (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \to (\eta\Upsilon(1S, 2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

✧ Can we understand almost complete absence of exotics in most of other meson systems?



✧ Looking forward to new challenges ...

Thanks to those who sent me the material

Christine Davies, Carleton DeTar, Derek Leinweber, Keh-Fei Liu, Daniel Mohler, Colin Morningstar, Raul Briceno, Takeshi Yamazaki, Andrea Guerrieri, Christopher Thomas

And to my collaborators :

Christian B. Lang	(Graz)
Daniel Mohler	(Fermilab)
Luka Leskovec	(Ljubljana)
Richard Woloshyn	(Vancouver)

Apologies to those whose work I was not able to present.

Backup slides

Interpolators in Zc channel

[S.P., Lang, Leskovec, Mohler, 1405.7623

$$\mathcal{O}_1 = \mathcal{O}_1^{\psi(0)\pi(0)} = \bar{c}\gamma_i c(0)\ \bar{d}\gamma_5 u(0)\,,$$

$$\mathcal{O}_2 = \mathcal{O}_2^{\psi(0)\pi(0)} = \bar{c}\gamma_i\gamma_t c(0)\ \bar{d}\gamma_5 u(0)\,,$$

$$\mathcal{O}_3 = \mathcal{O}_3^{\psi(0)\pi(0)} = \bar{c}\overleftarrow{\nabla}_j\gamma_i\overrightarrow{\nabla}_j c(0)\ \bar{d}\gamma_5 u(0)\,,$$

$$\mathcal{O}_4 = \mathcal{O}_4^{\psi(0)\pi(0)} = \bar{c}\overleftarrow{\nabla}_j\gamma_i\gamma_t\overrightarrow{\nabla}_j c(0)\ \bar{d}\gamma_5 u(0)\,,$$

$$\mathcal{O}_5 = \mathcal{O}_5^{\psi(0)\pi(0)} = |\epsilon_{ijk}||\epsilon_{klm}|\ \bar{c}\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c(0)\ \bar{d}\gamma_5 u(0)\,,$$

$$\mathcal{O}_6 = \mathcal{O}_6^{\psi(0)\pi(0)} = |\epsilon_{ijk}||\epsilon_{klm}|\ \bar{c}\gamma_t\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c(0)\ \bar{d}\gamma_5 u(0)\,,$$

$$\mathcal{O}_7 = \mathcal{O}_7^{\psi(0)\pi(0)} = R_{ijk}Q_{klm}\ \bar{c}\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c\ \bar{d}\gamma_5 u(0)\,,$$

$$\mathcal{O}_8 = \mathcal{O}_8^{\psi(0)\pi(0)} = R_{ijk}Q_{klm}\ \bar{c}\gamma_t\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c\ \bar{d}\gamma_5 u(0)\,.$$

$$\mathcal{O}_9 = \mathcal{O}^{\psi(1)\pi(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k)\ \bar{d}\gamma_5 u(-e_k)\,,$$

$$\mathcal{O}_{10} = \mathcal{O}^{\eta_c(0)\rho(0)} = \bar{c}\gamma_5 c(0)\ \bar{d}\gamma_i u(0)\,,$$

$$\mathcal{O}_{11} = \mathcal{O}_1^{D(0)D^*(0)} = \bar{c}\gamma_5 u(0)\ \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,$$

$$\mathcal{O}_{12} = \mathcal{O}_2^{D(0)D^*(0)} = \bar{c}\gamma_5\gamma_t u(0)\ \bar{d}\gamma_i\gamma_t c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,$$

$$\mathcal{O}_{13} = \mathcal{O}^{D(1)D^*(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_5 u(e_k)\ \bar{d}\gamma_i c(-e_k) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,$$

$$\mathcal{O}_{14} = \mathcal{O}^{D^*(0)D^*(0)} = \epsilon_{ijk}\ \bar{c}\gamma_j u(0)\ \bar{d}\gamma_k c(0)\,,$$

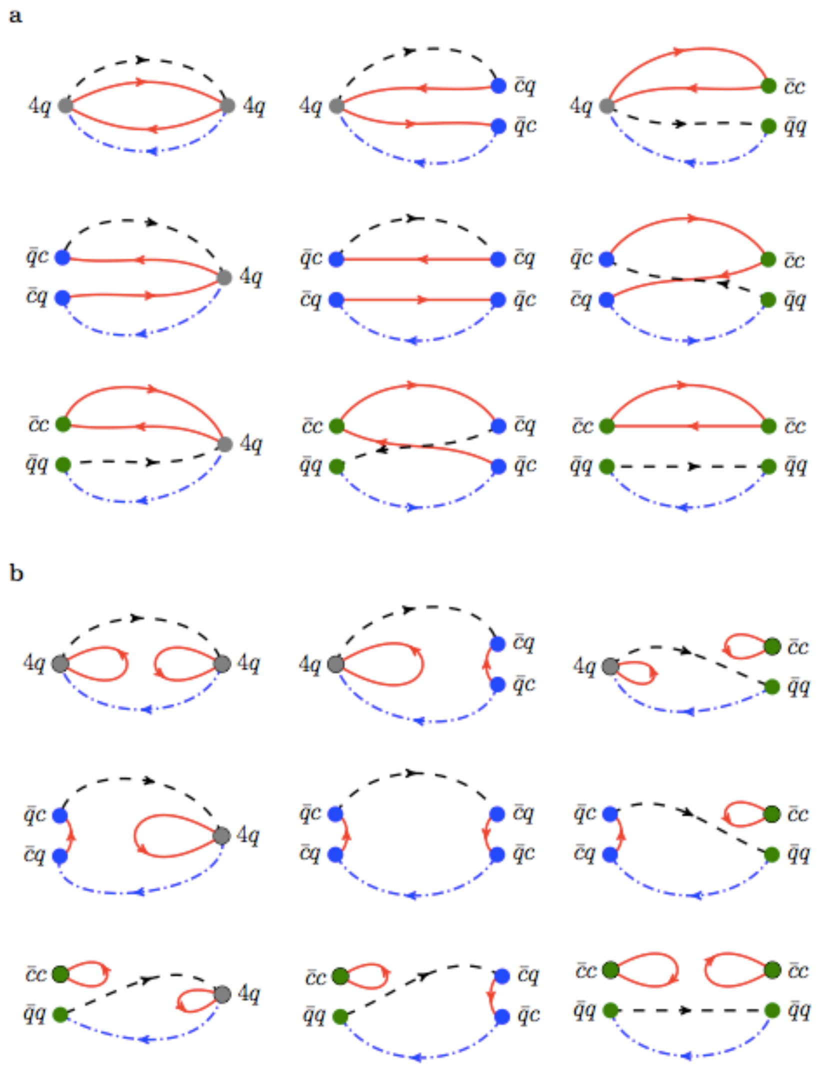
$$\mathcal{O}_{15} = \mathcal{O}_1^{4q} = N_L^3\ \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\gamma_5\bar{d}_c\ c_{b'}\gamma_i C u_{c'} - \bar{c}_b C\gamma_i\bar{d}_c\ c_{b'}\gamma_5 C u_{c'})$$

$$\mathcal{O}_{16} = \mathcal{O}_2^{4q} = N_L^3\ \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\bar{d}_c\ c_{b'}\gamma_i\gamma_5 C u_{c'} - \bar{c}_b C\gamma_i\gamma_5\bar{d}_c\ c_{b'} C u_{c'})$$

$$\mathcal{O}_{17} = \mathcal{O}_3^{4q} = \mathcal{O}_1^{4q}(N_v{=}32)\,,$$

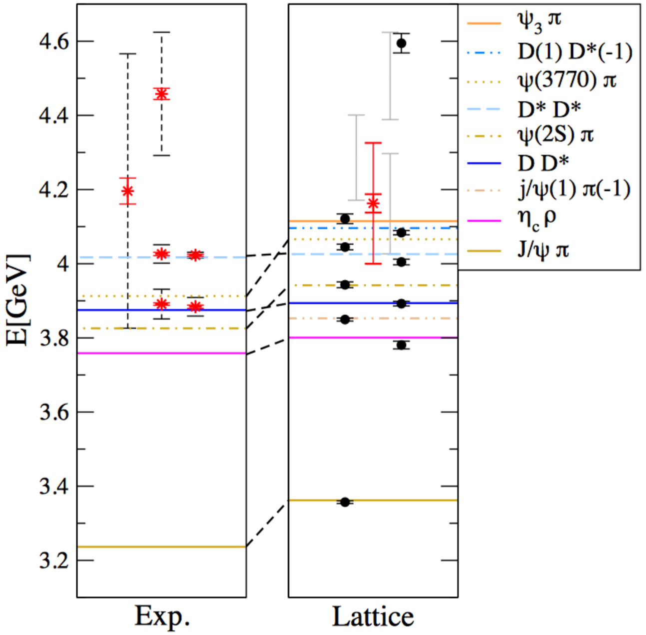
$$\mathcal{O}_{18} = \mathcal{O}_4^{4q} = \mathcal{O}_2^{4q}(N_v{=}32)\,.$$

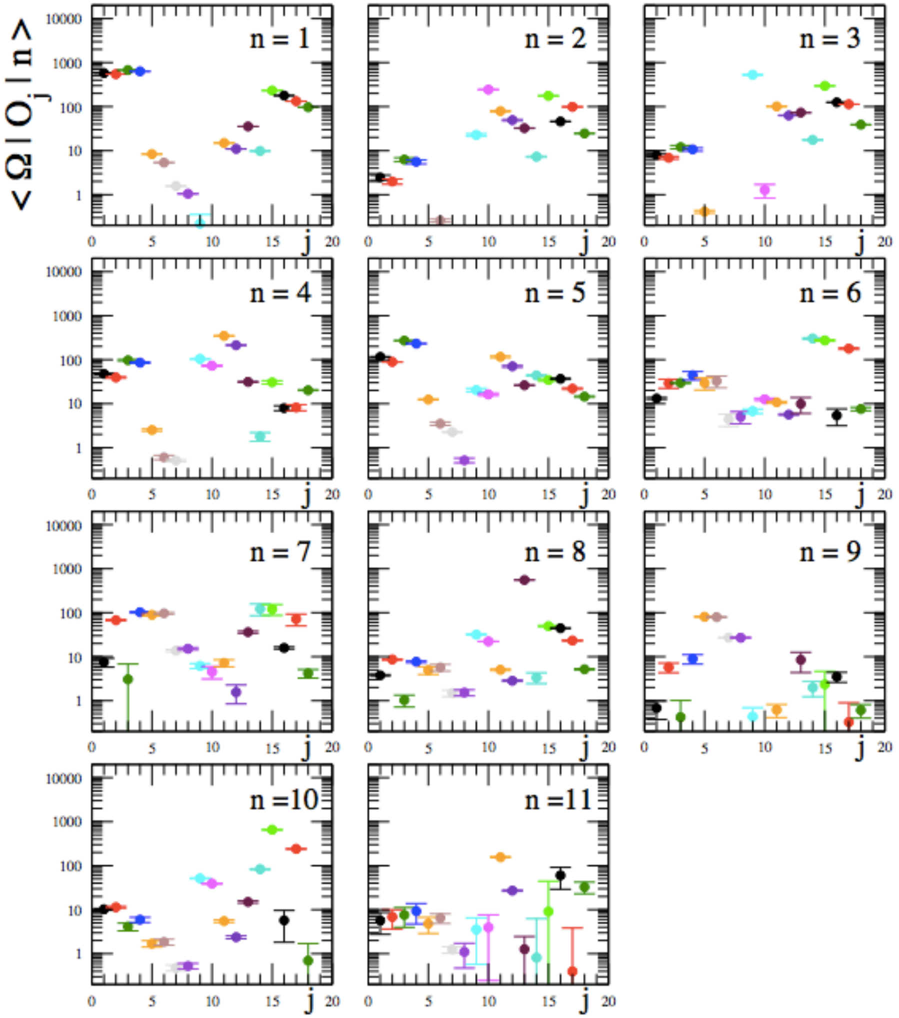
Wick contractions for Zc+



[S.P., Lang, Leskovec, Mohler, 1405.7623]

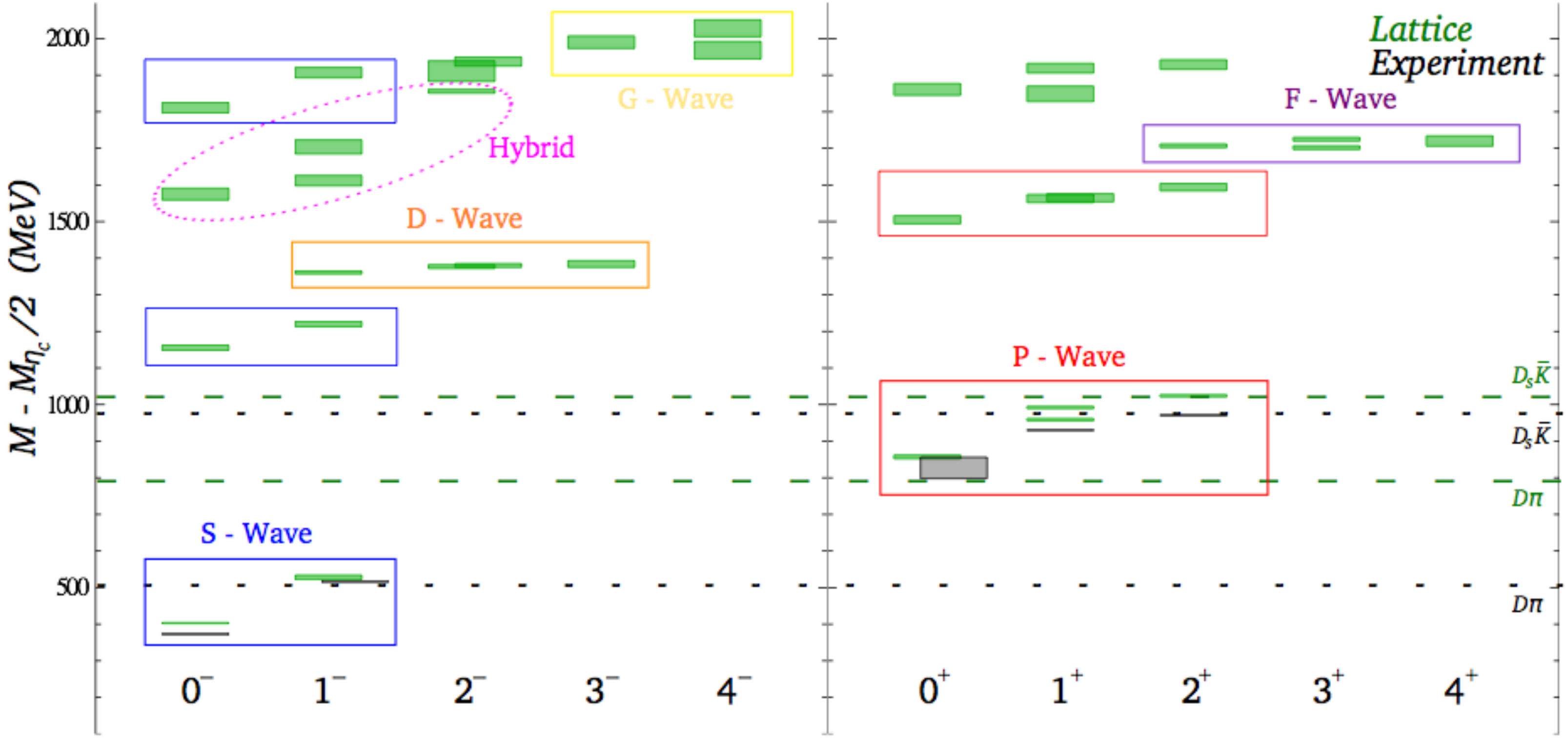
Overlaps of all states in Zc+ channel





[S.P., Lang, Leskovec, Mohler, 1405.7623]

D spectrum: single hadron approximation



1S-2S:

~ 700 MeV

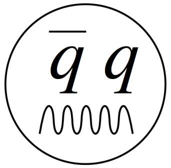
[G. Moir et al, HSC (Hadron Spectrum Coll.): 1301.7670, JHEP]

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, $N_f = 2+1$
- reliable J^P determination; many excited states
- identification with $n\ ^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

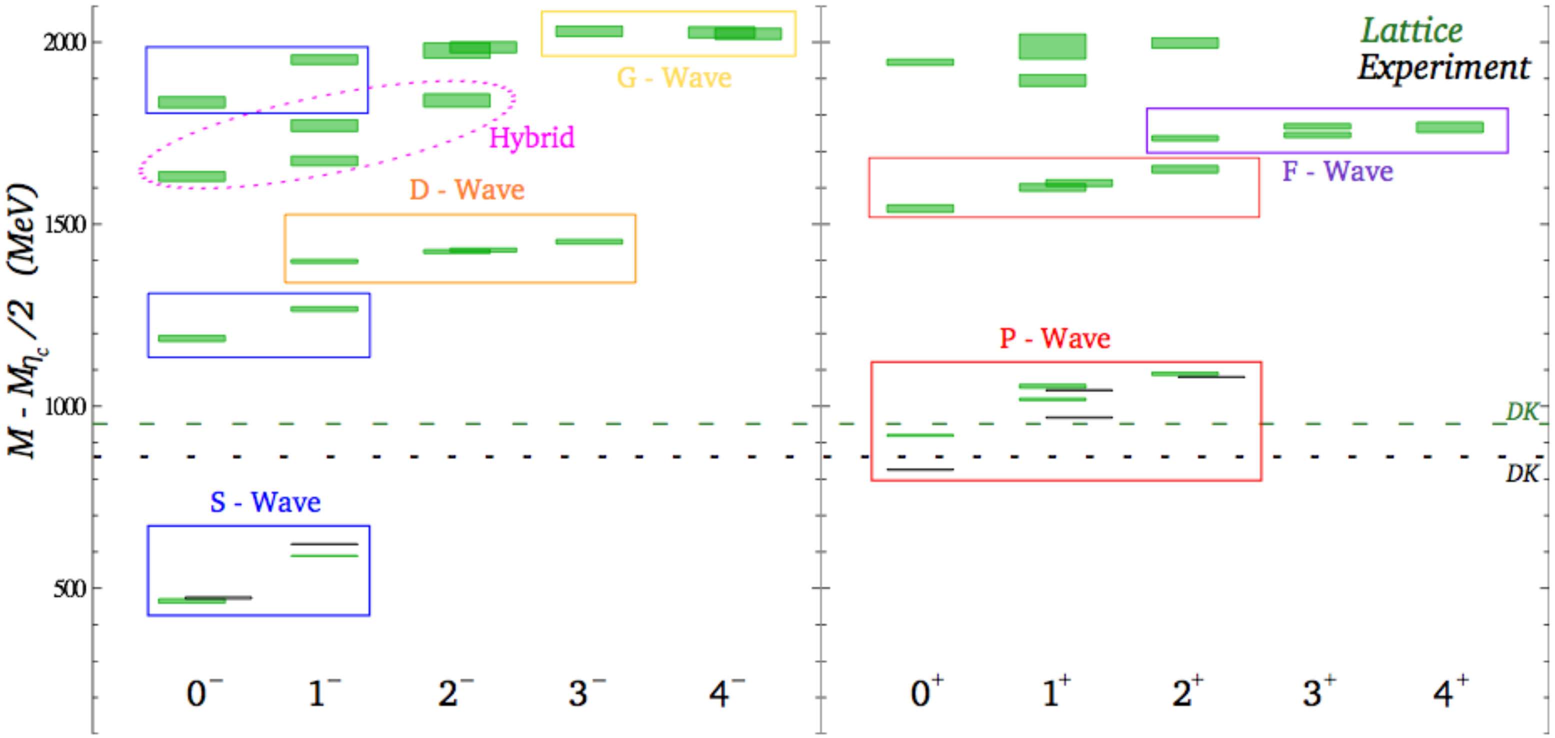
Hybrids:

large overlap with $O = \underline{q}\, F_{ij}\ q$

gluonic tensor $F_{ij} = [D_i\ , D_j\ ]$



D_s spectrum: single hadron approximation



[G. Moir et al., HSC : 1301.7670, JHEP]

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, Nf=2+1
- reliable J^{PC} determination
- identification with $n\ {}^{2S+1}L_J$ multiplets using $\langle O | n \rangle$
- green: lat, black: exp

Hybrids:

large overlap with $O = \underline{q}\ F_{ij}\ q$

gluonic tensor $F_{ij} = [D_i\ ,\ D_j\ ]$