Fermion loops in nuclear phenomenology & Λ(1405)

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Overview

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    - SU(3) flavour-symmetry constraints
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    - Lattice QCD
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- The light-quark sector of the $\Lambda(1405)$
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Aims

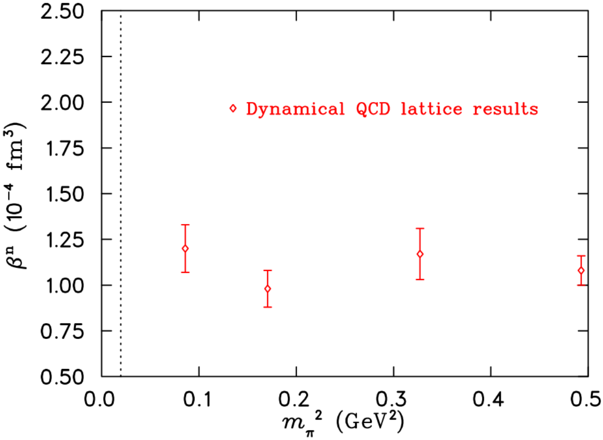
- To understand the role of quark loops in contributions to nuclear observables.
- To correct lattice QCD results by accounting for quenching effects.
- To unveil the nature of the $\Lambda(1405)$ with evidence from the strange quark sector.
- To interpret the insight gained from separate quark-sectors phenomenologically.

Fermion loops in observables

Case study: the neutron polarizability

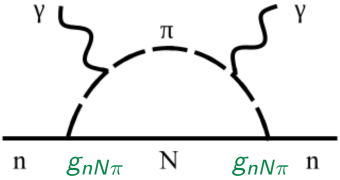
- State-of-the-art lattice QCD results for the magnetic polarizability of the neutron (β_n) have been produced by CSSM (T. Primer, *et. al.*, PRD, **arXiv:1307.1509**).
- To be able to compare with experiment, one needs to account for finite-volume corrections, extrapolating to the physical pion mass, and missing disconnected loops (PRD, **arXiv:1312.5781**).
- We focus on the identification and unquenching of the missing loops from lattice QCD.
- This will also give us insight into the role of quark-loops in observables, along the way.

Lattice QCD



2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
β^n (10^{-4} fm^3)
◇ Dynamical QCD lattice results
0.0
0.1
0.2
0.3
0.4
0.5
$m_\pi{}^2$ (GeV^2)

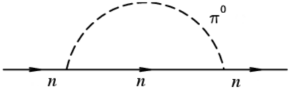
Example: 1-loop integral contribution



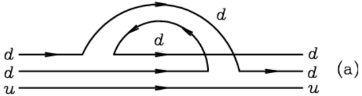
- This leading 1-loop integral corresponds to the process $n \rightarrow p\pi^-$, with a coupling of $g^2_{np\pi^-}$; the only non-zero process in QCD.
- If photon couplings to sea-quark loops are missing, there will be non-zero contributions from $n \rightarrow n\pi^0$ and $n \rightarrow n^-\pi^+$!

Example: 1-loop integral contribution

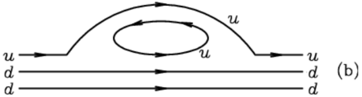
- Each meson loop can be written as individual quark-flow diagrams:



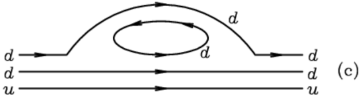
- Completely connected $\longrightarrow$



- Disconnected u quark $\longrightarrow$

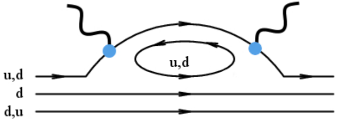


- Disconnected d quark $\longrightarrow$

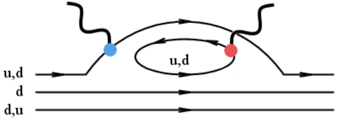


Case study: the neutron polarizability

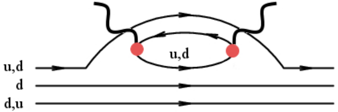
- Valence-valence: present in the lattice calculation:



- Valence-sea: not present in the lattice calculation:



- Sea-sea: not present in the lattice calculation:



Missing loops modifying the coupling

- The coupling for each process, $g^2_{nN\pi}$, is modified due to the missing loops:

$$g^2_{connected} = g^2_{nN\pi} - g^2_{VS} - g^2_{SS}.$$

- To solve this, g^2_{VS} and g^2_{SS} can be obtained through SU(3) symmetry.
- We treat the internal sea-quark as a ‘new’ flavour (e.g. a strange quark) and calculate these ‘kaon’ couplings.
- quark charges *remain the same* as a usual u-(or d) quark, as it is only the disconnected portion of the full QCD coupling.

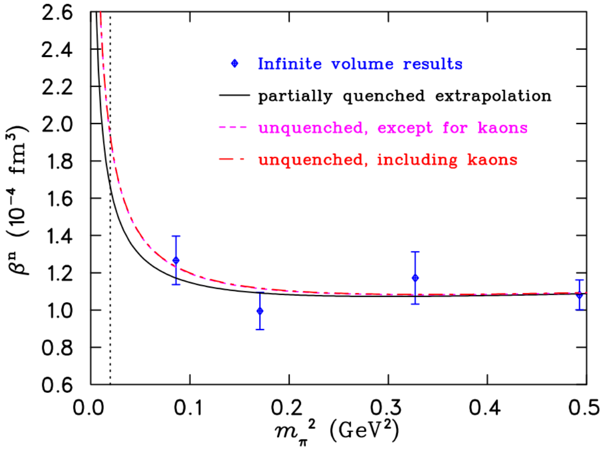
Missing loops modifying the coupling

$n \to N\pi$	Total	Valence-sea	Sea-sea
$n \to n\pi^0$	0	$2q_u q_{\bar{u}}\,\chi^2_{K^+\Sigma^-} +$ $2q_d q_{\bar{d}}\,(\chi^2_{K^0\Sigma^0} + \chi^2_{K^0\Lambda})$	$q^2_{\bar{u}}\,\chi^2_{K^+\Sigma^-} +$ $q^2_{\bar{d}}\,(\chi^2_{K^0\Sigma^0} + \chi^2_{K^0\Lambda})$
$n \to p\pi^-$	$2(D+F)^2$	$2q_d q_{\bar{u}}\,(\chi^2_{K^0\Sigma^0} + \chi^2_{K^0\Lambda})$	$q^2_{\bar{u}}\,(\chi^2_{K^0\Sigma^0} + \chi^2_{K^0\Lambda})$
$n \to n^-\pi^+$	0	$2q_u q_{\bar{d}}\,\chi^2_{K^+\Sigma^-}$	$q^2_{\bar{d}}\,\chi^2_{K^+\Sigma^-}$

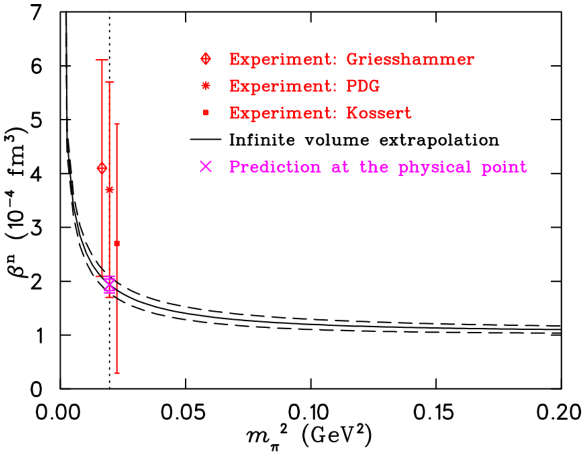
- $\chi^2_{K^+\Sigma^-} = 2(D-F)^2$, $\chi^2_{K^0\Sigma^0} = (D-F)^2$, $\chi^2_{K^0\Lambda} = (D+3F)^2/3$.
- The total change in coupling is: $\boxed{g^2_{connected} = 2g_A^2 - (D-F)^2 - \frac{7}{27}(D+3F)^2}$.

Unquenching the neutron polarizability

PRD, **arXiv:1312.5781**



Comparing with experiments

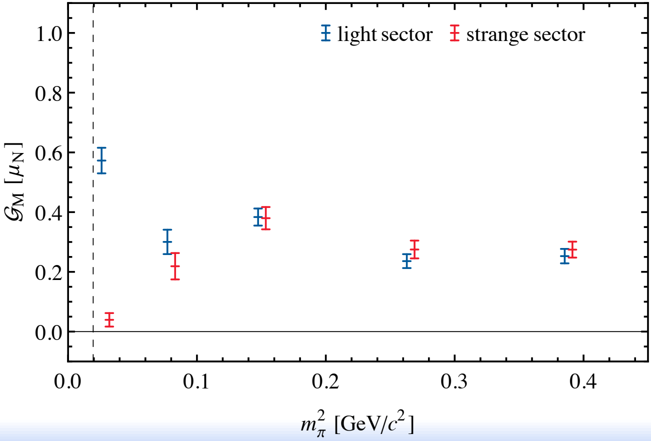


The extrapolated value is $\beta_n = 1.93(11)^{\mathrm{stat}}(11)^{\mathrm{sys}} \times 10^{-4}$ fm^3.

The anti-kaon–nucleon nature of the $\Lambda(1405)$

Lattice QCD

- The strange magnetic form factor of the Λ(1405) vanishes near the physical pion mass.



The finite-volume Hamiltonian model

- The matrix Hamiltonian model was developed for analysing scattering states on the lattice, as an alternative to Lüscher's method:
- the free parameters of the model (related to the low-energy coefficients of the chiral expansion) can be fit directly to lattice results $\longrightarrow$ more stable estimation of renormalised masses, decay widths, etc. by resumming the chiral expansion (FRR).
- But also: the matrix Hamiltonian model is more straightforwardly generalisable to multi-channel interactions,
- and has the ability to provide information on the output spectrum of states at any quark-mass/volume.

The finite-volume Hamiltonian model

- The finite-volume Hamiltonian includes the interactions between a bare Λ(1405) mass, $m_{\rm bare} = m_0 + \alpha_0 m_\pi^2$, and the leading octet states: $\pi\Sigma$, $\overline{K}N$, $K\Xi$, $\eta\Lambda$.

$$H = \begin{pmatrix} m_{\rm bare} & g_{\pi\Sigma}(k_0) & g_{\overline{K}N}(k_0) & g_{K\Xi}(k_0) & g_{\eta\Lambda}(k_0) & \cdots \\ g_{\pi\Sigma}(k_0) & \omega_{\pi\Sigma}(k_0) & 0 & \cdots & & \\ g_{\overline{K}N}(k_0) & 0 & \omega_{\overline{K}N}(k_0) & 0 & \cdots & \\ g_{K\Xi}(k_0) & 0 & 0 & \omega_{K\Xi}(k_0) & & \\ g_{\eta\Lambda}(k_0) & 0 & 0 & 0 & \omega_{\eta\Lambda}(k_0) & \\ \vdots & \vdots & \vdots & \vdots & & \ddots \end{pmatrix}$$

- The non-interacting energies are: $\omega_{MB}(k_n) = \sum_{i=M}^{B} \sqrt{k_n^2 + m_i^2}$.
- The momentum-squared values available in a finite volume are: $k_n^2 = \left(\frac{2\pi}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2) \equiv \left(\frac{2\pi}{L}\right)^2 n$.

The finite-volume Hamiltonian model

- The solution of the eigenvalue equation, $\det[H - \lambda \mathbb{I}] = 0$, takes the form:

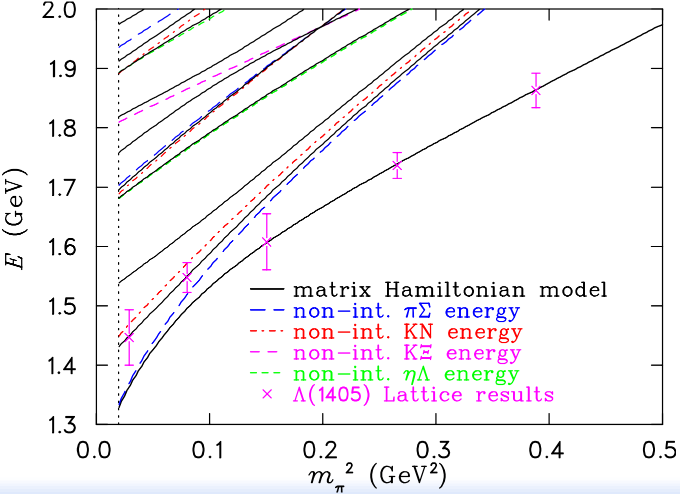
$$\lambda = m_0 + \alpha_0 \, m_\pi^2 - \sum_{M,B} \sum_{n=0}^{\infty} \frac{g_{MB}^2(k_n)}{\omega_{MB}(k_n) - \lambda} \,,$$

$$\text{with} \quad g_{MB}^2(k_n) = \frac{\kappa_{MB}}{16\pi^2 f_\pi^2} \frac{C_3(n)}{4\pi} \left(\frac{2\pi}{L}\right)^3 \omega_M(k_n) \, u^2(k_n).$$

- This is the finite-volume version of the S-wave loop integral: $\Sigma_{MB}^{\rm loop} = \frac{\kappa_{MB}}{16\pi^2 f_\pi^2} \mathcal{P} \int_0^\infty \mathrm{d}k \, \frac{k^2 \, \omega_M(k) \, u^2(k)}{E_{\Lambda 1405} - \omega_{MB}(k)}$, from effective field theory.
- $\kappa_{\pi\Sigma}$ is set so that the renormalised coupling reproduces the width $\Gamma_{\Lambda(1405)} = 50 \pm 2$ MeV.

Hamiltonian model: fit to lattice results

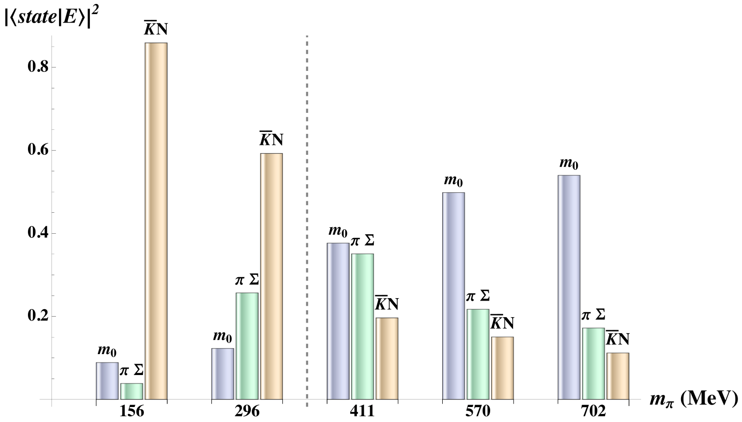
PRL, **arXiv:1411.3402**



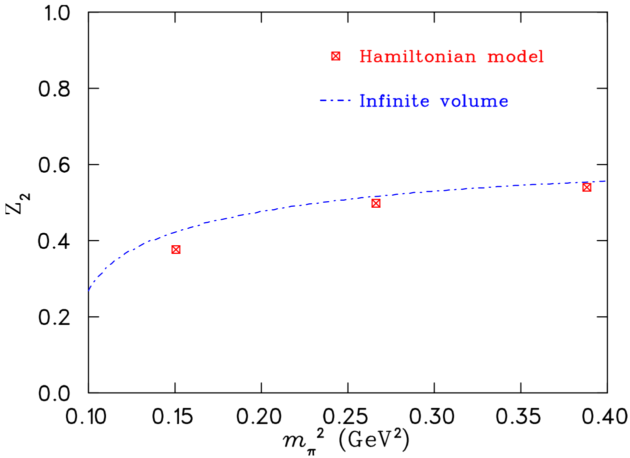
$\overline{K}N$ dominance near the physical point

PRL, **arXiv:1411.3402**

- Eigenvector overlap: content of the Λ(1405) from the model.



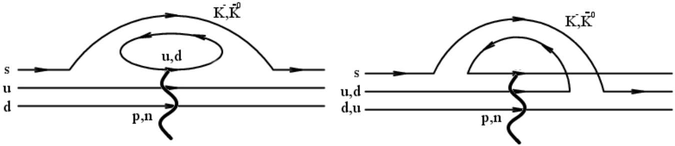
$\overline{K}N$ dominance near the physical point



The light-quark sector of the $\Lambda(1405)$

Loop contributions to magnetic moment

- If the Λ(1405) is dominated by $\overline{K}N$ loops near the physical pion mass, we expect its magnetic moment, μ, to be strongly related to that of the nucleon.
- We would like to do a lattice QCD calculation to compare with experiment, but we need to account for disconnected loops.

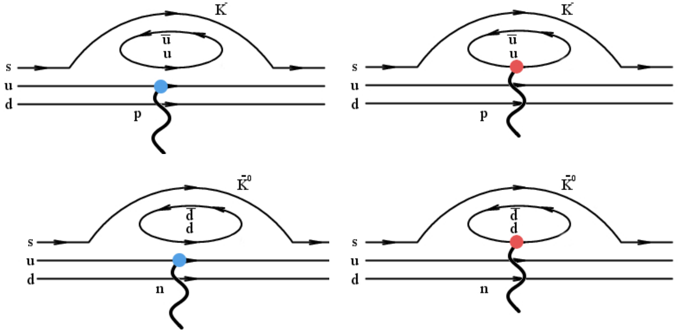


Λ*

Loop contributions to magnetic moment

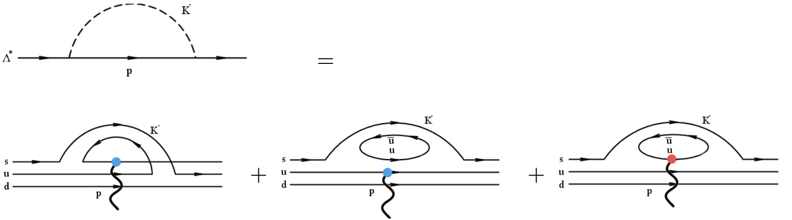
- For the disconnected loop contributions in both $\Lambda^* \to K^- p$ and $\Lambda^* \to \overline{K}^0 n$, valence couplings are present, but sea couplings are not.

present in lattice calc. not present in lattice calc.



Loop contributions to magnetic moment

- Therefore, to get the partially quenched coupling for the process $\Lambda^* \to K^- p$ for example:



- We need to cancel off the last graph, which represents half the disconnected u-sector. But how much of the coupling is in this graph?

The graded symmetry approach

- One can formalise adding in ‘new’ flavours to get the (partially)-quenched couplings by using the graded symmetry approach (Labrenz & Sharpe: arXiv:9605034).
- Fictitious ghost-quark loops are added, which cancel off the disconnected loops, to get the quenched couplings.
- The baryon matrix from chiral perturbation theory is promoted to a three-index tensor, $\mathcal{B}_{ijk}$, each index corresponding to one of six quarks $(u, d, s, \tilde{u}, \tilde{d}, \tilde{s})$:

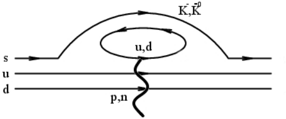
$$\mathcal{B}_{ijk} = \frac{1}{\sqrt{6}} \left(\epsilon_{ijk'} B_k^{k'} + \epsilon_{ikk'} B_j^{k'} \right), \quad (i, j, k = 1\text{–}6). \tag{1}$$

The graded symmetry approach

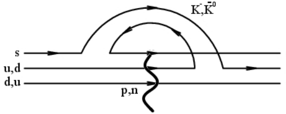
- The symmetrization in $\mathcal{B}_{ijk}$ removes the singlet term, which has an independent coupling from the octet's D and F.
- If we want to use the graded symmetry approach for a *singlet* Λ^*, we need to add the singlet coupling back in.
- We create an anti-symmetrized field $\mathcal{B}^S_{ijk}$ to couples to the octet field $\mathcal{B}_{ijk}$, and we isolate the singlet piece of this field.
- We can calculate the modification to the singlet coupling, g_s (corresponding to the process $\Lambda^* \to K^- p$).

The disconnected loop contribution

- The graded symmetry approach yields the following processes: $\Lambda^* \to \tilde{K}^- \tilde{\Lambda}^+_{p,\tilde{u}}$ and $\Lambda^* \to \overline{\tilde{K}}^0 \tilde{\Lambda}^0_{n,\tilde{d}}$ corresponding to the disconnected loop contributions, each with a strength of $\frac{2}{3} g_s^2$.



- $\sim \frac{2}{3} g_s^2$



- $\sim \frac{1}{3} g_s^2$

Comparing with lattice results

- Therefore, the light-quark sector magnetic moments of the Λ(1405) on the lattice are expected to be:

$$\langle \Lambda^* | \mu_u | \Lambda^* \rangle^{\mathrm{conn}} = \frac{1}{2}\left(2u_p + u_n - \frac{2}{3}u_p\right)$$

$$\langle \Lambda^* | \mu_d | \Lambda^* \rangle^{\mathrm{conn}} = \frac{1}{2}\left(2d_n + d_p - \frac{2}{3}d_n\right)$$

- Checking, we find that: $\frac{1}{2}(2u_p + u_n) = 1.03(2)\,\mu_N$,
- $\frac{1}{2}(2u_p + u_n - \frac{2}{3}u_p) = 0.63(2)\,\mu_N$,
- Whereas the Λ(1405) magnetic moment on the lattice is: $\mu_{\Lambda*}^{conn} = 0.58(5)\,\mu_N$. Success!

Conclusion

Insights from the role of fermion loops

- We investigated how missing fermion loops can alter chiral dynamics in surprising and often numerically significant ways.
- We examined how unquenched loop contributions are an important part of lattice QCD-based predictions.
- We uncovered the role of strange and light quark loops in determining the structure of the $\Lambda(1405)$.
- Further evidence was found to suggest that the $\Lambda(1405) \to \overline{K}N$ process plays a dominant role at the physical point.

Acknowledgments

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