Review of Hadronic Structure in Lattice QCD

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A Nucleon Sector
- Axial charge
- Electromagnetic form factors
- Dirac & Pauli radii
- Quark momentum fraction
- Nucleon Spin

B Hyperon Form Factors
- Hyperon EM form factors
- Axial form factors

C Mesons
- Pion momentum fraction
- $\rho$-meson EM form factors

D Conclusions
LQCD meets Nature

LQCD:
- estimates for experimentally well known quantities
- input for not well known quantities

Rich experimental activities in major facilities: JLab, MAMI, MESA, etc
- Investigation of baryon and meson structure
- Origin of mass and spin
- New physics searches: $(g - 2)_\mu$, dark photon searches
- Proton radius puzzle
- The list is long...
Proton Radius Puzzle

\[ < r_p^2 > \text{ from muonic hydrogen } \mu p \ 7.7\sigma \text{ smaller than elastic } e - p \text{ scattering} \]

CREMA Collaboration

- measured energy difference between the 2P and 2S states of muonic hydrogen
- \( \mu p \): 10 times more accurate than other measurements
- very sensitive to the proton size
- no obvious way to connect with other measurements (4% diff)

Physics Program for CLAS12 (Selected Hadron Experiments)

- The Longitudinal Spin Structure of the Nucleon
- Nucleon Resonance Studies with CLAS12
- Meson spectroscopy with low $Q^2$ electron scattering
- High Precision Measurement of the Proton Charge Radius
- and many more....
Light-by-Light scattering at LHC


- Never observed directly
- Indirectly observed by its effects on anomalous magnetic moments of electrons and muons
- Photon-photon collisions in ultraperipheral collisions of proton have been detected
- arXiv:1305.7142: LCH could detect LbyL (5.5-14 TeV) due to:
- 'quasireal' photons fluxes in EM interactions of protons and lead ions
A NUCLEON SECTOR
Nucleon on the Lattice in a nutshell

LQCD:

- estimates for experimentally well known quantities
- input for not well known quantities

**Contributing diagrams:**

- Connected
- Disconnected

**Computation of 2pt- and 3pt-functions:**

2pt:

\[ G(q', t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f \cdot \vec{q}} \Gamma^0_{\beta\alpha} \langle J_{\alpha}(\vec{x}_f, t_f) \overline{J}_\beta(0) \rangle \]

3pt:

\[ G_\mathcal{O}(\Gamma^\kappa, q', t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x} \cdot \vec{q}} e^{-i\vec{x}_f \cdot \vec{p}'} \Gamma^\kappa_{\beta\alpha} \langle J_{\alpha}(\vec{x}_f, t_f) \mathcal{O}(\vec{x}, t) \overline{J}_\beta(0) \rangle \]

\[
\Gamma^0 \equiv \frac{1}{4} (1 + \gamma_0) \\
\Gamma^2 \equiv \Gamma^0 \cdot \gamma_5 \cdot \gamma_i \\
\text{and other variations}
\]
Construction of optimized ratio:

\[ R_{\mathcal{O}}(\Gamma, \vec{q}, t) = \frac{G_{\mathcal{O}}(\Gamma, \vec{q}, t)}{G(\vec{0}, t_f)} \times \sqrt{\frac{G(-\vec{q}, t_f - t)G(\vec{0}, t)G(\vec{0}, t_f)}{G(\vec{0}, t_f - t)G(-\vec{q}, t)G(-\vec{q}, t_f)}} \]

\[ \lim_{t_f \to t \to \infty} \Pi(\Gamma, \vec{q}) \]

Plateau Method: Most common method

Renormalization: connection to experiments

\[ \Pi^R(\Gamma, \vec{q}) = Z_{\mathcal{O}} \Pi(\Gamma, \vec{q}) \]

Extraction of form factors

e.g. Axial current:

\[ A^3_\mu \equiv \bar{\psi} \gamma_\mu \gamma_5 \frac{\tau^3}{2} \psi \Rightarrow \bar{u}_N(p') \left[ G_A(q^2) \gamma_\mu \gamma_5 + G_P(q^2) \frac{q_\mu \gamma_5}{2 m_N} \right] u_N(p) \]

Isovector Combination: (u-d)

disconnected contributions cancel out

Simpler renormalization
A1. NUCLEON AXIAL CHARGE
The chosen one

Axial current: \[ \overline{\psi} \gamma_\mu \gamma_5 \frac{\tau^3}{2} \psi \]

\[ g_A \equiv G_A(0) \]

- \( g_A^{\text{exp}} = 1.2701(25) \) [PRD’12]
- governs the rate of \( \beta \)-decay
- determined directly from lattice data (no fit necessary)
- \( m_\pi > 200 \text{MeV} \): lattice results below exp.: \( \sim 10\%-15\% \)

Selected Works:
- T. Yamazaki et al. (RBC/UKQCD), [arXiv:0801.4016]
- T. Yamazaki (RBC/UKQCD), [arXiv:0904.2039]
- J.D. Bratt et al. (LHPC), [arXiv:1001.3620]
- C. Alexandrou et al. (ETMC), [arXiv:1012.0857]
- S. Collins et al. (QCDSF/UKQCD), [arXiv:1101.2326]
- B.B. Brandt et al. (CLS/MAINZ), [arXiv:1106.1554]
- G.S. Bali et al. (QCDSF), [arXiv:1112.3354]
- S. Capitani et al. (CLS/MAINZ), [arXiv:1205.0180]
- J.R. Green et al. (LHPC), [arXiv:1209.1687]
- J.R. Green et al. (LHPC), [arXiv:1211.0253]
- B.J. Owen et al. (CSSM), [arXiv:1212.4668]
- R. Horsley et al. (QCDSF), [arXiv:1302.2233]
- C. Alexandrou et al. (ETMC), [arXiv:1303.5979]
- T. Bhattacharya et al. (PNDME), [arXiv:1306.5435]
- S. Ohta et al. (RBC/UKQCD), [arXiv:1309.7942]
- G.S. Bali et al. (RQCD), [arXiv:1311.7041]
- A.J. Chambers et al. (QCDSF/UKQCD), [arXiv:1405.3019]

★ Lattice data from 'plateau' methods
★ Latest achievement: lattice results at physical \( m_\pi \)
★ No necessity of chiral extrapolation
★ Different strategies for addressing systematic uncertainties
**A1. NUCLEON AXIAL CHARGE**

The chosen one

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**Possible origin of systematics**

→ **Cut-off Effects**
  - adjustment of source-sink separation
  - 2-state fit
  - summation method

→ **Excited State Contamination**
  - investigation of volume effects as lattice box increases

→ **Finite Volume Effects**

- not being at the physical point

★ Lattice data from 'plateau' methods
★ Latest achievement: lattice results at physical \( m_\pi \)
★ No necessity of chiral extrapolation
★ Different strategies for addressing systematic uncertainties
Cut-off effects

- Continuum extrapolation requires 3 lattice spacings

[C. Alexandrou et al. (ETMC), arXiv:1012.0857] [G. Bali et al. (RQCD), 2014]

1st Conclusion: $a < 0.1$ fm is sufficient
Excited State Contamination

Plateau Method: single-state fit

\[ R(t_i, t, t_f) \xrightarrow{(t_f-t) \Delta >> 1} \Delta >> 1 \quad \mathcal{M} \left[ 1 + \alpha e^{-(t_f-t) \Delta (p')} + \beta e^{-(t-t_i) \Delta (p')} + \ldots \right] \]

2-state fit on 3pt-functions

\[ C^{(3)}_\Gamma(T(t_i, t, t_f; \vec{p}_i, \vec{p}_f) \approx |A_0|^2 \langle 0 | O_\Gamma | 0 \rangle e^{-M_0 T_{\text{sink}}} + |A_1|^2 \langle 1 | O_\Gamma | 1 \rangle e^{-M_1 T_{\text{sink}}} + \mathcal{A}_0 \mathcal{A}_1^* \langle 0 | O_\Gamma | 1 \rangle e^{-M_0 (t-t_i)} e^{-M_1 (t_f-t)} + \mathcal{A}_0^* \mathcal{A}_1 \langle 1 | O_\Gamma | 0 \rangle e^{-M_1 (t-t_i)} e^{-M_0 (t_f-t)} \]

Summation Method

\[ \sum_{t=t_i}^{t_f} R(t_i, t, t_f) = \text{const.} + \mathcal{M} T_{\text{sink}} + \mathcal{O} \left( e^{-(T_{\text{sink}} \Delta (p'))} \right) + \mathcal{O} \left( e^{-(T_{\text{sink}} \Delta (p'))} \right) \]

- suppressed excited states (exponentials decaying with \( T_{\text{sink}} \))
- Matrix element extracted from the slope
- Alternatively: sum over \( t_i + 1 \leq t \leq t_f - 1 \)
Plateau Method: single-state

[RQCD (2014):]

- $m_\pi = 285$ MeV
- $g_A$ not sensitive on $T_{\text{sink}}$: 0.49-1.19 fm

ETMC (2013): ⇒

[S.Dinter et al. (ETMC), arXiv:1108.1076]

- $m_\pi = 373$ MeV
- $g_A$ not sensitive on $T_{\text{sink}}$

Summation Method

[ETMC (2013):]

- $m_\pi = 373$ MeV
- $T_{\text{sink}}$: 0.3 fm-1.3 fm
- No curvature is seen in slope
- No detectable excited states
Plateau Method: single-state

\[ m_\pi = 285 \text{ MeV} \]

\[ g_A \text{ not sensitive on } T_{\text{sink}}: 0.49-1.19 \text{ fm} \]

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Summation Method

\[ m_\pi = 373 \text{ MeV} \]

\[ T_{\text{sink}}: 0.3 \text{ fm}-1.3 \text{ fm} \]

No curvature is seen in slope

No detectable excited states
Two-state fit on 3pt-functions

\[ T_{\text{sink}} (\text{fm}) \]

- 2-states simultaneous fit
- 1-state fit
- 2-states fit

\[ g_{A, \text{bare}} \]

\[ m_\pi = 310 \text{MeV} \]

- Largest difference for \( T_{\text{sink}} < 1 \text{ fm} \)
- All fits in agreement

2nd Conclusion: \( T_{\text{sink}} > 1 \text{ fm safe}^* \)

* based on \( m_\pi \approx 300 \text{MeV} \)

Feynman-Hellmann Approach:

\[ S \rightarrow S(\lambda) = S + \lambda \sum_x \bar{q}(x)i\gamma_5\gamma_3 q(x) \]

\[ \Delta q = \left. \frac{\partial E(\lambda)}{\partial \lambda} \right|_{\lambda=0} = \frac{1}{2M} \langle N | \bar{q}i\gamma_5\gamma_3 q | N \rangle \]

- External spin operator in \( S_{\text{fermion}} \)
- \( \Delta q \): linear response of nucleon energies
- Statistical Precision

\[ m_\pi = 470 \text{MeV} \]

Talk by J. Zanotti
**Improvement Technique: All-Mode-Avaraging (AMA)** [E.Shintani et. al. arXiv:1402.0244]

**signal/noise** \(\sim \sqrt{N_{\text{meas}}} \times e^{-(m_N + 3 m_\pi / 2)}\)

- Reduction of statistical error for a given number of gauge configurations
- Significant increase of \(N_{\text{meas}}\) at low computational cost
- Improved operator:

\[
\langle O^{\text{impr}} \rangle = \langle O^{\text{approx}} \rangle + \langle O^{\text{rest}} \rangle
\]

\(O^{\text{approx}}\): not precise but cheap
\(O^{\text{rest}}\): correction term

\[
O^{\text{rest}} = O^{\text{exact}} - O^{\text{approx}}
\]

- AMA result:

\[
O^{\text{AMA}} = \frac{1}{N_{\text{approx}}} \sum_{i=1}^{N_{\text{approx}}} O^{i}_{\text{approx}} + \frac{1}{N_{\text{exact}}} \sum_{j=1}^{N_{\text{exact}}} \left( O^{j}_{\text{exact}} - O^{j}_{\text{approx}} \right)
\]

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**RBC/UKQCD (2014): DWF \(N_f = 2+1\)**

- A factor of 20 improvement in computational efficiency
- A sloppy calculation costs \(\sim 1/65\) of an exact calculation
- The speedup with AMA: \(\sim 15-29\) times

Talk by S.Ohta
Finite Volume Effects

Lattice data for plateau method
No volume corrections

Systematics not fully understood:

- $g_A^{\text{PNDME}}$ ($L m_\pi \sim 3.7$) agrees with $g_A^{\text{exp}}$
- $g_A^{\text{ETMC}}$ ($L m_\pi \sim 3$) agrees with $g_A^{\text{exp}}$
- $g_A^{\text{QCDSF}}$ ($L m_\pi \sim 2.7$) close to $g_A^{\text{exp}}$
- $g_A^{\text{LHPC}}$ ($L m_\pi \sim 4$) lower than $g_A^{\text{exp}}$

- **PNDME** ($m_\pi = 128\text{MeV}$) : $L_s = 5.76$ fm, $a = 0.09$ fm
- **ETMC** ($m_\pi = 135\text{MeV}$) : $L_s = 4.37$ fm, $a = 0.091$ fm
- **LHPC** ($m_\pi = 149\text{MeV}$) : $L_s = 5.57$ fm, $a = 0.116$ fm
- **RQCD** ($m_\pi = 150/157\text{MeV}$) : $L_s = 4.48/3.36$ fm, $a = 0.07$ fm
- **QCDSF** ($m_\pi = 158\text{MeV}$) : $L_s = 3.41$ fm, $a = 0.071$ fm
- **QCDSF/UKQCD** ($m_\pi = 170\text{MeV}$) : $L_s = 3.36$ fm, $a = 0.07$ fm
- **RBC** ($m_\pi = 170\text{MeV}$) : $L_s = 4.6$ fm, $a = 0.141$ fm

[S. Collins et al. (QCDSF/UKQCD), arXiv:1101.2326]:
'Simulations in the region $L m_\pi > 3$ are expected to have sufficiently small finite size effects'
Finite Volume Effects

128 MeV ≤ m_π ≤ 300 MeV

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No volume corrections

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- **PNDME** (\( m_\pi = 128 \text{MeV} \)) : \( L_s = 5.76 \text{ fm} \), \( a = 0.09 \text{ fm} \)
- **ETMC** (\( m_\pi = 135 \text{MeV} \)) : \( L_s = 4.37 \text{ fm} \), \( a = 0.091 \text{ fm} \)
- **LHPC** (\( m_\pi = 149 \text{MeV} \)) : \( L_s = 5.57 \text{ fm} \), \( a = 0.116 \text{ fm} \)
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- **RBC** (\( m_\pi = 170 \text{MeV} \)) : \( L_s = 4.6 \text{ fm} \), \( a = 0.141 \text{ fm} \)

Volume effects still unclear

[S. Collins et al. (QCDSF/UKQCD), arXiv:1101.2326]:
'Simulations in the region \( L m_\pi > 3 \) are expected to have sufficiently small finite size effects'
Axial Charge: Summary

High statistical analyses to date reveal:

- Cutoff effects small for: $a < 0.1 \text{ fm}$
- No excited states for: $T_{\text{sink}} > 1 \text{ fm}$
- Finite Volume effects: $L m_\pi > 3$
A2. Nucleon EM form factors

\[ \langle N(p', s') | \gamma_\mu | N(p, s) \rangle \sim \bar{u}_N(p', s') \left[ F_1(q^2) \gamma_\mu + F_2(q^2) \frac{i \sigma^{\mu \rho} q_\rho}{2m_N} \right] u_N(p, s) \]

LHPC: \( m_\pi = 149 \text{ MeV}, \ a = 0.116 \text{ fm}, \ \mathcal{O}(7800) \) stat.

- Summation method goes either direction
- Errors are large

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Talk by G. Koutsou

[J.R. Green et al. (LHPC), arXiv:1211.0253]

[AMA Talk by G. Koutsou]
Disconnected Insertion

Sachs FFs: \( G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2), \ G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \)

Quark loops with hierarchical probing

- Gain depends on observable: for EM significant improvement
- Allows to increase the level of spatial dilution at any stage while reusing existing data
- Improves the stochastic estimator \( \text{Tr}[A^{-1}] = E\{z^\dagger A^{-1} z\} \) (\( z \): noise vector)
- Deterministic orthonormal vectors (Hadamard)
- Optimal distance \( k \) for \( A^{-1}_{i,j} \approx 0 \) obtained using probing
- Recursive probing (results from level \( i-1 \) is used at level \( i \))
- Multi coloring of sites is done hierarchically
- Bias is removed by using a random starting vector
- Up to factor of 10 speed up (32^3 \times 64\ clover lattice)
A3. Dirac & Pauli radii

\[ F_i(Q^2) \sim F_i(0) \left( 1 - \frac{1}{6} Q^2 \langle r_1^2 \rangle + O(Q^4) \right) \]

\[ \langle r_1^2 \rangle = -\frac{6}{F_i(Q^2)} \frac{dF_i(Q^2)}{dQ^2} \bigg|_{Q^2=0} \]

\[ F_i(Q^2) \sim \frac{F_i(0)}{\left( 1 + \frac{Q^2}{m_i^2} \right)^2} \]

\[ \Rightarrow \langle r_1^2 \rangle = \frac{12}{m_i^2} \]

Lattice data for plateau method

★ Estimation of radii strongly depends on small \( Q^2 \)
★ Need access for momenta close to zero ⇒
★ larger volumes
Avoid model dependence-fits:

\[ y\text{-summation, } g_{\text{max}} = 4, 300 \text{ confs} \]
\[ y\text{-summation } G_M(k^2 = 0) \]
\[ \text{sequential method, 1200 confs} \]

- Position space method

Poster by K. Ottnad (ETMC)

**Systematic Effects**

- Upward tendency with increase of \( T_{\text{sink}} \)
- Summation agrees with larger \( T_{\text{sink}} \) value
- Chiral extrapolation of summation method agrees with exp

[J.R. Green et al. (LHPC), arXiv:1404.4029]
A4. Quark Momentum Fraction

1-D Vector current: \( O^{\mu\nu} \equiv \bar{\psi} \gamma^{\mu} \overleftrightarrow{D}^{\nu} \psi \Rightarrow A_{20}(q^2), B_{20}(q^2), C_{20}(q^2) \)

\[ \langle x \rangle_q = A_{20}(0) \]

- Measured in DIS experiments. Value uses input from phen. models
- \( \langle x \rangle_{\text{phen}} = 0.1646(27) \) (MS(2 GeV)) [S. Alekhin et al., arXiv:0908.2766]
- Scheme and scale dependence
- All lattice results overestimate phen. value
- Chiral behavior: \( m_{\pi}^2 \log(m_{\pi}^2) \)
Excited States Investigation

**TMF,** $m_\pi = 373 \text{MeV}$
[S.Dinter et al. (ETMC), arXiv:1108.1076]

- $O(23,000)$ measurements
- $0.94 \text{ fm} \langle T_{\text{sink}} \rangle \approx 1.87 \text{ fm}$

**Clover,** $m_\pi = 149 \text{MeV}$
[J.R.Green et al. (LHPC), arXiv:1209.1687]

- $O(7,800)$ measurements
- $T_{\text{sink}} = 0.9, 1.2, 1.4 \text{ fm}$

**ALL WORKS AGREE:**
- Contaminated by excited states
- Convergence by varying $T_{\text{sink}}$
- Downward shift

**Clover,** $m_\pi = 340 \text{MeV}$
[T.Rae et al. (Mainz Group), 2014]

- $O(3,800)$ measurements
- $0.6 \text{ fm} \langle T_{\text{sink}} \rangle \approx 1.4 \text{ fm}$

- $O(2,800)$ measurements
- $0.63 \text{ fm} \langle T_{\text{sink}} \rangle \approx 1.05 \text{ fm}$
Renormalization

RI’ scheme:

\[ Z_q = \frac{1}{12} \left[ \left( S^L(p) \right)^{-1} S^{\text{Born}}(p) \right] \bigg|_{p^2 = \bar{\mu}^2} \]

\[ Z_q^{-1} Z_O \frac{1}{12} \left[ \left( \Gamma^L_O(p) \left( \Gamma^{\text{Born}}_O(p) \right)^{-1} \right) \bigg|_{p^2 = \bar{\mu}^2} = 1 \]

★ Tension between \( Z_O^{\text{pert}} \) and \( Z_O^{\text{non-pert}} \) up to 15%

either direction

Non-perturbative renormalization

★ Conversion to \( \overline{\text{MS}}(\mu = 2\text{GeV}) \)

\[ \overline{Z}_{DVI} \]

ETMC (\( N_f = 2 \), TMF & clover)

- 3-loop (Gracey)
- 3-loop (RGI)

\[ Z_{DVI} \text{ : Z-factor of } \langle x \rangle_{u - d} \]

- Systematic due to conversion insignificant
Lattice Artifacts

A. Subtraction of $O(g^2 \, a^2)$ perturbative terms

[C. Alexandrou et al. (ETMC), arXiv:1006.1920]
[M. Constantinou et al. (ETMC), arXiv:0907.0381]

B. Complete Subtraction of $O(g^2)$ artifacts

[M. Constantinou et al. (QCDSF), 2014]

Control of lattice artifacts (non-Lorentz invariant):

$\frac{\sum_\rho p_\rho^4}{\left(\sum_\rho p_\rho^2\right)^2} < 0.4$

( empirically)

Usage of momentum-source method:

- Dirac equation solved with momentum source
- # of inversion depends on # of momenta considered
- Application of any operator
- High statistical accuracy
A5. Nucleon Spin

Spin Sum Rule:

\[
\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left( L^q + \frac{1}{2} \Delta \Sigma^q \right) + J^G
\]

Extraction from LQCD:

\[
J^q = \frac{1}{2} \left( A_{20}^q + B_{20}^q \right), \quad L^q = J^q - \Sigma^q, \quad \Sigma^q = g_A^q
\]

★ Individual quark contributions ⇒ disconnected insertion contributes

Renormalization of Disconnected Contributions

▷ Requirement of renormalization for the singlet operators
  \( Z_{O}^{\text{singlet}} \) unknown non-perturbatively
  \( Z_{O}^{s} - Z_{O}^{ns} \) first appears to 2 loops in perturbation theory

▷ Recent perturbative results for [H. Panagopoulos et al. (Cyprus Group), 2014]
  Axial: \( Z_{A}^{s} - Z_{A}^{ns} \)  Scalar: \( Z_{S}^{s} - Z_{S}^{ns} \)
  Applicable for various actions: (Wilson, Clover, SLiNC, TM)_F \& (Wilson, t.l. Symanzik, Iwasaki, DBW2)_G

Tree-level Symanzik gluons:

\[
Z_{A}^{s} - Z_{A}^{ns} = g_A^4 C_f \frac{N_f}{16 \pi^2} \left( -2.0982 + 12.851 c_{SW} + 3.3621 c_{SW}^2 - 1.7260 c_{SW}^3 - 0.0164 c_{SW}^4 - 6 \log(a^2 \mu^2) \right)
\]

Talk by H. Panagopoulos
Nucleon Spin
Disconnected Contributions

\[ g_A^s : \langle N(p') | \bar{s} \gamma_\mu \gamma_5 s | N(p) \rangle \bigg|_{q^2=0} \]

Agreement between different discretizations:
- [R.Babich et al. (DISCO), arXiv:1012.0562]
- [G.S.Bali et al. (QCDSF), arXiv:1112.3354]
- [M.Engelhardt, arXiv:1210.0025]
- [A.Abdel-Rehim et al. (ETMC), arXiv:1310.6339]
- [S.Meinel et al. (LHPC), 2014], bare results
- [J.Zanotti et al. (CSSM/QCDSF/UKQCD), 2014]
- [M.Gong et al. (χ QCD), 2014]

\[ \langle x \rangle_{u+d} \]

 Talks by:
- M.Gong (χ QCD)
- A.Vaquero (ETMC)
- J.Zanotti (CSSM/QCDSF/UKQCD)

Di for \( g_A \) lower the total value
**Nucleon Spin**

**Results**

![Graph showing contributions to nucleon spin](image)

**Lattice results:**
- Most results only CI
- TMF: include $Z_A^s - Z_A^n$
- $m_\pi = 135$ MeV: $J^u \sim 0.25$, $J^d \sim 0$
- $L^u + L^d \sim 0$ ($L^u$, $L^d$ cancel out)
- $m_\pi = 135$ MeV: $\Delta \Sigma^u$, $\Delta \Sigma^d$ agrees with exp.

[S.N.Syritsyn et al. (LHPC), arXiv:1111.0718]
[A.Sternbeck et al. (QCDSF), arXiv:1203.6579]
[C.Alexandrou et al. (ETMC), arXiv:1303.5979]
Nucleon Spin

Contributions to nucleon spin

\[ m_\pi^2 (\text{GeV}^2) \]

\[ \frac{1}{2} \Delta \Sigma_{u+d} \]

\[ L_{u+d} \]

\[ \nabla : \text{DI (ETMC)} \]

\[ m_\pi = 135 \text{ MeV}: \text{Agreement with exp} \]

\[ \star \text{DI: lowers the total value} \]
HYPERON
FORM
FACTORS
Hyperon EM form factors

\[ \langle B(p', s') | j_\mu(q) | B(p, s) \rangle = \bar{u}(p', s') \left[ \gamma_\mu F_1(Q^2) + \frac{i\sigma_\mu\nu q^\nu}{2m_B} F_2(Q^2) \right] u(p, s) \]

Sachs FFs: \( G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \)

D1. \( G_E, G_M \) of Hyperons

- 'Connected \( \chi \)PT':
  - valence and sea quarks are treated separately
  - disconnected contractions may be omitted
- extrapolation on each \( Q^2 \) separately
- \( N_F=2+1 \) Clover, \( a = 0.074 \) fm

D2. \( G_M^s \) of \( \Lambda(1405) \)

- contains strange quark, but lighter than other excited spin-1/2 baryons
- superposition of molecular meson-baryon states (\( \pi \Sigma & KN \)?)
- 1\textsuperscript{st} lattice computation of the EM FFs of \( \Lambda(1405) \) (variational approach)
- in \( KN \): s-quark does not contribute in \( G_M \)

\[ [P.E. Shanahan et al. (CSSM & QCDSF/UKQCD), arXiv:1401.5862, 1403.1965] \]

[Approaching the physical point: \( G_M^s \rightarrow 0 \)]

[Talk by D.Leinweber]

[Approaching the physical point: \( G_M^s \rightarrow 0 \)]

[D.Leinweber et al. (CSSM), 2014]

[B.J.Menadue et al., arXiv:1109.6716]
Axial charges of hyperons

Axial matrix element:

\[
\left\langle B(p') | \bar{\psi}(x) \gamma_\mu \gamma_5 \psi(x) | B(p) \right\rangle \bigg|_{q^2 = 0}
\]

- **Connected part**

- First promising results at the physical point
  - **SU(3) breaking** \( \delta_{SU(3)} = g_A^N - g_A^\Sigma + g_A^{\Xi} \) versus \( x = (m_K^2 - m_\pi^2) / (4\pi^2 f_\pi^2) \)

Talk by C. Alexandrou
E1. Pion Quark distribution function

[C.Urbach et al. (ETMC), 2014]: $N_f=2$, $2+1+1$ TMF, $N_f=2$ TMF & clover

Lowest moment with $H(4)$-operator:

$$\mathcal{O}_{44}(x) = \frac{1}{2} \bar{u}(x) \{ \gamma_4 D \leftrightarrow 4 - \frac{1}{3} \sum_{k=1}^{3} \gamma_k D \leftrightarrow k \} u(x)$$

$$\langle x \rangle^{\text{bare}}_{\pi^+} = \frac{1}{2 m_{\pi}^2} \langle \pi, \bar{0} | \mathcal{O}_{44} | \pi, \bar{0} \rangle$$

- No external momentum is needed in the calculation
- Stochastic time slice sources:
  - less inversions
  - statistical accuracy
- disconnected contributions not included

phenomenology: $\langle x \rangle^{\text{bare}}_{\pi^+} = 0.0217(11)$

[K. Wijesooriya et al., nucl-ex/0509012]

[R. Baron et al. (ETMC), arXiv:0710.1580]

[D. Brommel (QCDSF/UKQCD) Pos(LATTICE) 2007, 140]

[G. Bali et al. (RQCD), arXiv:1311.7639]

[C. Urbach et al. (ETMC), 2014]
E2. $\rho$-meson EM form factors

[B.J.Owen et al. (CSSM), 2014] $N_f = 2+1$ Clover

$$\langle \rho(p', s') | j_\mu | \rho(p, s) \rangle: G_C(q^2), G_M(q^2), G_Q(q^2)$$

Variational approach

- automatic method for suppressing excited state effects
- separation of the correlators for individual energy eigenstates
  ⇒ rapid ground state dominance
  ⇒ access to excited states

- Set of operators: various source and sink smearings
  $\chi^i_\rho(x) = \bar{d}(x) \gamma^i u(x)$
- 4 levels of smearing ⇒ $4 \times 4$ correlation matrix
- substantial improvement for $G_M$ and $G_Q$

**Blue** points: variational method (VM)
**Red** points: standard method (SM)

- $G_M, G_Q$ (VM): plateau right after the current insertion
- $G_M$ (SM): plateau at later timeslices
- $G_Q$ (SM): No plateau identification
- $G_C$: plateau of VM earlier than in the SM
CONCLUSIONS

Breakthrought: Simulating the physical world!

- **Dedication of human force and computational resources on:**
  - Control of statistical uncertainties $\Rightarrow$ noise reduction techniques crucial
  - comprehensive study of systematic uncertainties
  - removal of excited states where necessary
  - cross-checks between methods
  - Simulations at different lattice spacings and volumes
  - study of DI at lower masses (Target: physical $m_\pi$!)
    - challenging task
    - exploid techniques: AMA, hierarchical probing, others
    - usage of GPUs
    - current computations of DI provide bounds

- **Nucleon spin: include dynamical simulations for gluon angular momentum**
  - Difficulties with renormalization and mixing
  - rely on perturbation theory

- **Exciting results emerging from other particles**
THANK YOU
BACKUP SLIDES
Plateau Method: single-state fit


- $m_\pi \geq 149\text{MeV}$
- light $m_\pi$: $g_A \swarrow$ with $T_s \nearrow$
- $L_t/a \geq 48$: $g_A \nearrow$ with $T_s \nearrow$
- Indication of thermal pion states
Finite Volume Effects

![Graph showing finite volume effects](image)

- **Black diamond**: summation (LHPC)
- **Black triangles**: volume corrected (QCDSF)
B2. Nucleon Axial form factors

*TMF, $N_f = 2$, $N_f = 2 + 1 + 1$ and TMF & clover, $N_f = 2*

\begin{align*}
G_A(Q^2) &= \frac{g_A}{\left(1 + Q^2/m_A^2\right)^2} \\
G_P(Q^2) &= \frac{G_A(Q^2) G_P(0)}{(Q^2 + m_P^2)}
\end{align*}

$G_P$ strongly dependent on the lowest values of $Q^2$

$G_A(Q^2)$ and $G_P(Q^2)$ vs. $Q^2$ (GeV$^2$)

★ Dipole fits:

\begin{align*}
G_A(Q^2) &= \frac{g_A}{\left(1 + Q^2/m_A^2\right)^2} \\
G_P(Q^2) &= \frac{G_A(Q^2) G_P(0)}{(Q^2 + m_P^2)}
\end{align*}

$m_A^{\exp} = 1.069\text{GeV}^\dagger$

1.2GeV $\langle m_A^{\text{lattice}} \rangle = 1.45\text{GeV}^*$

0.3GeV $\langle m_P^{\text{lattice}} \rangle = 0.5\text{GeV}^*$

$\dagger$ [V. Bernard et al., hep-ph/0607200]

★ TMF, $m_\pi = 135\text{MeV}$ (ETMC 2014)
B2. Nucleon Axial form factors

\[ T_{\text{MF}}, N_f = 2, N_f = 2 + 1 + 1 \text{ and TMF & clover}, N_f = 2 \]

\[
\begin{align*}
G_A(Q^2) &= \frac{g_A}{\left(1 + \frac{Q^2}{m^2_A}\right)^2} \\
G_p(Q^2) &= \frac{G_A(Q^2) G_p(0)}{(Q^2 + m_p^2)}
\end{align*}
\]

\[ m_{\pi} = 135 \text{MeV} \]

\[ m_{\text{exp}} = 1.069 \text{GeV} \]

\[ 1.2 \text{GeV} \langle m^\text{lattice}_A \rangle \langle 1.45 \text{GeV} \]

\[ 0.3 \text{GeV} \langle m^\text{lattice}_p \rangle \langle 0.5 \text{GeV} \]

\[ \star \text{ Dipole fits:} \]

\[ Q^2 \text{ strongly dependent on the lowest values of } Q^2 \]

\[ \star \text{ [V. Bernard et al., hep-ph/0607200]} \]

\[ \star \text{ TMF, } m_\pi = 135 \text{MeV (ETMC 2014)} \]
Generalized pencil-of-function

- Better extraction of states contributing to a correlator
- Variational method using 3pt-functions with 3 equally spaced sink locations

$$C^3_{-pt}(t_i, t, t_f) = \begin{pmatrix} C^3_{-pt}(t_i, t, t_f) & C^3_{-pt}(t_i, t, t_f + \tau) \\ C^3_{-pt}(t_i, t + \tau, t_f + \tau) & C^3_{-pt}(t_i, t + \tau, t_f + 2\tau) \end{pmatrix}$$

- Computational cost $\times 3$, but better ground signal
B

NUCLEON

CHARGES
B1. Scalar Charge

\[ g_s \equiv \langle N | \bar{u}u - \bar{d}d | N \rangle \]

- \( g_s, g_T \) provide constrains for scalar interactions at the TeV scale

**LHPC:** \( m_\pi = 149 - 356 \text{MeV} \)

[J.R.Green et al. (LHPC), arXiv:1206.4527]

**PNDME:** \( m_\pi = 310 \text{MeV} \)

[T. Bhattacharya (PNDME), arXiv:1306.5435]

- Severe contamination of excited states
- Confidence in results requires:
  Dedicated study with high statistics of plateau, 2-state fit, summation method
**TMF**: $N_f=2+1+1$, $m_{\pi}=373\text{MeV}$

[A. Abdel-Rehim et al. (ETMC), arXiv:1310.6339]

- $0.98\text{ fm} \langle T_{\text{sink}} \rangle < 1.48\text{ fm}$
- $T_{\text{sink}} \leq 1.31\text{ fm}$: agreement with SM

- Stat. errors must come down

**TMF & $c_{SW}$**: $N_f=2$, $m_{\pi}=135\text{MeV}$

[C. Alexandrou et al. (ETMC), 2014]

- $0.98\text{ fm} \langle T_{\text{sink}} \rangle < 1.48\text{ fm}$
- $T_{\text{sink}} \leq 1.3\text{ fm}$: agreement with SM

(similar to $m_{\pi}=373\text{MeV}$)

- Stat. errors must come down
B2. Tensor Charge

\[ \langle N(p', s') | \sigma^{\mu \nu} | N(p, s) \rangle \Rightarrow A_{T10}(q^2), B_{T10}(q^2), C_{T10}(q^2) \]

\[ \delta q = A_{T10}(0) \]

At scale \( \mu^2 = 110 \text{ GeV}^2 \):

- \( A^\text{exp}_{T10}(0.8 \text{ GeV}^2) = 0.64^{+0.35}_{-0.16} \) (†)
- \( A^\text{exp}_{T10}(0.8 \text{ GeV}^2) = 0.77^{+0.13}_{-0.27} \) (∗)

Evolution to 4 GeV^2 (NNLO):

- \( A^\text{exp}_{T10}(0.8 \text{ GeV}^2) = 0.72^{+0.39}_{-0.18} \) (†)
- \( A^\text{exp}_{T10}(0.8 \text{ GeV}^2) = 0.87^{+0.15}_{-0.30} \) (∗)

★ probes the transverse spin structure of the nucleon
★ Agreement among most lattice points
★ Mild \( m_\pi \) dependence

**PNDME** \( m_\pi = 310 \text{ MeV} \) ⇒